# Chapter 5
# Conclusions

In this thesis, we prove fixed points theorems and convergence theorems for some generalized nonexpansive mappings and nonspreading mappings in CAT(0) spaces. First, we prove the existence of common fixed point for some generalized nonexpansive mappings and nonspreading mappings in CAT(0) spaces. Second, we prove $\Delta-$convergence theorems for some generalized nonexpansive mappings and nonspreading mappings in CAT(0) spaces. Finally, we prove strong convergence theorems for some generalized nonexpansive mappings and nonspreading mappings in CAT(0) spaces.

(1) Let $E$ be a nonempty bounded closed convex subset of a complete CAT(0) space X, and let $T : E \to E$ satisfies condition (C) and $S : E \to E$ is a nonspreading mapping. Let $T$ and $S$ are commuting mappings on $E$. Then $T$ and $S$ have a common fixed point.

(2) Let $E$ be a nonempty closed convex subset of a complete CAT(0) space X, and let $T : E \to E$ satisfies condition (C) and $S : E \to E$ is a nonspreading mapping such that $F(T) \cap F(S) \neq \emptyset$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be two sequences in $(0, 1)$. Let $\{x_n\}$ be defined as (A). If $\liminf_{n\to\infty} \alpha_n(1-\alpha_n) > 0$ and $\liminf_{n\to\infty} \beta_n(1-\beta_n) > 0$, then $\Delta-\lim_n x_n = w \in F(T) \cap F(S)$.

(3) Let E be a nonempty closed convex subset of a complete CAT(0) space X, and let $T : E \to E$ satisfies condition (C) and $S : E \to E$ is a nonspreading mapping such that $F(T) \cap F(S) \neq \emptyset$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be two sequences in $(0, 1)$. Let $\{z_n\}$ be defined as (B). If $\liminf_{n\to\infty} \alpha_n(1-\alpha_n) > 0$ and $\liminf_{n\to\infty} \beta_n(1-\beta_n) > 0$, then $\Delta-\lim_n z_n = v \in F(T) \cap F(S)$.

(4) Let $E$ be a nonempty compact convex subset of a complete CAT(0) space X, and let $T : E \to E$ satisfies condition (C) and $S : E \to E$ is a nonspreading mapping such that $F(T) \cap F(S) \neq \emptyset$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be two sequences in $(0, 1)$. Let $\{x_n\}$ be defined as (A). If $\liminf_{n\to\infty} \alpha_n(1-\alpha_n) > 0$ and $\liminf_{n\to\infty} \beta_n(1-\beta_n) > 0$, then $x_n \to w \in F(T) \cap F(S)$.

(5) Let E be a nonempty compact convex subset of a complete CAT(0) space X, and let $T : E \to E$ satisfies condition (C) and $S : E \to E$ is a nonspreading mapping such that $F(T) \cap F(S) \neq \emptyset$.Let $\{\alpha_n\}$ and $\{\beta_n\}$ be two sequences in $(0, 1)$. Let $\{z_n\}$ be defined as (B). If $\liminf_{n\to\infty} \alpha_n(1-\alpha_n) > 0$ and $\liminf_{n\to\infty} \beta_n(1-\beta_n) > 0$, then $z_n \to v \in F(T) \cap F(S)$.