

## **Chapter 5**

### **Modeling Dependence Dynamics of Air Pollution: Time Series Analysis Using a Copula Based GARCH Type Model**

Previous chapter exhibits a full map of volatility spillovers between local and regional/national air pollution indices. Among 42 sample groups, Guangzhou group and Shenzhen group exhibit a similar, unique feature that is different from other 40 groups. Further research the dependence structure is important.

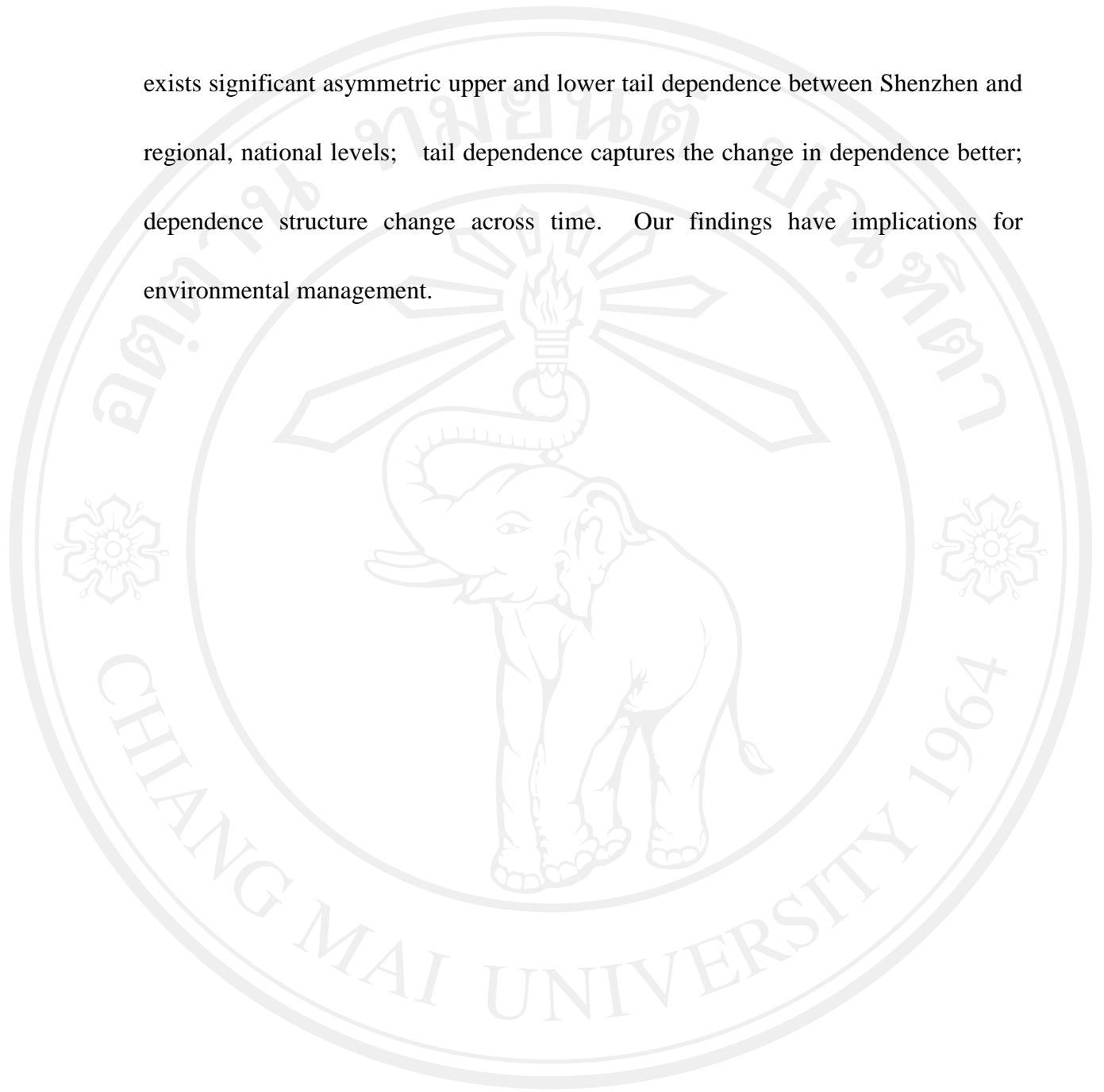
This chapter is developed from the original paper ‘Modeling Dependence Dynamics of Air Pollution: time series analysis using a copula based GARCH type model’ by He et al. (2013) presented at the 6<sup>th</sup> Conference of the Thailand Econometric Society, published in “Uncertainty Analysis in Econometrics with Applications”, Springer-Verlag (2013), pp.215-226. Van-Nam Huynh et al. (Eds.)

### **Abstract**

This paper investigates the dependence structure between the Air Pollution Index (API) of Shenzhen and corresponding regional, national levels based on copula based GARCH models. In particular, time varying normal copula and time varying SJC copula are compared and employed to model the dependence structure.

Comparison with the results of DCC-GARCH model is made. We find that there

exists significant asymmetric upper and lower tail dependence between Shenzhen and regional, national levels; tail dependence captures the change in dependence better; dependence structure change across time. Our findings have implications for environmental management.



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## 5.1 Introduction

Air pollution in china is attracting the focus of not only the Chinese government and people but also researchers worldwide. There are disputes about the reliability of the Air Pollution Index (API) the government released and the pollutant detected and included to get the API. Despite the disputes, since the composite and method of computing the integrated air pollution APIs are unchanged during our observation, exploring spatial dependence dynamic through examine the conditional dependences of urban API and regional, national levels are feasible and meaningful.

Some previous studies examined spatial contagion of air pollution. But they mostly focus on some pollutants and dust (Yongxin Zhang et al. 2010; Tracey Holloway et al. 2008; Lee et al. 2010; Chung-Ming Liu et al. 2006; F. Cousin et al. 2005), studies concerning API, an integrated index are not noticed. And for geographical scale, most previous studies examined contagion between regions within one county, or between countries (Feng Xiao et al. 2006; Guor-Cheng Fang et al. 2010; Paul J. Miller et al. 1998; Suhejla Hoti et al. 2005), spatial contagion between city, regional and national API is yet to be carried out.

In our previous studies, based on the DCC-GARCH model (Engle, 2002) which is an econometric model widely used by many researchers in time varying correlations finance research, we investigated the dynamic correlation of Air Pollution Indices (APIs) between 42 Chinese sample cities and their corresponding regional and

national levels for a duration of 10 years. Some meaningful findings were drawn which were shared by most sample cities, for example, the correlations of local APIs between regional and national levels are time varying; most cities exhibit positive conditional correlations with both regional and national APIs, and the conditional correlations of most cities with regional and national APIs are only slightly different and are mostly stable. Interesting thing is that, the behavior of Shenzhen and Zhuhai, two cities with the shortest distance among the subjected cities, exhibit unique characteristic between each other: a decrease of dynamic correlation with both regional and national levels after spring 2001, and then increase again after autumn 2004. It is not surprising that Shenzhen and Zhuhai behave similarly. Since they are so close to each other geometrically, with 56.4 kilometers of direct distance, and according to our integrating method of city related regional and national APIs, they are expected to have similar regional and national APIs. To further explore the dynamic spatial contagion feature of these two cities, we focus on Shenzhen in this study. Shenzhen lies in Pearl River Delta, one of the three key regions required to carry out inter-region cooperation to cut and improve air quality. The reason why we choose Shenzhen is that Shenzhen has higher population density, higher GDP per capita (18 thousand USD for Shenzhen while 14 thousand for Zhuhai in 2011), and experiences higher API.

DCC-GARCH model, as a conventional linear-based correlation method is



somewhat restrictive due to its requirements of normality for the joint distribution and of linear relationships among variables. More flexible copula-based models have become a common practice to cope with dependence between random variables. Moreover, methods have emerged for dealing with non-normality and dependence dynamics with asymmetry over time using copula-based GARCH models (Patton 2006; Jondeau and Rockinger 2006). Most studies focused on the financial market. But this method has not been used in the API dependence study.

To assess the changing dependence structures over time, following our previous research, this paper attempts to investigate time varying air pollution dependence between Shenzhen and its corresponding regional, national levels. Comparison between the DCC-GARCH model based result and copula based result will be made.

This study contribute to the existing literature not only by focusing on the dependence structure of urban and regional, national air pollution, but also by trying to apply the copula based GARCH type models to air pollution co-movement study.

The rest of the paper is set up in the following manner. Section 2 presents the econometric model. Section 3 contains the description of the data. The empirical results are in Section 4, followed by conclusion in the last section.

## **5.2 Model**

### **5.2.1 DCC-GARCH model**

Time varying correlations are often estimated with multivariate

generalized autoregressive conditional heteroskedasticity (MGARCH) models. DCC models proposed by Engle which can be estimated very simply with univariate or two step methods based on the likelihood function, is an important one. In this paper, we employed the DCC model, after Engle (2002) to examine the existence of volatility in each series and the dynamic correlations between urban APIs, regional APIs and national APIs.

Let us consider the APIs  $Y_t = (Y_{1t}, \dots, Y_{kt})'$ , for  $t = 1, \dots, T$ . The following mean equation was estimated for each series given as:

$$\begin{aligned} Y_{it} &= \mu_i + a Y_{it-1} + \varepsilon_{it} \\ \varepsilon_{it} &\sim N(0, H_t) \end{aligned} \quad (5.1)$$

where  $Y_{it}$  is API in series  $i$  at time  $t$ ,  $i$  is either city, regional or national API,  $\varepsilon_{it}$  is the error term for the API  $i$  at time  $t$ . All estimated series exhibited evidence of ARCH effects. DCC (Engle 2002) parameterization of conditional covariance metrics is given as:

$$H_t \equiv D_t R_t D_t \quad (5.2)$$

where  $D_t$  is the  $k \times k$  diagonal matrix of time varying standard deviations from univariate GARCH models with  $\sqrt{h_{it}}$  on the  $i$  th diagonal, and  $R_t$  is the time varying correlation matrix.

The elements of  $D_t$  is  $\sqrt{h_{it}}$ . For simplicity,  $h_{it}$  can be expressed for the univariate form as:

$$h_{it} = \omega_i + \sum_{p=1}^{p_i} a_{ip} r_{it-p}^2 + \sum_{q=1}^{Q_i} \beta_{iq} h_{it-q} \quad (5.3)$$

for  $i = 1; 2, \dots, k$  with the usual GARCH restrictions for non-negativity and stationarity being imposed, such as non-negativity of variances and  $\sum_{p=1}^{p_i} a_{ip} + \sum_{q=1}^{Q_i} \beta_{iq} < 1$ .

The proposed dynamic correlation structure is:

$$Q_t = (1 - \sum_{m=1}^M \theta 1_m - \sum_{n=1}^N \theta 2_n) \bar{Q} + \sum_{m=1}^M \theta 1_m (\varepsilon_{t-m} \varepsilon'_{t-m}) + \sum_{n=1}^N \theta 2_n Q_{t-n}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \quad (5.4)$$

where  $\bar{Q}$  is the unconditional covariance of the standardized residuals resulting from the first stage GARCH estimation, and

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{q_{kk}} \end{bmatrix} \quad (5.5)$$

so that  $Q_t^*$  is a diagonal matrix composed of the square root of the diagonal elements of  $Q_t$ . The typical element of  $R_t$  will be of the form  $\rho_{ijt} = \frac{q_{ijt}}{\sqrt{q_{ii}q_{jj}}}$ .

To investigate the seasonal effect of mean and variance, and the effect on the dynamic correlation between local APIs, regional APIs and national APIs, we set three seasonal dummy in both mean and variance equations, so that equation (5.1) now becomes:

$$Y_{it} = \mu_i + S_2 D_2 + S_3 D_3 + S_4 D_4 + a Y_{it-1} + \varepsilon_{it} \quad (5.6)$$

Equation (5.3) becomes:

$$h_{it} = \omega_i + S'_2 D_2 + S'_3 D_3 + S'_4 D_4 + \sum_{p=1}^{p_i} a_{ip} r_{it-p}^2 + \sum_{q=1}^{Q_i} \beta_{iq} h_{it-q} \quad (5.7)$$

so that  $D$  is seasonal effect vector where  $D_2, D_3, D_4$  equals 1 when  $t$  is in summer, autumn, or winter respectively, other equations same. Spring includes March, April and May; summer includes June, July and August; autumn includes September, October and November; winter includes December, January and February.

### 5.2.2 Copula concept

Copulas are functions that join or couple multivariate distribution functions to their uniform one-dimensional marginal distribution functions (Roger B. Nelsen, 2006). Sklar(1959) showed that a joint distribution can be factored into the margins and a dependence function called a copula. For bivariate case, let  $X$  and  $Y$  be two continuous random variables with margins  $F(x)$  and  $G(y)$  and with a joint distribution function  $H(x, y)$ , Sklar's theorem states that the standard representation for the joint distribution  $H$  is:

$$H(x, y) = C(F(x), G(y)) \quad (5.8)$$

where  $C(u, v)$ ,  $u=F(x)$  and  $v=G(y)$  is the copula that captures the dependence structure between  $X$  and  $Y$ . If the margins are continuous, then  $C$  is uniquely determined, otherwise, the copula  $C$  is uniquely determined on  $\text{Ran}(F) \times \text{Ran}(G)$ . Thus, copulas can be used to link margins to a multivariate distribution function, which, in turn, can be decomposed into its univariate marginal distributions and a copula capturing the dependence structure between the two variables. In terms of construction, the copula

is a multivariate distribution function, with uniform (0, 1) margins that relate the quantiles of the distributions rather than the original variables. It is therefore unaffected by a monotonically increasing transformation of the variables.

Patton (2006) extended Sklar's theorem for conditional distributions.

By extending Sklar's theorem, the conditional copula function can be written as:

$$H(x, y|w) = C(F(x|w), G(y|w)|w) \quad (5.9)$$

where  $W$  is the conditioning variable,  $F(x|w)$  is the conditional distribution of  $X/W=w$ ,  $G(y|w)$  is the conditional distribution of  $Y/W=w$  and  $H(x, y|w)$  is the joint conditional distribution of  $(X, Y)/W=w$ .

Given the condition that  $F$  and  $G$  are differentiable,  $H$  and  $C$  are twice differentiable, the unconditional and conditional joint densities are given by:

$$\begin{aligned} f(x, y) &= f(x) \cdot g(y) \cdot c(u, v) \\ f(x, y) &= f(x) \cdot g(y) \cdot c(u, v|w) \end{aligned} \quad (5.10)$$

About the models for the marginal distributions, APIs series in this study exhibit volatility clustering feature. To capture the most important features of air pollution index, such as fat tails or leverage effects, and seasonality of the first and the second moments, the marginal models of the APIs are estimated by three most widely used models and then choose one outperform other two:

Mean equations are specified as an ARIMA(p, d, q) process as:

$$(1 - \sum_i^p \theta_i L^i)(1 - L)^d y_t = (1 + \sum_i^q \phi_i L^i) \varepsilon_t \quad (5.11)$$

where  $p$ ,  $d$ , and  $q$  are integers greater than or equal to zero and refer to the order of the autoregressive, integrated, and moving average parts of the model respectively.

In this paper, volatility models to be estimated are associated with a stationary AR (1) conditional means given by:

$$Y_t = \mu + \theta Y_{t-1} + \varepsilon_t \quad |\theta| < 1 \quad (5.12)$$

where,  $Y_t$  is Air Pollution Index,  $\varepsilon_t$  is shock to API.

### **GARCH**

Generalised autoregressive conditional heterocedasticity (GARCH)

model (Bollerslev, 1986) is as the follow:

$$\begin{aligned} \varepsilon_t &= \eta_t \sqrt{h_t} \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \end{aligned} \quad (5.13)$$

where  $h_t$  is conditional volatility, conditional on the information of period  $t-1$ ;  $\eta_t$  is standardized shock to API. Notice that GARCH model is lack of asymmetric and leverage.

### **GJR-GARCH**

The threshold GARCH (TGARCH) (Glosten, Jaganathan, & Runkle, 1993) is a simple extension of the GARCH scheme with extra term(s) added to account for possible asymmetries:

$$\begin{aligned} \varepsilon_t &= \eta_t \sqrt{h_t} \\ h_t &= \omega + \alpha \varepsilon_{t-1}^2 + \gamma I(\varepsilon_{t-1}) + \beta h_{t-1} \end{aligned} \quad (5.14)$$

where  $\omega > 0$ ,  $\alpha \geq 0$ ,  $\alpha + \gamma \geq 1$  and  $\beta \geq 0$  are sufficient conditions to ensure that the conditional variance  $h_t > 0$ .  $I(\varepsilon_{t-1})$  is an indicator function, taking the values of 1 if  $\varepsilon_{t-1} < 0$  (good news in this study) and 0 if  $\varepsilon_{t-1} > 0$ . The impact of bad news and good news on the conditional variance in this model is different, if  $\gamma > 0$ , the positive innovations have a higher impact than negative ones. The GJR is asymmetric as long as  $\gamma$  is significant different from zero.

### EGARCH

The EGARCH (p, q) model of Nelson(1991) can also accommodate asymmetry and specifies the conditional variance in a different way:

$$\begin{aligned}\varepsilon_t &= \eta_t \sqrt{h_t} \\ \log h_t &= \omega + \alpha |\varepsilon_{t-1}| + \gamma \varepsilon_{t-1} + \beta \log h_{t-1}\end{aligned}\quad (5.15)$$

EGARCH models the logarithm of conditional volatility, thereby removing the need for constraints on the parameters to ensure a positive conditional variance (Long more & Robinson, 2004).  $|\varepsilon_{t-1}|$  and  $\varepsilon_{t-1}$  capture the size and sign effects of standardized shocks respectively. The presence of leverage effects can be tested by the hypothesis that  $\gamma < 0$  and  $\gamma < \alpha < -\gamma$ . The model permits asymmetries via  $\gamma$  and if  $\gamma < 0$ , negative shocks lead to an increase in volatility. Good news generate less volatility than bad news. The model is asymmetric as long as  $\gamma \neq 0$ .

To capture the seasonal effect in our data, we include seasonal dummy  $D$  in both mean equations and variance equations so that  $D$  is seasonal effect vector where



$D_2, D_3, D_4$  equal 1 when  $t$  is in summer, autumn, or winter respectively, other things equal.

### 5.2.3 Copula models

Copula methods have advantages over linear correlation in that the copula-based GARCH models allow for better flexibility in joint distributions than bivariate normal or Student-t distributions.

In this study, we are interesting in the time varying dependence of air pollution, especially time varying dependence of the propensity of air pollution to improve or deteriorate. So we focus on the conditional Symmetrized Joe-Clayton copula and conditional Gaussian copula of Patton (2006).

The conditional Gaussian copula function is the density of the joint standard uniform variables  $(u_t, v_t)$ , as the random variables are bivariate normal with a time-varying correlation,  $\rho_t$ . Moreover, let  $x_t = \Phi^{-1}(u_t)$  and  $y_t = \Phi^{-1}(v_t)$ , where  $\Phi^{-1}(\cdot)$  denotes the inverse of the cumulative density function of the standard normal distribution. The density of the time-varying Gaussian copula is then:

$$c_t^{\text{Gau}}(u_t, v_t | \rho_t) = \frac{1}{\sqrt{1-\rho_t}} \exp \left\{ \frac{2\rho_t x_t y_t - x_t^2 - y_t^2}{2(1-\rho_t^2)} + \frac{x_t^2 + y_t^2}{2} \right\} \quad (5.16)$$

Tail dependence captures the behavior of random variables during extreme events. In our study, it measures the propensity of Shenzhen air pollution to improve or deteriorate simultaneously with regional and national air pollution. The Gumbel, Clayton and SJC copulas efficiently capture the tail dependences arising

from the extreme observations caused by the asymmetry.

The density of the time-varying Clayton copula is:

$$c_t^{\text{clay}}(u_t, v_t | \theta_t) = (\theta_t + 1)(u_t^{-\theta_t} + v_t^{-\theta_t} - 1)^{-\frac{2\theta_t - 1}{\theta_t}} u_t^{-\theta_t - 1} v_t^{-\theta_t - 1} \quad (5.17)$$

where  $\theta_t \in [0, \infty)$  is the degree of dependence between  $u_t$  and  $v_t$ ,  $\theta_t = 0$  implies no dependence and  $\theta_t \rightarrow \infty$  a fully dependent relationship. The lower-tail dependence measured by the Clayton copula is  $\lambda_t^L = 2^{-\frac{1}{\theta_t}}$

The SJC copula is Patton's (2006a) modification of the Joe–Clayton (JC) copula. It is more general because the symmetry property of the JC copula is only a special case. The density of the JC copula is:

$$C_{\text{JC}}(u, v | T^U, T^L) = 1 - \left( 1 - \left\{ [1 - (1 - u)^k]^{-\gamma} + [1 - (1 - v)^k]^{-\gamma} - 1 \right\}^{-\frac{1}{\gamma}} \right)^{\frac{1}{k}} \quad (5.18)$$

where  $k = 1/\log_2(2 - T^U)$ ,  $\gamma = -1/\log_2(T^L)$ ,  $T^U \in (0, 1)$  and  $T^L \in (0, 1)$  are the measures of the upper and lower-tail dependencies respectively. The density of the generalized SJC copula is:

$$C_{\text{SJC}}(u, v | T^U, T^L) = 0.5[C_{\text{JC}}(u, v | T^U, T^L) + C_{\text{JC}}(1 - u, 1 - v | T^U, T^V) + u + v - 1] \quad (5.19)$$

The SJC copula is symmetric when  $T^U = T^L$  and asymmetric otherwise.

The dependent process of the time varying Gaussian copula has the following form:

$$\rho_t = \Lambda_1 \left( \omega + \beta \Lambda_1^{-1}(\rho_{t-1}) + \alpha \frac{1}{m} \sum_{i=1}^m \phi^{-1}(U_{1,t-1}) \phi^{-1}(U_{2,t-i}) \right)$$

$$\Lambda_1(x) = \frac{1 - \exp(-x)}{1 + \exp(-x)} \quad (5.20)$$

where  $\Lambda_1(\cdot)$  is a transformation function which holds the correlation parameter  $\rho_t$  in the interval  $(-1,1)$ ,  $\phi(\cdot)$  is the standard normal *cdf* and  $m$  is an arbitrary window length.

The upper and lower-tail dependences of the conditional SJC copula is

as:

$$T^U = \Pi \left( \beta_U^{\text{SJC}} T_{t-1}^U + \omega_U^{\text{SJC}} + \gamma_U^{\text{SJC}} \frac{1}{10} \sum_{i=1}^{10} |u_{t-1} - v_{t-1}| \right)$$

$$T^L = \Pi \left( \beta_L^{\text{SJC}} T_{t-1}^L + \omega_L^{\text{SJC}} + \gamma_L^{\text{SJC}} \frac{1}{10} \sum_{i=1}^{10} |u_{t-1} - v_{t-1}| \right) \quad (5.21)$$

where  $\Pi$  is the logistic transformation to keep  $T^U$  and  $T^L$  within the  $(0, 1)$  interval.

### 5.3 Data description

The data series for this study comprises of 3 series of daily average Air Pollution Index (APIs) during the period from June 5th, 2000 to March 04th, 2010:

API of Shenzhen; API of region to which Shenzhen longs and national API with

3560 observations each series. Data on Shenzhen API comes from the data base of

Ministry of Environmental Protection of the People's Republic of China

(<http://www.zhb.gov.cn/>) (MEPPRC).

The data of regional and national levels are integrated from APIs of the other cities within the region and nation respectively, by calculating inverse distance

weighted average of city APIs for all other cities in the region and in the nation. Figure 5-1 exhibits the plot of APIs for Shenzhen and corresponding regional, national levels. Obvious volatility clustering feature can be noticed in all the three series. JB test shows that normality hypothesis is significantly rejected. Both the ADF unit root tests and PP test show that all the series are statistically significant. Rejecting the hypothesis that there exists unit root.

#### 5.4 Results

Table 5-1 reports the estimation result of the two-step DCC model based on the univariate GJR-GARCH (1, 1) for each series, with the error skewed-t distribution assumption in all cases (We estimated with normal and student-t distribution assumption, skewed-t distribution outperform the other two).  $\theta_1$  and  $\theta_2$ , are statistically significant at 1% level, indicating that the assumption of constant conditional correlation for all shocks to APIs is not supported empirically. Both the condition mean and variance of spring are significantly lower than summer, but are higher than that of winter. The autumn is special, higher in mean but lower in variance compare with spring.

Table 5-2 reports the model specification for marginal distributions. Based on the log-likelihood value and Akaike, Schwarz information criteria, AR(1)-EGARCH model for Shenzhen and AR(1)-GJR-GARCH model for regional and national series outperform other models, with the error skewed-t distribution assumption in all cases.

So an AR-skewed-t-EGARCH model was employed for the marginal distributions of Shenzhen API and an AR-skewed-t-GARCH model employed for regional and national APIs. ARCH effect tests of residual did not reject the null hypothesis of no serial correlation in the squared standardized residuals at 1% level, suggesting that the models listed capture the time varying volatility in the data very well.

Table 5-3 reports the univariate estimation result for each series chosen in previous step. Except several seasonal dummies, other estimates are significant at 1% level. The asymmetric effects in three series are all significant. The parameter  $\gamma$  of the conditional volatility equation in GJR-GARCH model is negative and highly significant, implying that negative shocks (good news) exert smaller impact on regional and national air pollution volatility than positive shocks (bad news) of the same magnitude. Similarly,  $\gamma$  in EGARCH model in Shenzhen API is positive and highly significant, implying that positive shock (bad news) exert bigger impact on Shenzhen air pollution.

From table 5-4, we notice that the log-likelihood of time varying normal copula is higher than that of time varying SJC copula for both Shenzhen-regional estimation and Shenzhen-national estimation. But we hope to examine whether the feature of linear dependence also exists in tail dependence, so we focus on time varying conditional SJC copula, and compare the dependence behaviors implied by DCC, time varying normal copula and time varying SJC copula.

Figure 5-2 presents the plot of time varying dependence implied by DCC-GARCH model. We notice decline of dependence both between Shenzhen and regional, Shenzhen and national air pollution from the end May, 2001 and reach a bottom in October 2004 and then gradually increase to a high level, over 0.7.

In figure 5-3, we present plots of conditional dependence based on the time varying normal copula. Compare with figure 2, the time varying dependence implied by time varying normal copula exhibits a lower dependence during June 2001 to November 2004, but is not that significant as in figure 2.

Figure 5-4 is the plot of conditional tail dependence implied by the time varying SJC copula model. The dynamics of conditional upper and lower tail dependence were confirmed. Further, we notice that both upper and lower tail dependence exhibit a decline of dependence in late spring 2001, and then increase again autumn 2004, which is in conform to what we notice from the dependence implied by DCC-GARCH. This feature is not such clear for the dependence from the time varying normal copula.

Figure 5-5 confirms that for both Shenzhen-regional dependence and Shenzhen-national dependence, lower tail dependence is higher than upper tail dependence most of time, especially during the period with relative low dependence, indicating the existence of asymmetry in tail dependence. Upper (lower) tail dependence measures the dependence between APIs of Shenzhen and regional (or



national) on days when the air pollution deteriorate (improve).

## 5.5 Conclusions

In this study, we examine time varying dependence between air pollution of Shenzhen and corresponding regional, national levels by modeling the conditional dependence structure via copula time varying SJC copula. The Engle DCC and time varying normal copula are estimated as comparison.

Univariate estimations reveal that in all three seasons, seasonal effect exists in both mean and variance equations, with lower mean and variance in summer and higher that in winter. The asymmetric effects are all significant, bad news exert bigger impact on air pollution of Shenzhen, regional and national levels.

The change in dependence during the time period from end May 2001 to October 2004 we found in DCC model also takes place in dependence implied by time varying normal copula and time varying SJC copula. This feature is not very obvious in time varying normal copula, but it is very clear in both upper and lower tail dependence implied by time varying SJC copula. This may imply that the change come mostly from extreme value. Further, the existence of asymmetry is confirmed.

We notice that lower tails are higher than upper tails in both Shenzhen-regional and Shenzhen-national relationships, indicating Shenzhen will benefit from the improved regional and national air quality; the decline of regional and national air quality will affect the contemporaneous air quality of Shenzhen, but with lower impact.



Asymmetry increase after October 2004 and increasing with the level of dependence, suggesting the change of dependence structure over time. DCC, time varying copula and time varying SJC copula all reveal that the conditional dependence between Shenzhen and national is slightly higher than that of Shenzhen and regional.

These results have strong policy implications. When capturing the regional and inter-region relationship, seasonal variation should be taken into consideration.

Spring and Winter exhibit higher volatility, which means higher uncertainty; regional or single city settlement in air pollution control is important but not enough, inter-region cooperation and national decision are important; The cooperation mechanism should be able to respond the time varying nature of conditional correlation; regional heterogeneity should be considered in cooperation policy decision, cooperation among regions with higher correlation and similar correlation feature is better.

Table 5-1 DCC Estimation Results

API	c	S <sub>2</sub>	S <sub>3</sub>	S <sub>4</sub>	AR	ω	S' <sub>2</sub>	S' <sub>3</sub>	S' <sub>4</sub>	ARCH (α)	GARCH (β)	GJR (Gamma)	θ <sub>1</sub>	θ <sub>2</sub>	θ	Q(5)	Q(10)	
Shenzhen	53.44*** (38.13)	-11.27*** (-6.44)	6.75*** (3.44)	10.28*** (4.96)	0.68*** (49.15)	80.90*** (3.86)	-28.84*** (-2.90)	-7.98 (-0.99)	35.91*** (2.86)	0.26*** (4.91)	0.50*** (4.33)	-0.31*** (-5.88)	0.99*** (358.8)	0.01*** (4.70)	3.06***	12.32***		
Regional	49.43*** (40.67)	-10.70*** (-7.24)	1.891 (1.16)	9.70*** (5.82)	0.72*** (54.55)	62.66*** (4.78)	-26.84*** (-3.57)	-10.53 (-1.60)	5.23 (0.76)	0.33*** (6.10)	0.24* (1.86)	-0.28*** (-4.91)			12.32***	26.71		
National	50.89*** (41.90)	-10.68*** (-7.25)	1.47 (0.91)	9.15*** (5.52)	0.73*** (56.60)	52.31*** (4.71)	-23.44*** (-3.65)	-10.12* (-1.84)	4.50 (0.77)	0.33*** (6.09)	0.27** (2.19)	-0.28*** (-4.97)			5.65***	33.84		

Notes: This table reports the estimation results of DCC-GARCH models for the city APIs against regional and national APIs. Absolute t-ratios are indicated in parentheses. The Q(5) and Q(10) are, respectively, the Ljung-Box autocorrelations test (1978) of five and lags in the standardized squared residuals from the regression. \*\*\*, \*\*, \* denote statistical significance at 1%, 5% and 10% level respectively.

Table 5-2 Model Specification for the Marginal Distributions

Shenzhen						
	AR(1)-G arch(1,1) Skew T	AR(1)- Garch(1, 1)- T	AR(1)-E garch Skew T	AR(1) Egarch(1,1) T	AR(1)-J GR(1,1) Skew T	AR(1)- JGR(1,1) T
Log-likelihood	-14359.15	-14374.31	-14301.84	-14326.90	-14312.31	-14338.17
Akaike	8.07	8.08	8.04	8.06	8.05	8.06
Schwarz	8.10	8.10	8.07	8.08	8.07	8.09
ARCH 1-2 test	0.43[0.64]	0.88[0.41]	0.46[0.63]		1.57[0.20]	1.16[0.31]
ARCH 1-5 test	1.20[0.30]	2.18[0.05]	0.38[0.86]		1.24[0.28]	1.14[0.33]
ARCH 1-10 test	1.40[0.17]	1.63[0.08]	1.03[0.40]		1.23[0.26]	1.21[0.27]
Regional						
Log-likelih ood	-12937.2	-12951.75	-	-	-12907.44	-12930.25
Akaike	7.28	7.28			7.26	7.27
Schwarz	7.30	7.30			7.28	7.29
ARCH 1-2 test	0.12[0.88]	0.089[0.9]			0.47[0.620]	0.47[0.621]
ARCH 1-5 test	0.26[0.93]	0.28[0.92]			0.37[0.865]	0.34[0.888]
ARCH 1-10 test	1.42[0.16]	1.38[0.17]			1.81[0.053]	1.73[0.068]
National						
Log-likelih ood	-12700.72	-12712.35	-12664.63	-	-12670.05	-12688.92
Akaike	7.14	7.15	7.12		7.13	7.14
Schwarz	7.17	7.17	7.15		7.15	7.16
ARCH 1-2 test	0.06[0.93]	0.04[0.95]	0.10[0.90]		0.35[0.70]	0.35[0.70]
ARCH 1-5 test	0.27[0.92]	0.30[0.91]	0.33[0.89]		0.35[0.87]	0.31[0.90]
ARCH1-10 test	1.35[0.19]	1.34[0.20]	1.45[0.15]		1.82[0.05]	1.76[0.06]

Note: - no convergence

Table 5-3 Results for the Marginal Distributions

Parameters	Shenzhen	Regional	National
Cst(M)	49.46 <sup>***</sup> (1.18)	50.79 <sup>***</sup> (1.20)	52.96 <sup>***</sup> (1.37)
d2(M)	-11.72 <sup>***</sup> (1.35)	-11.55 <sup>***</sup> (1.36)	-11.58 <sup>***</sup> (1.69)
d3(M)	2.028(1.69)	1.72(1.70)	7.97 <sup>***</sup> (2.10)
d4(M)	10.61 <sup>***</sup> (1.61)	10.10 <sup>***</sup> (1.67)	11.60 <sup>***</sup> (2.07)
AR(1)	0.73 <sup>***</sup> (0.01)	0.75 <sup>***</sup> (0.01)	0.69 <sup>***</sup> (0.01)
Cst(V)	57.51 <sup>***</sup> (9.07)	48.51 <sup>***</sup> (7.94)	5.35 <sup>***</sup> (0.08)
d2(V)	-30.38 <sup>***</sup> (6.23)	-26.08 <sup>***</sup> (5.37)	-0.48 <sup>***</sup> (0.11)
d3(V)	-6.77(5.53)	-6.86(4.63)	-0.02(0.10)
d4(V)	8.92(6.25)	7.79(5.31)	0.48 <sup>***</sup> (0.09)
ARCH(Alpha1)	0.43 <sup>***</sup> (0.06)	0.42 <sup>***</sup> (0.06)	0.13(0.19)
GARCH(Beta1)	0.27 <sup>***</sup> (0.09)	0.30 <sup>***</sup> (0.09)	0.56 <sup>***</sup> (0.14)
GJR(Gamma)		-0.40 <sup>***</sup> (0.06)	-0.39 <sup>***</sup> (0.06)
EGARCH(Gamma)	0.40 <sup>***</sup> (0.03)		

Notes: Standard errors for the estimators are included in parentheses. <sup>\*\*\*</sup>indicate significant at the 1% level.

Table 5-4 Copula Estimation Results

	Shenzhen-Regional	Shenzhen-National
Time-varying normal copula		
Constant	0.0244 (0.16054)	0.0205 (0.23302)
$\alpha$	0.2681 (0.00208)	0.2638 (0.04716)
$\beta$	1.9531 (0.20404)	1.9756 (0.31045)
LL	-595.0485	-624.5461
AIC	1220.63	1281.1339
BIC	1273.20	1338.3642
Time-varying SJC copula		
Constant <sup>U</sup>	1.2044 (0.00368)	1.1301 (0.00043)
$\alpha^U$	-11.1693 (0.00254)	-11.2216 (0.00103)
$\beta^U$	0.3192 (0.00016)	0.4682 (0.00019)
Constant <sup>L</sup>	-1.6446 (0.00013)	-1.7493 (0.17075)
$\alpha^L$	-1.4649 (0.00007)	-1.0727 (0.21451)
$\beta^L$	3.7598 (0.00012)	3.8549 (0.00031)
LL	-657.3164	-697.9039
AIC	1348.3557	1431.613
BIC	1408.6636	1495.6345

Note: AIC: Akaike Information Criteria; BIC: Bayesian Information Criteria; LL-copula log-likelihood. Standard error are in parenthesis.

Figure 5-1 Dynamic Correlation Estimated from DCC-GARCH Model.

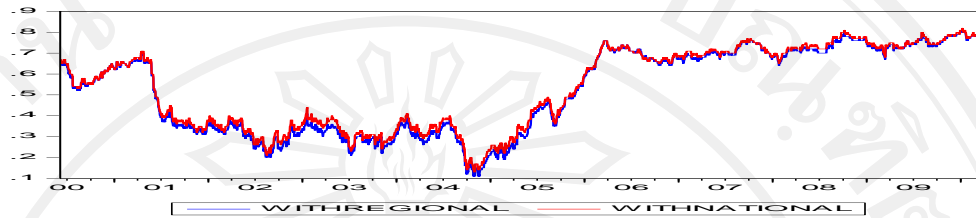


Figure 5-2 Dynamic Correlation Estimated from DCC-GARCH Model.

*Note: Red line displays the implied time paths of the conditional dependence between Shenzhen API and national API, blue line between Shenzhen API and regional API.*

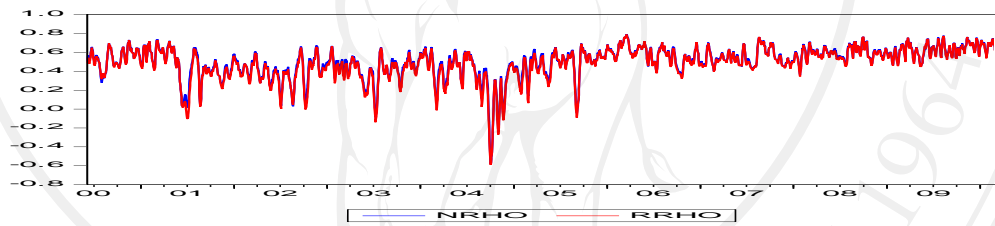
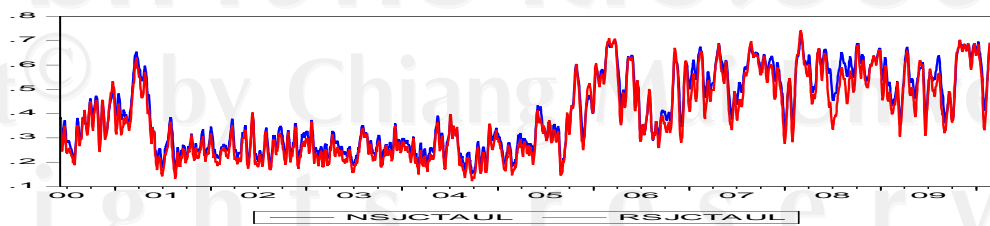


Figure 5-3 Conditional Dependence Implied by Time Varying Normal Copula.

*Note: Red line displays the implied time paths of the conditional dependence between Shenzhen and regional, blue line between Shenzhen and national.*

Conditional dependence in lower tail implied by time varying SJC copula



Conditional dependence in upper tail implied by time varying SJC copula

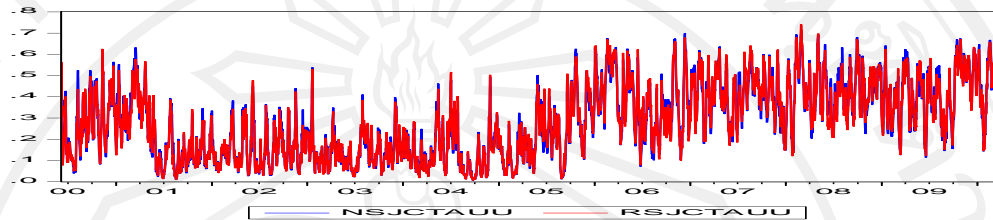
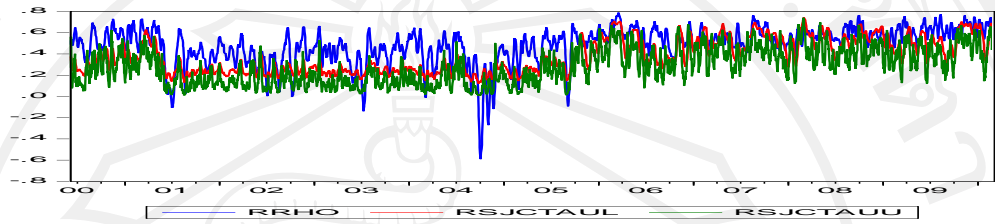


Figure 5-4 Conditional Dependence Implied by Time Varying SJC Copula

Note: Red lines represents the dependence between Shenzhen and regional APIs, while blue lines display dependence between Shenzhen and national APIs.



Conditional dependence between Shenzhen and regional API



Conditional dependence between Shenzhen and national API

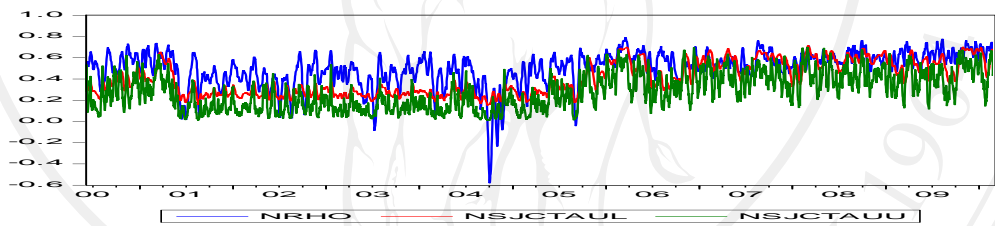


Figure 5-5 Conditional Dependence Estimate from the Copula Models

Note: Blue lines display the time paths of conditional dependence estimated from time varying normal copula. Red lines and blue lines display the lower tail and upper tail dependence implied by time varying SJC copula model respectively.