CHAPTER 2

Methodology of the Study

Power distribution load flow study is an important part of distribution systems analysis and it is used in operational as well as planning stages. Many real time applications in the distribution automation system such as networks optimization, reactive power planning, switching, state estimation and so forth, need the support of a robust and efficient power flow method. Such a power flow solution method must be able to model the special features of distribution system. This chapter will present two sections relating to distribution load flow as followings:

- Modeling of system elements.
- Distribution load flow technique.

2.1 Modeling system elements

2.1.1 Distribution network models

For purpose of calculating the Technical Loss, the Distribution System shall be represented by Distribution Network Models that are appropriate for Three Phase Load Flow simulations. All equipment, devices and conductors of the Distribution System shall be characterized to capture the unbalances due to equipment construction, installation configurations, and connection and due to the unbalanced loading. In addition, the models must capture the Load-Losses and No-Load (or Fixed) Losses of all Distribution System equipment, devices and conductors except for metering burdens which are estimated separately.

The Distribution System shall be modeled by an interconnected network represented by series and shunt impedances and/or admittance-parameter network in which a common node is used as a reference as illustrated in Figure 2.1. Self and mutual impedance and/or admittances of each Distribution System element (e.g., line, transformer, etc.) shall be included.

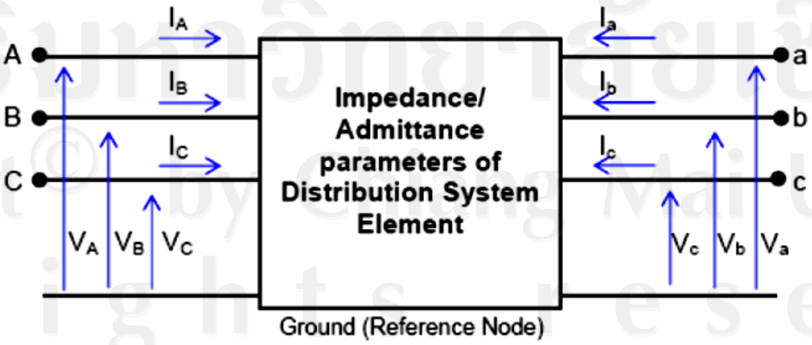


Figure 2.1 Distribution Network Model

2.1.2 Line modeling

Distribution feeder consists of three-phase overhead or underground cable. The feeder can be represented by a single line π or T representation for balanced load. Figure 2.2 shows the single-phase π representation of a line section.

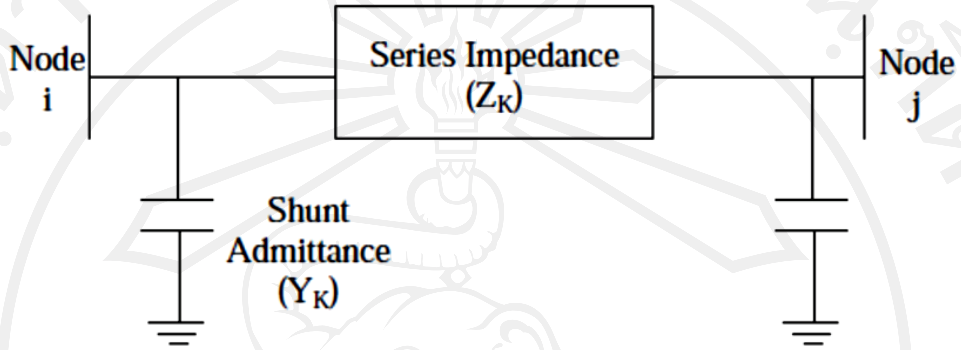


Figure 2.2 Model of a line section for single-phase (π) representation

2.1.3 Line flows and loss

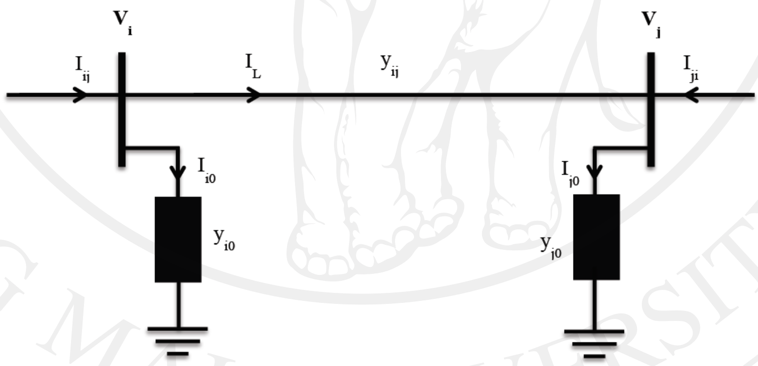


Figure 2.3 Transmission line model for calculating line flows and loss

- Bus i current:

$$I_{ij} = I_L + I_{io} = y_{ij}(V_i - V_j) + y_{io}V_i \tag{2.1}$$

- Bus j current:

$$I_{ji} = -I_L + I_{jo} = y_{ij}(V_j - V_i) + y_{jo}V_j \tag{2.2}$$

Where,

y_{ij} = charging admittance between I –th and the j –th node
y_{io} = charging admittance at I –th node
V_i = voltage at I –th bus
I_{ij} = current direction flow from I –th bus to j –th bus
I_{ji} = current direction flow from j –th bus to I –th bus

- Line flows from the complex power:

$$S_{ij} = V_i . I_{ij}^* \;\&\; S_{ji} = V_j . I_{ji}^* \tag{2.3}$$

The loss in line i – j is the algebraic sum of power flows determined from:

$$S_{L,ij} = S_{ij} + S_{ji} \tag{2.4}$$

2.2 Distribution load flow technique

Distribution system losses shall be classified into 2 categories as follow:

1) Technical losses.
2) Non-Technical losses.

The technical loss is composed of distribution system losses that are inherent in the physical delivery of electricity. The distribution system losses will be presented as following figure 2.4 below.

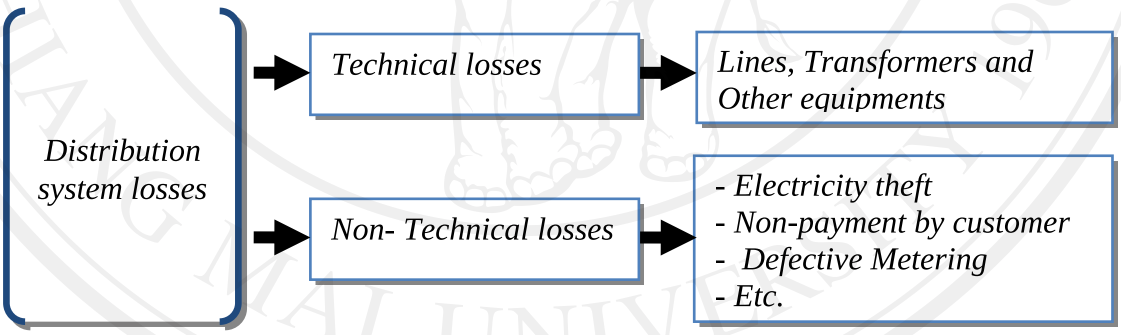


Figure 2.4 Loss classification.

2.3 Technical line losses

2.3.1 Line loss calculation

Line losses in the distribution system occur in the primary as well as the secondary feeders. They are functions of the square of permissible current through the conductors of resistance R, i.e. line loss is directly proportional to the size of the conductor and square of the current passing through it.

Line losses, $L_{loss} = I^2 \times R_T$ Watts (2.5)

Where,

I = Phase current flowing in the line in Ampere (A)

R_T = Total resistance of the line in Ohm (Ω)

Θ = angle between current and voltage

$$I = I\,cos\theta + jI\,sin\theta \text{ or } I = \sqrt{(I\,cos\theta)^2 + (I\,sin\theta)^2}$$

So, line losses,

$$L_{loss} = R_T[(Icos\theta)^2 + (I\,sin\theta)^2] \quad (2.6)$$

Average loss in the line is then calculated as follows:

$$L_{Average} = L_{peak} \times L_sF \quad (2.7)$$

Where,

L_{Peak} = Loss at peak load

L_sF = Loss factor

$L_sF = 0.2\,LF + 0.8(LF)^2$

Where,

LF is the annual load factor of the line, which is calculated from:

$$Annual\ load\ factor\ (LF) = \frac{Total\ annual\ energy}{Annual\ peak\ load \times 8760} \quad (2.8)$$

Loss reduction can be achieved by the following methods:

(a) Load balancing
(b) Capacitor bank and power factor (PF) correction
(c) Switching optimization or Reconfiguration

The distribution transformer loss may be determined based on the number of transformers in the line as indicated below.

♦ The losses can be considered as a concentrated load at the receiving end of the line when one transformer is measured.

$$L_{Annual} = L_{peak} \times L_sF \times 8760 \quad (2.9)$$

$$= I_m^2 \times R_T \times (0.2LF + 0.8(LF)^2) \times 8760\ \ (Single\ phase) \quad (2.10)$$

$$= 3I_m^2 \times R_T \times (0.2\,LF + 0.8(LF)^2) \times 8760\ (Three\ phase) \qquad (2.11)$$

The total percentages of losses per annum are calculated as follows.

$$Loss\ \% = \frac{3\times I_m^2 \times R}{E}\,(0.2LF + 0.8(LF)^2) \qquad (2.12)$$

Where,

I_m = Maximum current recorded at the substation (A)

E = Unit energy sent out from the substation in the particular year (kWH)

R_T = Total resistance of the line (Ω)

♦ With a relatively longer line serving a few transformers substantially widely spaced, the value of load in that branch may be considered a distributed load:

$$L_{Annual} = \sum_{i=1}^{n} I_m^2 \times R_i \times (0.2LF + 0.8(LF)^2) \times 8760\ \ (Single\ phase) \qquad (2.13)$$

$$L_{Annual} = 3\sum_{i=1}^{n} I_m^2 \times R_i \times (0.2LF + 0.8(LF)^2) \times 8760\ (Three\ phase) \qquad (2.14)$$

The total percentage of losses per annum is then:

$$Loss\ \% = \frac{3\sum_{i=1}^{n}(I_i^2 \times R_i) \times (0.2LF + 0.8(LF)^2) \times 8760 \times 100}{E} \qquad (2.15)$$

Where,

I_i = current (A) in a part of the line

R_i = resistance (Ω) of a portion of the line

E= unit energy sent out from the substation in kWH in a year

n= part of the line

♦ In case, where the line length is short and there are a number of transformers, the line is considered as a uniformly distributed load and the total load of which can be assumed at a point one third of the distance. The annual losses are the calculated as follows:

$$L_{Annual} = \frac{I_m^2 \times R_T}{3}(0.2LF + 0.8(LF)^2 \times 8760\ \ (Single\ \ phase) \qquad (2.16)$$

$$L_{Annual} = I_m^2 \times R_T(0.2LF + 0.8(LF)^2 \times 8760\ \ (Three\ phase) \qquad (2.17)$$

The annual percentage of losses is then:

$$Loss\ \% = \frac{I_m^2 \times R_T}{E}(0.2LF + 0.8(LF)^2 \times 8760 \times 100 \tag{2.18}$$

Where,

I_m = Maximum current (A) recorded at the substation in a year.

E = Unit energy (kWH) sent out from the substation in the particular year

R_T = Total resistance of the line (Ω)

$$R_T = \sum_{i=1}^{n} L_i R_i \tag{2.19}$$

L_i = Portion of the line length (km)

R_i = Resistance of the corresponding section of the line (Ω/km)

n= Part of the line

Technical losses in power transmission and distribution systems are caused mainly by the resistance of conductors, the magnetic consuming energy following a hysteresis loop of an iron core of transformer or eddy current flows. The other miscellaneous losses such as corona losses and leak current through insulators are relatively small. Distribution technical losses reduction usually focuses on the reduction in conductor losses sharing a large part of transmission and distribution losses.

The electric current goes through the conductors of distribution feeders with the technical losses in proportion to the square of current expressed by the following the well known formula.

$$P_l = \sum_{i=1}^{n} |I|^2 R \tag{2.20}$$

Where "n" is total number of branches in the system, "R" is the resistance of conductor in branch "i" and "$|I|$" is the magnitude of current flow in branch "I", for three phase system the losses of feeder becomes $3I^2$ R.

The formula about the conductor losses make clear that power distribution loss reduction can be achieved by the decrease in a conductor resistance, or in a current. The decrease in conductor resistance can be achieved through system reinforcement such as re-conductor, or adding new lines. The decrease in current can be achieved by the installation of capacitors through power factor correction and Leveling of distribution load or switching optimization.

Peak loss will be performed by three phase load flow calculation. Peak load data of each point in a distribution system for line losses calculation can be estimated by allocating a peak load (identified by measurement) of lines that is measured at substation. The load allocation depends on the size of transformer.

Medium voltage (MV) load data collection is conducted by utilizing the existing meters at MV substations. The load data of each feeder should be recorded at every hour through a year. Usually the loss calculation is carried out at the peak period of time. Yearly loss can be grasped by multiplying the peak loss and the load factor that can be estimated by the hourly data. The required load data on the MV feeders for technical loss estimation are the current and the power factor at the sending end of substations.

The line constants such as conductor resistance, reactance and capacitance can be calculated by the information about conductor's configurations and the types of conductors.

In equation form, the technical loss shall be computed as follows [3]:

$$Technical\ loss = \sum[\ lines\ loss)] + \sum[Transformers\ loss] \tag{2.21}$$

In general, loss of other equipments can be negligible. However, some utilities may include the loss of this group in the calculation for better accuracy.

2.3.2 Load balancing

The load balancing technique is the interesting loss reduction technique for 3 phase 4 wire system. In general, the effect of load balancing is more obvious during the peak load period. Although this concept is simple and easy to perform, utility will not benefit much from this technique.

Load balancing is one of the low cost techniques for technical loss reduction because it does not require additional network equipment. It only requires load level and phasor information at each load point. To get more benefit from this technique, load information and processes of load balancing have to be considered. If customer can be categorized by using load characteristic, energy usage of each customer is a good choice to use for load balancing. In radial distribution system, load balancing shall be done at the end of every lateral and energy loss of neutral wire in every lateral will decrease and utility will get more benefit than only applied load balancing in the main line [3].

Three analysis objectives which are minimize the kW losses, balance the load (kVA) and balance the current (A). We can reduce the processing time of the load balancing analysis by de-selecting single phase, two phases and three phase section options. For example, by de- selecting three phase sections, CYMDIST will not try at all to reconnect loads to different phases within three phase sections.

The minimum unbalance factor is used to exclude from consideration any three phase sections that are fairly well balanced already. Unbalance is defined as follows:

$$Unbalance = \left|\frac{(Phase\ load - average\ phase\ load)}{average\ phase\ load}\right| \times 100\% \tag{2.22}$$

2.3.3 Calculation of voltage drop

The voltage variation in the distribution network considerably influenced the quality of the electricity. It does have very significant importance as almost all equipment connected to a power utility system is designed to be used at a certain definite voltage. However, it is not practical, to serve every customer on a power distribution at the constant voltage corresponding to the nameplate voltage exactly, because voltage drop exists in each part of the power system starting from the generator to the customer’s meter. Voltage drop on the line can be calculated by the following equation.

$$Voltage\ drop\ (\Delta \mathrm{V}) = \mathrm{I}(\mathrm{R}\cos\theta + \mathrm{X}\sin\theta) \times \mathrm{L} \tag{2.23}$$

This formula gives the voltage drop on one conductor line- to neutral. The three phase line to line drop is $\sqrt{3}$ time the above value and the single phase drop is twice the above values.

$$Voltage\ drop\ (\Delta V) = \sqrt{3}I(R\cos\theta + X\sin\theta) \times L \tag{2.24}$$

Where,

I = current on the line (A)
R = resistance of the line (Ohm/ km)
X = reactance of the line (Ohm/km)
ϴ = angle between current and voltage
L = length of the line (km)

Actually, one primary feeder may supply many transformers so approximately voltage drop can be calculated.

1. Where only one transformer is involved, voltage drop may be calculated as a concentrated load at the end of the line by using the formulae (2.24) accordingly.

2. Where the branch is relatively long and serves a few transformers widely spaced, these values may be derived from a circuit considered to have distribution load.

$$Voltage\ drop\ (\Delta\, \mathrm{V}) = \sqrt{3}\sum_{i=1}^{n} \mathrm{I_i}\ (\mathrm{R_i} \cos\theta + \mathrm{X_i}\ \sin\theta) \times \mathrm{L_i} \qquad (2.25)$$

3. Where the length is short, or where a larger, more closely situated number of transformers exist, the circuit may be considered as supply a uniformly distributed load, the total load may be assumed to be connected at a point half (1/2) the length of the branch (from the tap off at the main to the last transformer) for calculating voltage drop in the line.

$$Voltage\ drop\ (\Delta\, \mathrm{V}) = \frac{\sqrt{3}}{2}\mathrm{I}(\mathrm{R_T} \cos\theta + \mathrm{X_T}\ \sin\theta) \qquad (2.26)$$

Where,

I = current sending out of substation (A)
$\mathrm{R_T}$ = total resistance of the line (Ohm)
$\mathrm{X_T}$ = total reactance of the line (Ohm)
Θ = angle between current and voltage

2.3.4 Capacitor bank and power factor (PF) correction

The capacitor is capable of compensating reactive current that is required by system load. By installing capacitor, current flow in the distribution system is reduced, thus, the system loss reduction as well. Capacitor also has voltage maintaining characteristic. It can reduce voltage drop during heavy load period. When capacitors are connected to the distribution feeder, they inject the reactive power (current) that reduce the current flow as shown in figure 2.5. Shown the image is of current flow reduction. During the light load period, capacitor may earn voltage rise that exceeds the permissible voltage rang. It should be cautioned. Capacitor is not so expensive that it is easily applied [4].

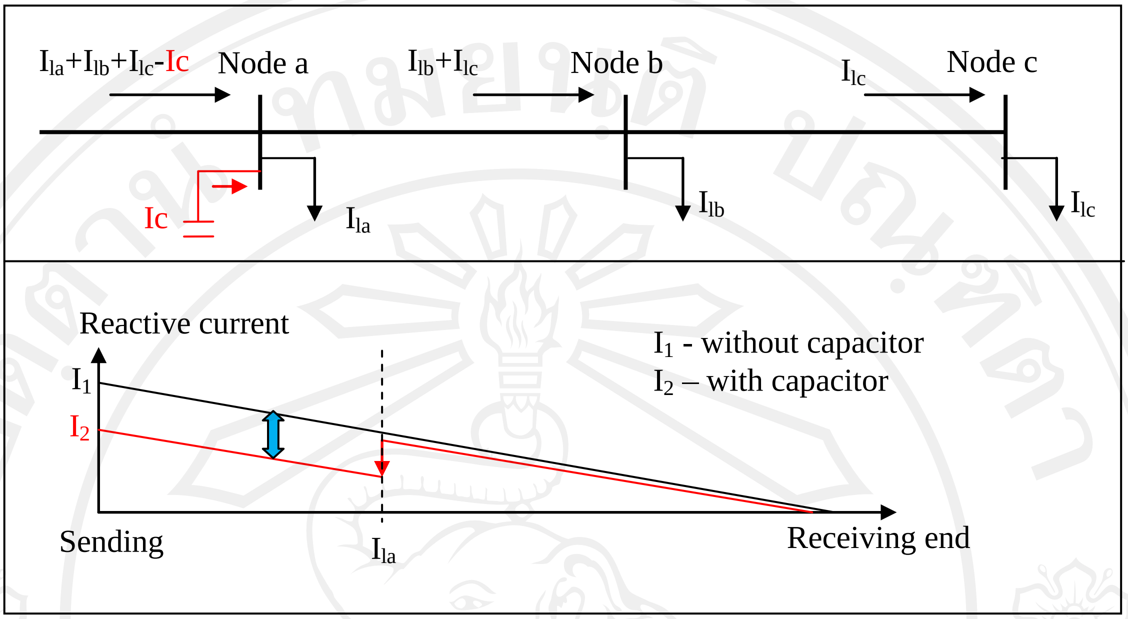


Figure 2.5 Image of reactive power compensation [4].

The capacitor bank used in parallel with an inductive load provide load with reactive power. They reduce the system's reactive and apparent power, and therefore cause its PF to increase [5].

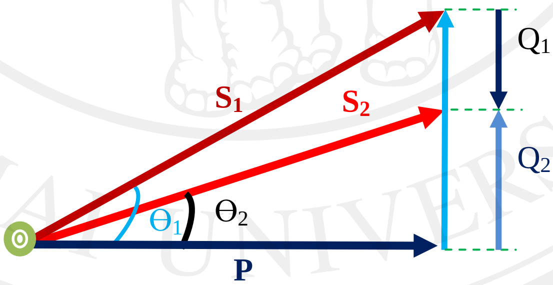


Figure 2.6 Power triangles for a power factor correction capacitor bank.

Furthermore, capacitor current causes voltage rise which results in lower line losses and voltage drops leading to an improved efficiency and voltage regulation. Based on the power triangle shown in figure, the reactive power delivered by the capacitor bank Q_c is

$$Q_c = Q_1 - Q_2 \tag{2.27}$$

$$= P(\tan\Theta_1 - \tan\Theta_2) \tag{2.28}$$

$$= P[\tan(\cos^{-1} PF_1) - \tan(\cos^{-1} PF_2)] \tag{2.29}$$

Where P is the real power delivered by the system and adsorbed by the load, Q_1 is the load's reactive power, and Q_2 is the system's reactive power after the capacitor bank connection.

As it can be observed from the following equation, since a low PF means a high current

$$I=\frac{P_{3\emptyset}}{\sqrt{3}}\ V_{L\text{-}L}\mathrm{Cos}\theta \tag{2.30}$$

The disadvantages of a low PF include: (i) increased line losses, (ii) increased generator and transformer rating, and (iii) extra regulation of equipment impedance of the case of low lagging PF.

2.3.5 Optimal capacitor placement (OCAP)

Figure 2.7 shows a brief summary of the procedure of OCAP analysis. OCAP will locate capacitors on distribution networks if profitable

2.3.5.1 Fixed capacitor allocation.

OCAP first considers fixed capacitors, which, by definition, are on during all load snapshots. OCAP will finds all the eligible buses to allocate fixed capacitors with the maximum savings, if all the below constraints meet; otherwise, the fixed capacitor allocation will stop.

- The present value of the savings is larger than that of the present value of costs.
- Fixed capacitors are available.
- The voltages at all buses is no more than the specified maximum voltage ($V \leq V_{max} = +5\%$)

2.3.5.2 Switched capacitor allocation

After the fixed capacitor allocation stops, OCAP will continue to place the switched capacitor. OCAP will find all the eligible buses to allocate switched capacitors with the largest savings, if all the below constraints meet; otherwise, the switched capacitor allocation will stop.

- The present value of the savings is larger than that of the present value of the costs.
- Switched capacitors are available.
- The voltages at all buses

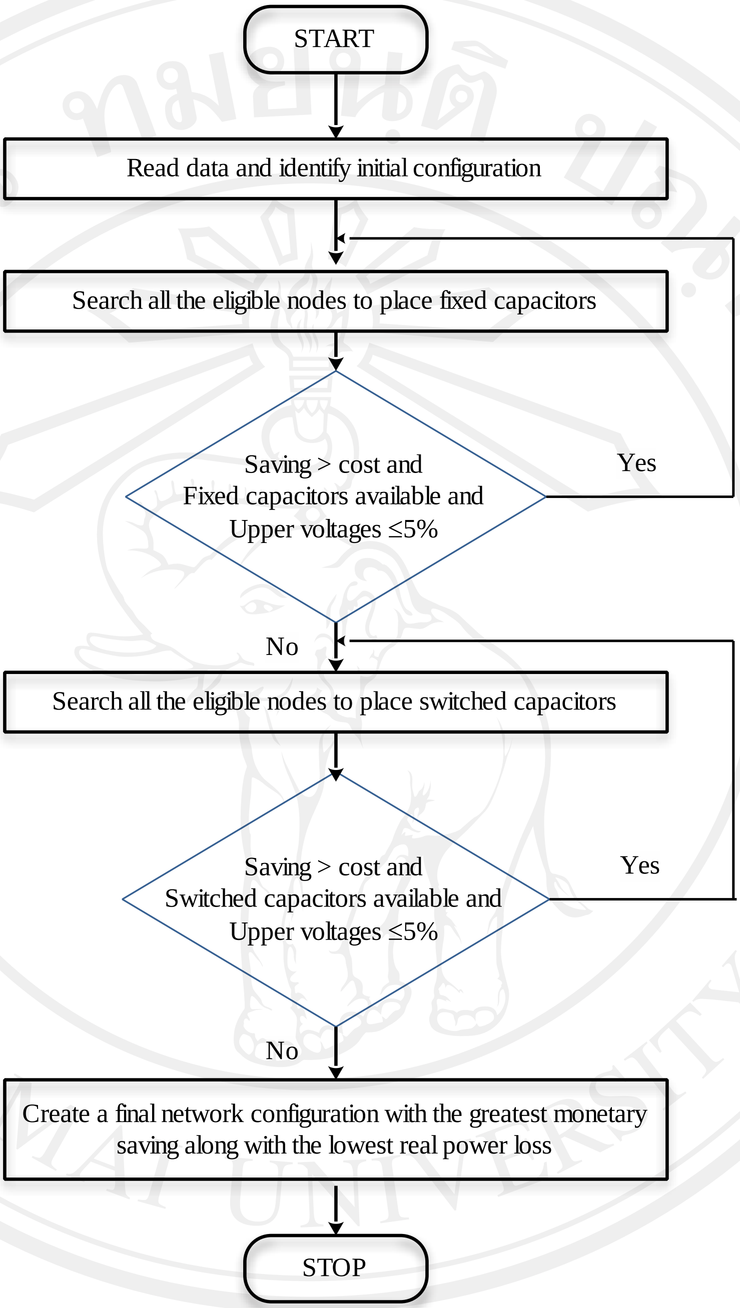
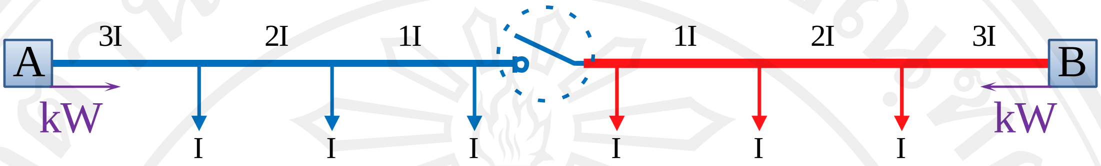


Figure 2.7 Optimal capacitor placement flowchart

2.3.6 Switching optimization or reconfiguration

One of the effective ways for technical loss reduction is to change the feeder configuration by switching devices, called switching optimization. This is the most cost effective countermeasure for technical loss reduction because it only needs to relocate the existing switching device or install new one to the adequate position. The cost of switching device is relatively low compared with re-conductor or new feeder installation, even if it requires several new devices. Switching optimization depends on the size of loads, size and length of conductor [4]. As seen in figure 2.8 shows the concept of switching optimization.

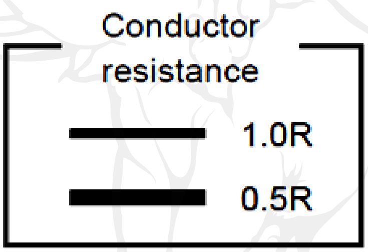
(a) Original system.



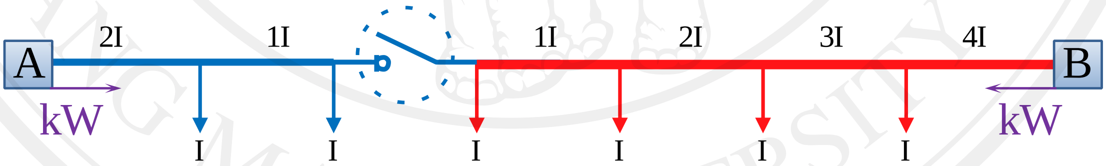
$$\text{Loss on feeder A} = ((3I)^2 + (2I)^2 + (I)^2)\ .\ R = 14I^2R \tag{2.31}$$

$$\text{Loss on feeder B} = ((3I)^2 + (2I)^2 + (I)^2)\ .\ 0.5R = 7I^2R \tag{2.32}$$

$$\text{Total losses} = 21.I^2R \tag{2.33}$$



(b) Optimized system by switching.



$$\text{Loss on feeder A} = ((2I)^2 + (I)^2)\ X\ R = 5I^2R \tag{2.34}$$

$$\text{Loss on feeder B} = ((4I)^2 + (3I)^2 + (2I)^2\ )\ X\ 0.5R + (I)^2\ X\ R = 15.5I^2R \tag{2.35}$$

$$\text{Total losses} = 20.5\ I^2R \tag{2.36}$$

Figure 2.8 Concept of switching optimization.

In the original system, both substations have the same amounts of load (3 X I), but the feeder A and feeder B supplies the power with thin conductor (high resistance) and thick conductor (low resistance) respectively. Now suppose that the conductor resistances are 1.0R and 0.5R, current of each load is I and all of the section lengths are same. Total losses on the original system would be $21I^2R$. On the other hand, the total loss in the optimum system is reduced to $20.5I^2R$. The better feeder configuration would be obtained by switching optimization due to inappropriate position of switching devices in the original system.

Switching optimization searches for the better configuration of the feeders based on the loss reduction by conducting one by one switching operation. The optimum system saves $0.5I^2R$ loss compared with the original system. In the optimum system, the location of switching device is changed to substation A-side and load is transferred from substation A to B. This load becomes far from substation in comparison with the original system and the power from substation B increases so that losses on feeder B would increase. However, the current of the load passes thick conductor from substation B, and the loads on feeder A become adequate size considering the conductor size, which means that total losses on feeder A and B would decrease. The total loads on the feeder A and B become and, respectively. Switching optimization minimizes the loss but overload of substation should be noted.

2.3.6.1 Problem formulation of network reconfiguration

The loss minimum formulation of network reconfiguration, generally, is mathematically expressed as follows [10]:

$$Min(P_{loss}) = Min\left(\sum_{i=1}^{N_b}(P_i^2 + Q_i^2).r_i\right) \tag{2.37}$$

Where r_i is the branch resistance $P_i + jQ_i$, is the power flow in branch $i, P_i + jQ_i = \sum_{t\in i}(P_{Lt} + jQ_{Lt})$, and $P_{Lt} + jQ_{Lt}$ is the power load connected at node t in the downstream of branch i .

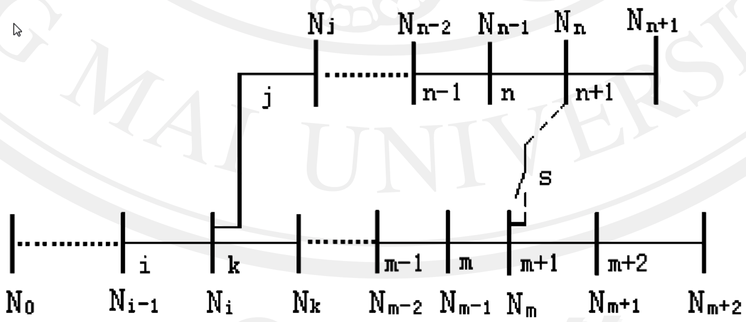


Figure 2.9 Simple distribution networks.

In a radial distribution network, when a tie switch is closed, a loop is formed and a sectionalizing switch in the loop should be opened to restore the radial structure of the system. As a result of this switching operation, the loads between these two switches associated branches will be transferred from one feeder to the other. In a loop reconfiguration, the branch exchange with the maximum loop power loss reduction just is the branch exchange with the maximum system power loss reduction. As a result, the loss-minimum formulation of the system network reconfiguration can be equivalently replaced by the loss-minimum formulation of each tie switch associated loop reconfiguration, that is

$$M_{in}(P_{loss}) \leftrightarrow Min(P_{L\_i}) \quad (i = 1, \dots\dots, N_t) \tag{2.38}$$

Where N_t is the total number of tie switches in the system; P_{L_i} is the real power losses of the ith tie switch associated loop network.

2.3.6.2 The optimal branch exchange in a loop

In figure 2.9, let branches $\{j, \dots\dots.., n\}$ in the tie switch s associated loop be denoted by set L and the other $\{k, \dots\dots.., m\}$ by set R. The resistance of tie switch s associated branch is r_s when tie switch s is closed and a sectionalizing switch in the loop is opened, there is a power load $P_s + jQ_s$ transferred between two branch sets. Assume that the transferred power load $P_s + jQ_s$ is a continuous variable and is transferred from set L to set R.

After branch exchange, the power loss reduction ΔP_{al} is given as follows:

$$\Delta P_{al} = P_{ori} - P_{ref} =$$

$$= 2\sum_{i \in L}[P_i P_s + Q_i Q_s].r - 2\sum_{i \in R}[P_i P_s + Q_i Q_s].r_i - (P_s^2 + Q_s^2).r_s \tag{2.39}$$

Where P_{oris} is the real power losses of the original network, and P_{refs} is the real power losses after reconfiguration.

To seek the optimal transfer load $P_s + jQ_s$ that makes ΔP_{al} a maximum, the partial differentials of ΔP_{al} to $P_s$andQ_s, respectively, are set to be zero as follows:

$$\frac{\partial \Delta P_{al}}{\partial P_s} = 0, \frac{\partial \Delta P_{al}}{\partial Q_s} = 0 \tag{2.40}$$

Solving (2.40) yields a solution of the optimal transfer load

$$P_s = \frac{\sum_{i \in L} P_i r_i \sum_{i \in R} P_i r_i}{\sum_{i \in L \cup R} r_i + r_s} \tag{2.41}$$

$$Q_s = \frac{\sum_{i \in L} Q_i r_i \sum_{i \in R} Q_i r_i}{\sum_{i \in L \cup R} r_i + r_s} \tag{2.42}$$

To find the branch where the power flow matches the optimal transfer load $P_s + jQ_s$, the rules are derived from the sign and magnitude of the optimal transfer load, as follows.

- If $(P_s + jQ_s) < (P_n + jQ_n)/2, or\ (P_s + jQ_s) < (P_m + jQ_m)/2$, the real power loss in the loop is already a minimum. The tie switch should keep open status.
- If $P_s + jQ_s > 0$, the optimal transfer load $P_s + jQ_s$ is really transferred from set L to set R . In set L, the branch with the power flow being closest to $P_s + jQ_s$ is to be open.
- If $P_s + jQ_s < 0$, the optimal transfer load $P_s + jQ_s$ is really transferred from set R to set L. I set R, the branch with the power flow being closest to $P_s + jQ_s$ is to be opened.

To eliminate the possible error caused by rounding a continuous solution, two branches, with branch power flows just bigger and smaller than$P_s + jQ_s$, exchange status with the tie branch. Also, the corresponding power loss reductions of these two branch exchanges are evaluated by using (2.37) to determine which one is the optimal branch exchange.

The procedure of the proposed branch exchange algorithm is given in figure 2.10, where k is the iteration count.

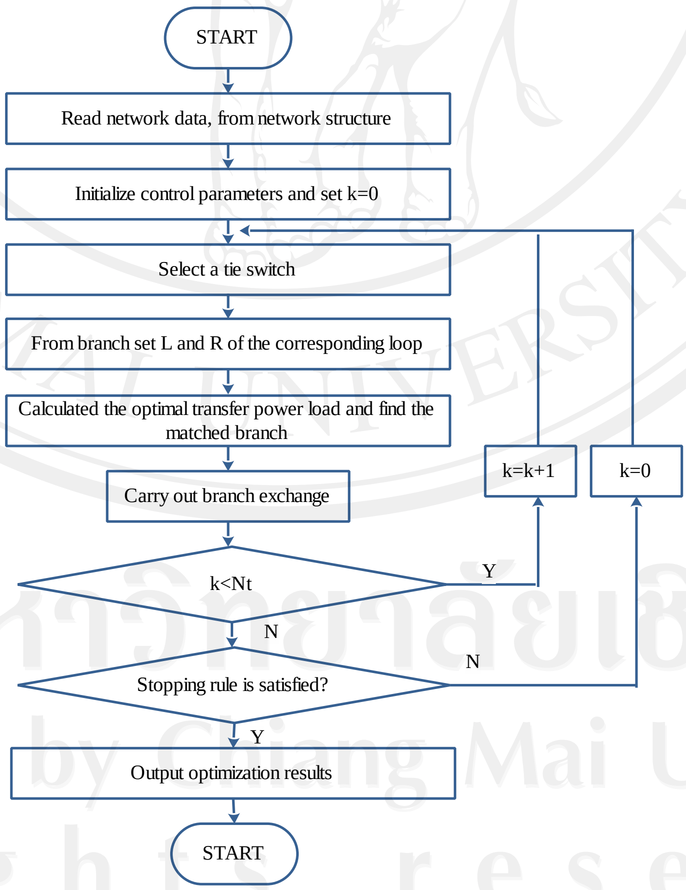


Figure 2.10 Flowchart of the proposed branch exchange algorithm [10]

2.3.7 Economic justification

Figure 2.11 shows a concept of seeking an appropriate technical loss reduction. The investment for loss reduction reduces power losses and their costs. We must study appropriate technical loss reduction taking into account the total cost, namely the sum of the cost of power losses and the cost of facilities. When reduction in the cost of power losses is larger than the increase in the cost of facilities, the total cost would reduce. Our target is not only loss reduction, but also reducing the total cost. Seeking too high loss reduction sometimes requires the huge cost making the countermeasures unfeasible.

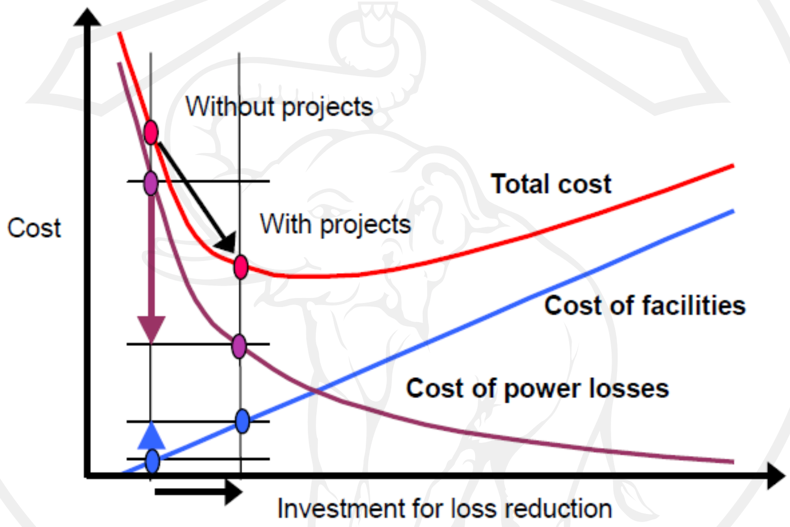


Figure 2.11 Concept of seeking appropriate technical loss reduction[4].

The evaluation of technical loss reduction appropriately taking into account the total cost including the cost of power losses and the cost of facilities. When the total cost is reduced with projects, the additional cost of facilities by the projects should be smaller than the cost saving by the loss reduction.

2.3.8 Financial analysis

Steps in financial analysis:

- Estimate the cost, or capital, of the project or planning.
- Estimate the cash flows from the project, including the value of the asset at a specific terminal date. The cash flow projections should cover both the revenues generated by project and the operating/maintaining cost (O&M), debt repayment schedule, taxes, tariff, etc.
- Determine the Discount Rate.
- Estimating the project economic life.
- Assessing project risks, and translating those risks into financial terms. Determine NPV, IRR, Payback Period

2.3.8.1 Net present value (NPV)

The net present worth in an engineering project is the different between the present values of the total profit and total cost of the project within its operational lifetime. Obviously the higher the NPV is, the higher is the economic benefit.

$$NPV(i) = \frac{-I_0}{(1+i)^0} + \frac{A_1}{(1+i)^1} + \frac{A_2}{(1+i)^2} + \frac{A_3}{(1+i)^3} + . \dots + \frac{A_N}{(1+i)^N}$$

$$\text{Or} \quad \mathrm{NPV(i)} = \sum_{n=1}^{N} \frac{A_n}{(1+i)^n} \tag{2.43}$$

Where:

A_n = the expected net cash receipt at the end of year n.

i = the discount rate = minimum attractive rate of return (MARR).

N = the planning's duration in years.

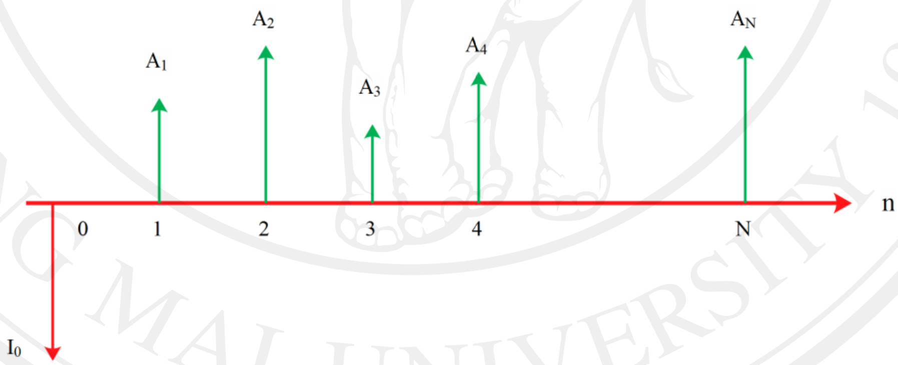


Figure 2.12 Net Present Value (NPV) current of time

Decision rule of NPV:
- NPV > 0; Accept the project
- NPV = 0; Remain in different
- NPV < 0; Reject the project

Positive NPV means attractive financial return, and large NPV means more attractive project alternative.

2.3.8.2 Internal rate of return

Internal rate of return (IRR) is the interest rate charged on the unrecovered project balance of the investment such that, when the project terminates, the unrecovered project balance will be zero. Large IRR indicates that the project is more attractive financially.

$$I_0 = \frac{A_1}{(1+R)^1} + \frac{A_2}{(1+R)^2} + \frac{A_3}{(1+R)^3} + \ldots + \frac{A_N}{(1+R)^N} = \sum_{n=1}^{N} \frac{A_n}{(1+R)^n}$$

$$\text{Or} \quad \sum_{n=1}^{N} \frac{A_n}{(1+R)^n} - I_0 = 0 \qquad (2.44)$$

Where, R denotes the internal rate of return (IRR).

Decision rule of IRR: - IRR > MARR; Accept the investment.
- IRR < MARR; Remain in different.
- IRR < MARR; Reject the investment.

2.3.8.3 Payback period

Determining the relative worth of a new project by calculating the time it will take to pay back what it cost is the single most popular method of project screening. If we assume that all projects have equal annual benefit the payback period can be calculated using the formula.

$$\text{Payback period} = N_p = \frac{\text{Initial cost}}{\text{Uniform annual benefit}} = \frac{I_0}{A} \qquad (2.45)$$

Decision rule of the Payback method:

- $N_p > N_0$; Accept all projects
- $N_p < N_0$; Reject all projects.

Where N_0 = some critical number of years.

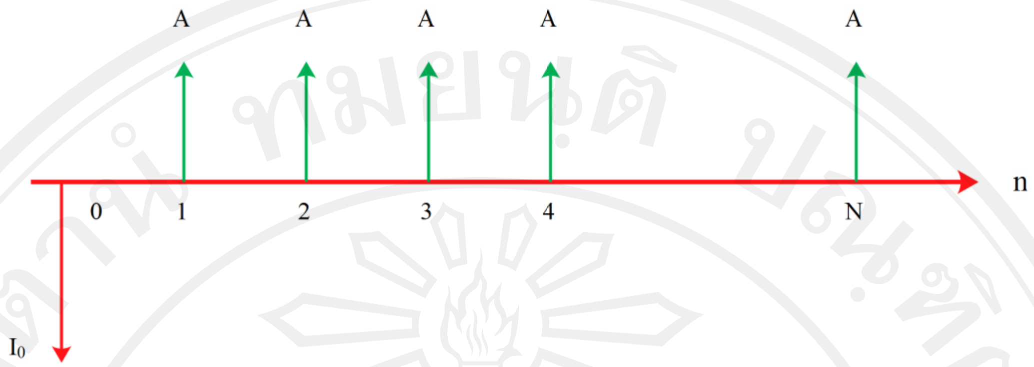


Figure 2.13 Payback period current of time