CHAPTER 1
INTRODUCTION

A switched system is a particular kind of hybrid systems. It consists of two or more subsystems and a switching law which determines the active subsystem at each time instant. Each switched system is a subject of interest in many fields of research in recent years. Many switched systems have been successfully applied in real-world processes such as in chemical processes, electrical circuits, computer-controller systems, and intelligent-control systems, etc.

Most of the works on switched systems are based on Lyapounov or multiple Lyapounov functions which can be found in the form of linear matrix inequality (LMI). Examples can be found in [2], [3], [4]. A switched system may be stable even if its subsystems are unstable or a switched system may be unstable even if all of its subsystems are stable as the following example [13].

Consider a switched system

$$\dot{x} = A_{\sigma(t)}x(t)$$
$$\sigma(t) : [0, \infty) \rightarrow \{1, 2\}, \qquad (1.1)$$

when $x \in \mathbb{R}^2$, $A \in \mathbb{R}^{2\times 2}$ and $\sigma(t)$ is a switching law determining the active subsystem at instant time with

$$A_1 = \begin{pmatrix} -0.1 & 1 \\ -10 & -0.1 \end{pmatrix} \text{ and } A_2 = \begin{pmatrix} -0.1 & 10 \\ -1 & -0.1 \end{pmatrix},$$

and a switching law $\sigma(t)$ defined as follow:

(i) use subsystem A_1 when $x_1x_2 \leq 0$
(ii) use subsystem A_2 when $x_1x_2 > 0$

We can easily see that each subsystems is stable since $\lambda_1 = -0.1 \pm 3.1623j, \lambda_2 = -0.1 \pm 3.1623j$ are the eigenvalues of A_1 and A_2, respectively. Hence, according to the figure 1.1, show that even if every subsystem is stable, switched system is unstable.

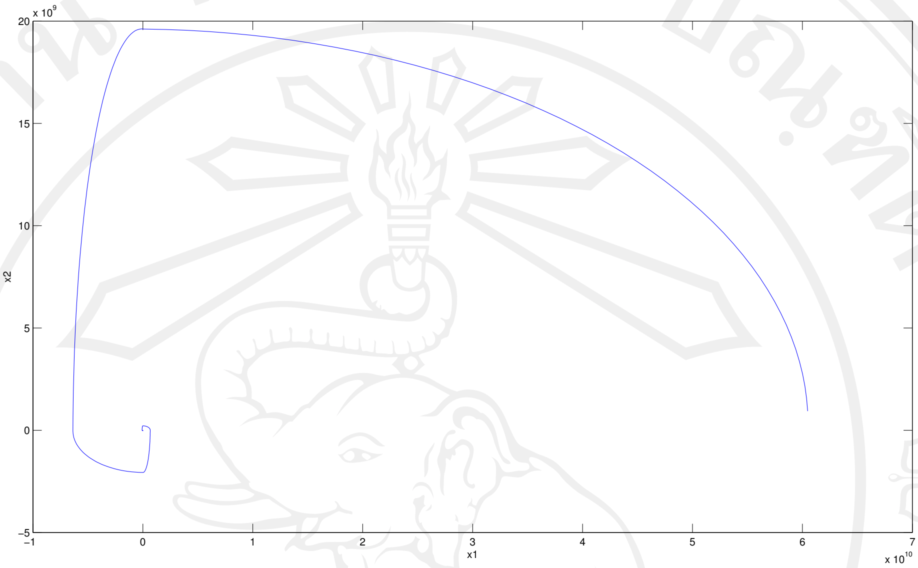

Figure 1.1: The phase plane of the solution of switched system given by a switching law

According to [10], the second-order autonomous system is in the form

$$\dot{x}_1 = f_1(x_1, x_2), \tag{1.2}$$

$$\dot{x}_2 = f_2(x_1, x_2), \tag{1.3}$$

with $x_0 = (x_{10}, x_{20})$; that is, $x(0) = x_0$. The locus in the x_1x_2 plane of the solution $x(t)$ for all $t \geq 0$ is a curve that passes through the point x_0. This curve is called a trajectories or an orbit of (1.2)-(1.3) starting from x_0. The plane is often called the state plane or phase plane. The right-hand side of (1.2)-(1.3) expresses the tangent vector $x(t) = (x_1(t), x_2(t))$ to the curve. By using the vector notation $\dot{x} = f(x)$ where $f(x)$ is the vector $(f_1(x), f_2(x))$, we consider $f(x)$ as vector field on the state plane. This means that each point x in the plane we assign a vector $f(x)$.

Consider a linear time invariant (LTI) system

$$\dot{x} = Ax, x(0) = x_0. \tag{1.4}$$

The solution is well known to be

$$x(t) = e^{At}x_0, \ t \geq 0 \tag{1.5}$$

when $x = (x_1, x_2)^T, A \in \mathbb{R}^{2\times 2}$ and the trajectories of the solution can be shown on the $\mathbb{R}^2$ plane.

X. Xu and P.J. Antsaklis [5] proposed a method that selects an active subsystem to minimize the distance from the current state to the origin. The authors proposed a switching law based on the angle of subsystem vector field and the geometric properties of $\mathbb{R}^2$. A subsequent study [6] used the result of the region separation to stabilize switched systems by a static output feedback.

In 2008, K.R. Santalli, A. Megretski and M.A. Dahleh [11], studied the stabilizability of two dimensional linear systems using switched output feedback. The authors studied a problem of second-order single-input single-output feedback (SISO) linear time-invariant (LTI) system of the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx. \end{aligned} \tag{1.6}$$

With a feedback control of the form $u(x) = v(x)Cx$, where $v(x)$ is a real-value function with $x \in \mathbb{R}^2$.

L. Zhang, Y. Chen and P. Cui [7] studied a problem on asymptotic stabilization of second-order linear time-invariant (LTI) autonomous switched system that consists of two subsystems with unstable foci. The authors derived the necessary and sufficient condition for the origin to be asymptotically stabilizable, and found a stabilizing switching law without using Lyapunov or multiple Lyapunov functions. The authors studied the locus in which two subsystems vector fields are parallel. From their work, a switched system can be stabilized with a certain condition.

In this work, we study the problem of a switched system that consists of two subsystems with unstable foci that cannot be stabilized the system. Hence we would like to find the third stable subsystem with has complex eigenvalues and a new switching law that guarantees the overall system asymptotically stabilizable.

The rest of the thesis is organized as follow. In chapter 2, we present some notations, lemma, definitions and theorems that used in this thesis. In chapter 3, we derive a switching law that suffices to make the overall system asymptotically stablilizable and a numerical example is given in the chapter 4. Finally, the summary is given in chapter 5.