Chapter 3

Sales forecast with limited observations

3.1 Introduction

Sales forecast of innovative agro-industrial product with limited information is a challenge for both scholars and practitioners. Traditional econometrics usually uses more than 30 observations to construct a time trend. However, practitioners cannot wait for 30 months to have the complete information for such a forecast. In this chapter, we propose a method to use limited information from only 3 observations of the sales values for the forecasting. The forecasts are adjusted according to the product life cycle theory, which suggests an S-shape evolution of sales over time that can be modeled using the logistic function, as suggested by Stoneman (2010). The results of the study will enhance practitioners in agro-industry to accurately forecast the sales of their new products.

3.2 Conceptual framework and literature review

This section will present the product life cycle theory. It will then describe the sales forecasting following the theory and explain the Bass model and the intuition behind it. Finally, the Logistic function will be introduced.

3.2.1 Product Life Cycle Theory

Raymond Vernon developed the product life cycle theory in the 1960s. This theory is used to compare and analyze various stages of products and industries. The introduction stage is the beginning of the sales of a product in the market. The product is not known well to consumers. The product needs to be introduced into the market through various channels such as advertising, etc. This process has high costs and low sales causing slow growth. In the growth stage, total sales rise significantly. The overall business grows rapidly. However, in this stage, it begins to face with new competition. In the maturity stage, the product captures enough market shares and gets stable in the market. Competitors will continue to enter the market. Firms should focus on reducing costs of production and marketing. Competitors are likely to implement marketing strategies to increase their sales. The firms should look for opportunities to develop new products and services to the market. The decline stage is the final stage in the life cycle of product. At this stage, the sales and profits decline until the business is no longer profitable. Firms that may consider shutting down the business should find an innovative way of presenting a new product to beat out the competition. The product life cycle is shown in Figure 3.1

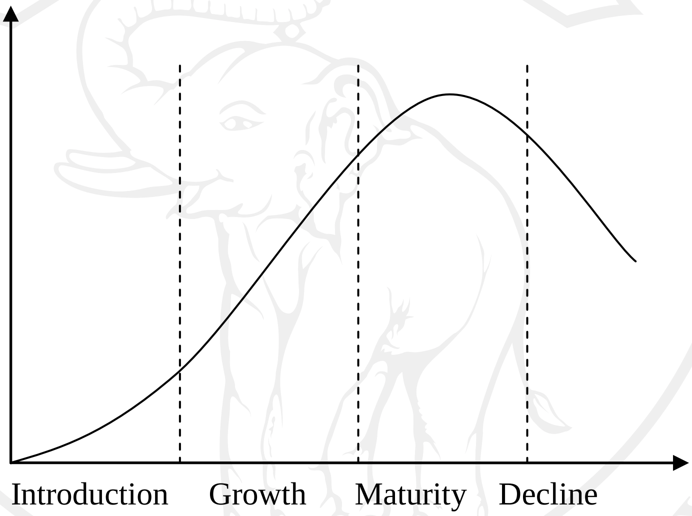

Figure 3.1: Stages of product life cycle

Source : Raymond Vernon (1960)

3.2.2 Sales forecasting

Meade and Islam (2006) mentioned that modeling and forecasting are very important in research about innovation diffusion. They reviewed the development of models since 1970 and the improvement of models in terms of accuracy in predicting or understanding the forecasting problem. They suggested that future forecasting of new product diffusion should be challenged by limited data.

Moreover, in forecasting, there are several functional forms or models to use. The Bass model introduced by Bass (1969) is the most famous one. However, the modern literature such as Stoneman (2010) suggests that the Logistic function may be an alternative functional form for the forecasts. In this chapter, we will attempt to find out which functional form is better for forecasting sales of feta cheese from buffalo milk.

3.2.3 Bass model

Bass (1969) and Srinivasan and Mason (1986) introduced a functional form to forecast sales of new products as follows:

$$V_T = \frac{M(1-\exp(-(p+q)T))}{1+\exp(-\left(\frac{q}{p}\right)(p+q)T)} \tag{3.1}$$

where V_T = Sales of innovative agro-industrial product;

M = Maximum sales of innovative agro-industrial product;

p = Coefficient of innovation;

q = Coefficient of imitation;

T = Time.

To interpret the coefficient of innovation, p, and the coefficient of imitation, q, Meade and Islam (2006) explained that individuals are driven by the need to introduce new things in their daily life and imitate other people in the society. The diffusion of a new product at time t is determined by p+qF(t), where F(t) is the proportion of adopters at time t. There is no pure innovation or imitation, therefore both p and q are positive. The combination of p and q controls the scale of sales whereas the ratio q over p controls the shape of the growth. It should be noted that the ratio q over p must be greater than 1 to ensure that the sales present as S-curve. Higher value of the ratio correspond to faster growth of the sales.

Meade and Islam (2006) also point out some interesting aspects of the ratio q over p as follows:

1) The ratio is positively correlated with collectivism, as people in collectivist societies tend to imitate one another easier than do people in more individualistic societies.

2) The ratio is positively correlated with the hierarchical nature of the culture, as people in the same class tend to use the product at the same time.
3) The ratio is also positively correlated with the domination of male in the society. The reason is unclear, but it may be linked to the discipline of the society, which encourages people to use the same products in a uniform manner.

Rogers (2003) explained the dynamism of the diffusion of a new product into two processes. First, people use the new product because of mass media. Second, they adopt the product by interpersonal communication. He interpreted the coefficient of innovation, p, as an indicator of the first process and the coefficient of imitation, q, as an indicator of the second process. This interpretation is somehow different from that of Meade and Islam (2006), as Rogers linksthe coefficient of innovation to a channel of adoption and not merely to the need to innovate.

In the diffusion process, Rogers (2003) believed that the effect of interpersonal communication is greater than that of the media. In the following figure, the adoption of the new product due to interpersonal communication starts at a higher point. Moreover, it rises overtime until it reaches its peak and then drops due to the decline of popularity of the product. However, the adoption which is influenced by mass media decreases over time after the product fades out of the media.

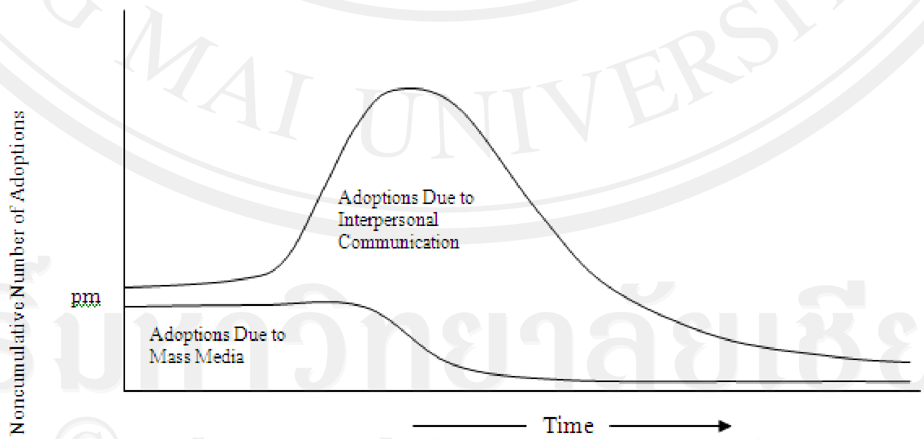

Figure 3.2: Influence of mass media and interpersonal communication on the adoption of a new product

The total effects of both sources are presented in Figure 3.3. The total sales reach a peak when the effect from interpersonal communication rises and the effect from mass media still persists.

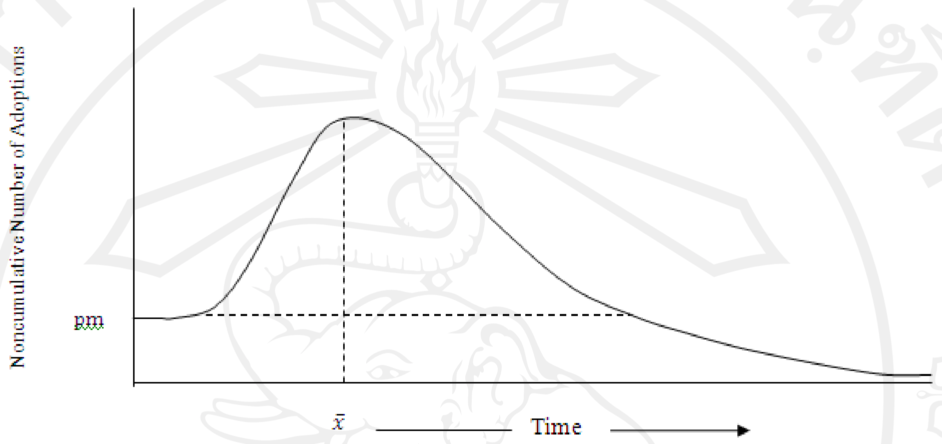

Figure 3.3: Total effects of mass media and interpersonal communication on the adoption of a new product

Overall sales will grow fast at the beginning and then slower overtime until they reach the maximum values at the end. It should be noticed that the growth curve may not present a clear S-curve (Figure 3.4).

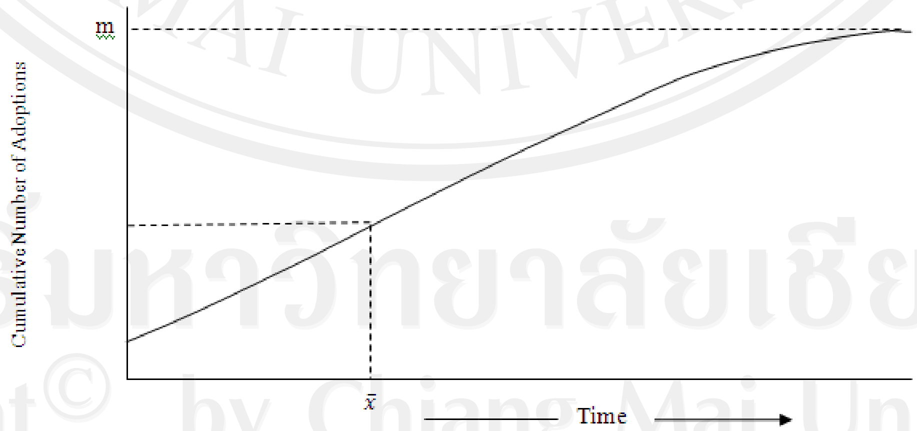

Figure 3.4: Overall sales of a new product over time

3.2.4 Logistic function

For real numbers *a*, *b*, and *c*, the function

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

is a logistic function. If $a>0$, a logistic function increases when $b>0$ and decreases when $b<0$. The coefficient c is called the *limiting value* or the *upper limit* of the function because the graph of a logistic function will have a horizontal asymptote at $y = c$.

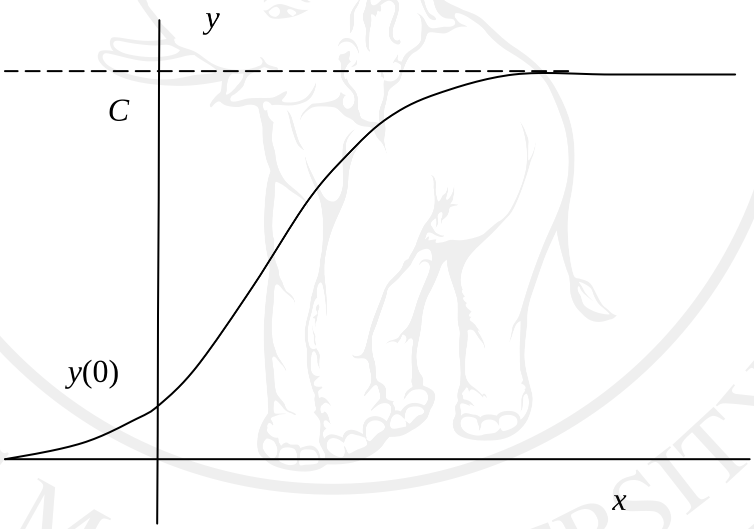

Figure 3.5: S-shaped of the logistic function

The S-shape in the graph of a Logistic function shows that the initial exponential growth is followed by a period in which growth slows and then levels off. The graph approaches but never reaches the maximum upper limit.

Stoneman (2010) suggested to use the logistic function for forecasting sales of new products as follows:

$$V_T = \frac{M}{1+A*\exp(-\beta T)} \tag{3.2}$$

where V_T = Sales of innovative agro-industrial product;

M = Maximum sales of innovative agro-industrial product;

β = Growth parameter;

A = Shift parameter;

T = Time.

3.3 Methodology

The estimation of parameters in the Logistic function can be performed in the four following ways:

Method 1: Least squares using quadratic interpolation algorithm

The parameter estimation includes the following steps.

Step 1: Set three initial values of parameter M. Transform the data by the Logistic transformation into a linear function.

$$ln\left(\frac{V_T/M}{1-V_T/M}\right) - \ln\left(\frac{1}{A}\right) = \beta T \tag{3.3}$$

Then, estimate parameter β using Ordinary Least Squares (OLS)

Step 2: Take parameter M and β to forecast sales by this formula.

$$V_T = \frac{M}{1 + A * \exp(-\beta T)}$$

The value of A is calculated by the following formula to fix the y-intercept at the first data of the series (Vo):

$$A = \frac{M}{V_0} - 1 \tag{3.4}$$

Step 3: Calculate the Sum Squared Error (SSE).

$$\sum e^2 = \sum_{i=1}^{N}\left(V_T - \widehat{V_T}\right) \tag{3.5}$$

Step 4: Calculate the SSE at the three points using the three initial M values.

Step 5: Search for a new M value by Quadratic Interpolation

Step 6: Include the new M with other two previous M values which are located nearest to the new M. Then, estimate parameter β and calculate the SSE again.

Step 7: Repeat steps 5 and 6 for 10,000 iterations.

Step 8: Summarize the values of parameter M and β.

It should be remarked that the estimation of β using OLS with logistic transformation may suffer from the heteroscedasticity problem (Judge et al, 1986). This study will address this problem using the Estimated Generalized Least Squares (EGLS which was also suggested by Judge et al (1986).

Method 2: Least squares using Quasi-Newton algorithm

The parameter estimation includes the following steps.

Step 1: Repeat steps 1 to 4 of method 1 (Least squares using quadratic interpolation algorithm). This will yield the values of M, β and SSE. Each parameter will contain three values.

Step 2: Calculate the slope between the values of M, β and SSE. Two slopes will be available for each parameter.

Step 3: Set the initial value of H (Ho) to the identity matrix with size of 2 ×2.

Step 4: Calculate a new H using the following formula:

$$H = H_o + \frac{vv'}{v'u} - \frac{H_o uu' H_o}{u' H_o u} \tag{3.6}$$

where v = Difference of the parameter;

u = Difference of the slope of the parameter.

Step 5: Calculate the increment of the parameter as follows:

$$d = -Hg \tag{3.7}$$

where d = The increment of the parameter;

g = Initial slope of the parameter.

Step 6: Calculate a new parameter by adding the increment to the previous parameter.

Step 7: Create two nearby values for parameter M. Repeat the process for parameter β.

Step 8: Calculate the SSE from the new parameter M and β.

Step 9: Repeat steps 4 to 8 for 10,000 iterations.

Step10: Summarize the values of parameter M and β.

Method 3: Maximum likelihood using quadratic interpolation algorithm

This method is similar to the least squares method using quadratic interpolation, except that. the objective function is now the likelihood function defined as follows:

$$L = \prod_{i=1}^{T} Pr(V_T|T) \tag{3.8}$$

and

$$Pr(V_T|T) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\left(-\frac{1}{2}\right)\frac{(V_T-F_T)^2}{\sigma^2}\right\} \tag{3.9}$$

where $Pr(V_T|T)$ = Probability of occurrence of a sales value at a time;

σ^2 = Variance;

V_T = Sales value;

F_T = Forecasted sales value.

It should be noted that, in this study, the normal distribution is assumed for the likelihood function. This is because when the new product is launched to the market, researchers have only limited information on its sales. No one really knows the true distribution of the sales at time T. However, for the calculation of the likelihood function, a distribution has to be assumed. The normal distribution is a simple model with a symmetry property. So, readers of this thesis should keep in mind that the predictions are based on the normality assumption. Moreover, the study has to assume that the variance (σ^2) is constant over time. This is due to the limited information on the sales of the innovative product too.

Method 4: Maximum likelihood using the Quasi-Newton Algorithm

This method is quite similar to method 3 (Maximum likelihood using quadratic interpolation algorithm). The objective function is the same, but the Quasi-Newton algorithm is used in place of the quadratic interpolation algorithm.

It should be noted that all the observations included in the study are deseasonalized data. This is to avoid the fluctuation of the seasonal effect of the sales. Moreover, it makes the smoothing of the model easier. In practice, after the forecasts are obtained, practitioners can add the seasonal effect to the forecasted volume to adjust for the sales in different seasons.

It also should be remarked that the study will use the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) to measure goodness of fit of the models. The in-sample data will be used for the calculation. Both criteria will be used in order to evaluate the consistency between them; this will allow usto double-check whether the results are consistent. However, to assess the accuracy of the prediction, we will use the Mean Absolute Percentage Error (MAPE). This indicator is more convenient to let analysts see which percentage of the forecasts miss the targets. It will be based on the out-sample data for the calculation.

3.4 Analysis

The study will conduct 6 major analyses as follows:

Analysis 1: Comparison between OLS and EGLS

This analysis will compare OLS and EGLS to estimate parameter β of the Logistic function after transformation into a linear model. The initiative is from the suggestion of Judge et al (1986) that OLS estimation may suffer from heteroscedasticity in the estimation of the logistic transformed function while EGLS may reduce this problem.

Analysis 2: Comparison between fixed and floating y-intercept

In the estimation of the model, it may be better to fix the y-intercept to be the first observation to improve the accuracy of the forecast. This analysis will compare the fixed y-intercept method with the traditional floating y-intercept in terms of out-of-sample MAPE.

Analysis 3: Sufficient observations for the sales forecast

This analysis will estimate the logistic function using different numbers of observations. It will then compare the Mean Absolute Percentage Error (MAPE) from the out-of-sample test to find the best model that uses the smallest number of observations. It will determine how many observations are needed to make the sales forecast accurate.

Analysis 4: Comparison between functional forms: Bass Model vs Logistic function

This analysis will compare two different functional forms: the classical Bass model and the Logistic function. It will use MAPE as a measure of accuracy. The t-test will be used to compare the average MAPE between models. MAPE will be estimated from the out-of-sample tests when the number of observations is varied for each model.

Analysis 5: Comparison between global and local forecasts: Cumulative observations vs. Rolling windows

This analysis will estimate the models by adding a number of observations to the previous set of observations, which is called the process of cumulative observations. This process includes the cumulative knowledge from the past. As an alternative approach, we will also estimate the model by using the rolling windows method, which deletes the oldest data after adding the newest ones. The analysis will compare both approaches in terms of MAPE.

Analysis 6: Comparison between estimation algorithms: Quasi-Newton, Gauss-Newton and Newton-Raphson

This analysis will compare estimation algorithms, namely, Quasi-Newton versus Gauss-Newton and Quasi-Newton versus Newton-Raphson. Gauss-Newton is well-known for its use in the non-linear least squares and Newton-Raphson is also widely used to find the maximum likelihood. However, Quasi-Newton can be used in place of both least squares and maximum likelihood. Therefore, it is interesting to compare these approaches in terms of forecasting accuracy.

It should be noted that this study focuses on only one product, the feta cheese from buffalo milk. It does not include other competitive products, i.e., feta cheese of other brands or other kinds of cheese. Therefore, it is a univariate analysis. It would be interesting to include competitors to make the study cover multivariate analysis. However, this is beyond the scope of this study and it is left for further research.

Moreover, it should also be noted that all parameters in the model are assumed to be constant over time, i.e., time-invariant.

3.5 Data

The data are obtained from the Royal Project Foundation. They are monthly sales of feta cheese. The data cover the period during January 2010 to August 2012. Totally, the model has 32 observations.

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3.6 Results

The results will be presented in six parts. First, the results from the comparison between OLS and EGLS will be reported. Second, the study will display the output of the comparison between the methods of fixed and floating y-intercept. Next, it will illustrate the results of the comparison between the Bass and Logistic models. Forth, the minimum number of observations needed for the accurate prediction of sales will be estimated. Fifth, we will figure out whether the method of rolling windows or cumulative observations provides a better forecast. Finally, the performances of the Quasi-Newton, Gauss-Newton and Newton-Raphson algorithms will be compared.

Analysis 1 : Comparison between OLS and EGLS

This section will show the results of the comparison between OLS and EGLS. Table 3.1 displays the results from the estimation of the Logistic function using OLS and quadratic interpolation. For comparison, the results presented in Table 5.2 use the EGLS estimation method. All calculations have been performed using Matlab.

The related indicators which appear in the tables are as follows:

Mstar = Peak sales estimate;

Beta = Growth parameter;

SSE = Sum Squared Error computed with in-sample observations;

MAPE = Mean Absolute Percentage Error measured by the out-of-sample test;

AIC = Akaike Information Criterion;

BIC = Bayesian Information Criterion.

It should be noted that the number of observations begins at 3 to ensure that the estimated curve is non-linear. The total number of observations is 32. The last row of the table does not present the MAPE because there is no observation left outside the model.

Table 3.1: Estimation results for the Logistic function using OLS and quadratic interpolation

No. of obs	Mstar	Beta	SSE (Million)	MAPE	AIC	BIC
3	1,801,249	0.2028	828	46.77	20.10	19.80
4	1,479,900	0.1343	965	46.49	19.80	19.65
5	1,481,169	0.1248	987	48.10	19.50	19.42
6	1,803,815	0.0980	1,110	49.09	19.37	19.33
7	1,154,894	0.0420	2,057	43.42	19.78	19.78
8	1,154,819	0.0477	2,013	45.38	19.59	19.60
9	1,170,708	0.0657	2,130	47.97	19.50	19.53
10	1,154,446	0.0486	2,501	45.96	19.54	19.57
11	1,152,311	0.0414	2,732	45.29	19.51	19.55
12	1,152,992	0.0432	2,703	47.69	19.40	19.44
13	1,151,351	0.0365	2,988	46.62	19.41	19.45
14	1,147,465	0.0492	5,261	49.36	19.89	19.93
15	1,156,423	0.0527	6,002	50.58	19.94	19.99
16	1,156,741	0.0520	6,247	51.80	19.91	19.96
17	1,153,404	0.0503	6,400	53.22	19.86	19.91
18	1,471,196	0.0431	6,379	53.38	19.80	19.85
19	1,474,993	0.0468	8,795	54.26	20.06	20.11
20	1,159,226	0.0496	11,191	54.90	20.24	20.29
21	1,481,511	0.0480	11,634	55.94	20.23	20.28
22	1,173,916	0.0501	14,895	56.08	20.42	20.47
23	1,253,215	0.0507	17,641	55.86	20.54	20.59
24	1,188,729	0.0502	19,173	56.12	20.58	20.63
25	1,100,433	0.0523	27,163	54.44	20.89	20.94
26	1,805,585	0.0484	26,768	57.26	20.83	20.88
27	1,807,158	0.0477	28,475	57.46	20.85	20.90
28	1,814,071	0.0475	31,563	56.29	20.91	20.96
29	2,227,125	0.0468	34,114	54.64	20.95	21.00
30	1,837,872	0.0474	40,498	47.21	21.09	21.14
31	3,407,578	0.0455	41,516	39.35	21.08	21.13
32	4,157,133	0.0434	41,343	-	21.04	21.09

Tables 3.1 and 3.2 show the estimated parameters Mstar and Beta as well as the related SSE, MAPE, AIC and BIC criteria when estimating the Logistic function using, respectively, the OLS and EGLS methods with quadratic interpolation.

Table 3.2: Estimation results for the Logistic Model using EGLS and quadratic interpolation

No. of obs	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
3	-	-	-	-	-	-
4	-	-	-	-	-	-
5	-	-	-	-	-	-
6	-	-	-	-	-	-
7	-	-	-	-	-	-
8	-	-	-	-	-	-
9	972,781	-	-	-	-	-
10	750,814	-	-	-	-	-
11	-	-	-	-	-	-
12	425,039	-	-	-	-	-
13	998,975	-	-	-	-	-
14	700,842	-	-	-	-	-
15	260,452	0.0384	5,423	48.67	19.84	19.89
16	299,801	-	-	-	-	-
17	558,969	0.1215	7,733	59.29	20.05	20.10
18	253,201	0.1329	7,795	63.92	20.00	20.05
19	480,827	0.0535	8,888	56.31	20.07	20.12
20	395,604	0.0581	11,551	57.22	20.27	20.32
21	344,415	0.0607	12,325	59.14	20.29	20.34
22	508,918	0.0599	15,663	58.37	20.47	20.52
23	987,656	0.0578	18,347	57.43	20.58	20.63
24	524,175	0.0538	19,619	57.18	20.61	20.65
25	942,445	0.0504	26,892	54.03	20.88	20.92
26	412,361	0.0325	23,772	50.88	20.71	20.76
27	384,275	0.0518	29,439	58.89	20.88	20.93
28	876,712	0.0470	31,554	56.34	20.91	20.96
29	130,547	0.0890	41,955	61.92	21.16	21.21
30	1,116,843	0.0564	42,685	49.69	21.14	21.19
31	804,151	0.0529	43,634	42.06	21.13	21.18
32	404,811	0.0486	43,377	-	21.09	21.14

It should be noted that the EGLS method does not work well and does not yield reliable estimation results with less than 17 observations. This is possibly caused by ratios V/M and 1-V/M that are too different. The ratio V/M with less than 17 observations may be very small and the ratio 1-V/M, therefore, is too large. This large difference may make the odd-ratio close to zero, causing log-odds to approach minus infinity.

For accuracy, the comparison should use the same number of observations, so that both methods produce proper results. To this end, observations 1-14 as well as observation 17 were deleted in both OLS and EGLS results.

Table 3.3: Data for the comparison between OLS and EGLS

Number of observations	OLS					
	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
15	1,156,423	0.0527	6,002	50.58	19.94	19.99
18	1,471,196	0.0431	6,379	53.38	19.80	19.85
19	1,474,993	0.0468	8,795	54.26	20.06	20.11
20	1,159,226	0.0496	11,191	54.90	20.24	20.29
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30	1,837,872	0.0474	40,498	47.21	21.09	21.14
31	3,407,578	0.0455	41,516	39.35	21.08	21.13
32	4,157,133	0.0434	41,343	-	21.04	21.09

Number of observations	EGLS					
	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
15	260,452	0.0384	5,423	48.67	19.84	19.89
18	253,201	0.1329	7,795	63.92	20.00	20.05
19	480,827	0.0535	8,888	56.31	20.07	20.12
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31	804,151	0.0529	43,634	42.06	21.13	21.18
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Table 3.4: Descriptive statistics of MAPE, AIC and BIC from OLS and EGLS

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	OLSMAPE	53.5847	15	4.75	1.23
	EGLSMAPE	55.4700	15	5.61	1.45
Pair 2	OLSAIC	20.5906	16	.428	.10706
	EGLSAIC	20.6269	16	.434	.10841
Pair 3	OLSBIC	20.6406	16	.428	.10706
	EGLABIC	20.6756	16	.433	.10832

Source: Calculation using SPSS version 11.0.

Table 3.4 shows that MAPE of OLS is smaller than that of EGLS. It also shows that AIC and BIC of OLS are slightly smaller than those of EGLS.

Table 3.5: Comparison of MAPE, AIC and BIC between OLS and EGLS using the t-test

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	OLSMAPE EGLSMAPE	-1.885	3.775	.975	-3.976	.205	-1.93	14	.074
Pair 2	OLSAIC EGLSAIC	-.036	.084	.021	-.081	.009	-1.73	15	.104
Pair 3	OLSBIC EGLABIC	-.035	.084	.021	-.080	.010	-1.66	15	.118

Source: Calculation using SPSS version 11.0.

Table 3.5 reveals that OLS is better than EGLS as it yields a smaller MAPE. This result is statistically significant at the confidence level of 90%. However, OLS and EGLS do not produce significantly different AIC and BIC.

It should be noted that the t-test is valid in this case because of the normality assumption that was made earlier. Therefore, with this assumed normal distribution, the use of the t-test is acceptable.

Analysis 2: Comparison between fixed and floating y-intercept

In this analysis, the study will compare the performance of the Logistic function using fixed and floating y-intercept. The main estimation method is OLS with quadratic interpolation. The results are shown in Tables 3.6 to 3.9.

Table 3.6: Estimation results for the Logistic function using OLS and quadratic interpolation and fixed y-intercept

No. of Obs	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
3	1,801,249	0.2028	828	46.77	20.10	19.80
4	1,479,900	0.1343	965	46.49	19.80	19.65
5	1,481,169	0.1248	987	48.10	19.50	19.42
6	1,803,815	0.0980	1,110	49.09	19.37	19.33
7	1,154,894	0.0420	2,057	43.42	19.78	19.78
8	1,154,819	0.0477	2,013	45.38	19.59	19.60
9	1,170,708	0.0657	2,130	47.97	19.50	19.53
10	1,154,446	0.0486	2,501	45.96	19.54	19.57
11	1,152,311	0.0414	2,732	45.29	19.51	19.55
12	1,152,992	0.0432	2,703	47.69	19.40	19.44
13	1,151,351	0.0365	2,988	46.62	19.41	19.45
14	1,147,465	0.0492	5,261	49.36	19.89	19.93
15	1,156,423	0.0527	6,002	50.58	19.94	19.99
16	1,156,741	0.0520	6,247	51.80	19.91	19.96
17	1,153,404	0.0503	6,400	53.22	19.86	19.91
18	1,471,196	0.0431	6,379	53.38	19.80	19.85
19	1,474,993	0.0468	8,795	54.26	20.06	20.11
20	1,159,226	0.0496	11,191	54.90	20.24	20.29
21	1,481,511	0.0480	11,634	55.94	20.23	20.28
22	1,173,916	0.0501	14,895	56.08	20.42	20.47
23	1,253,215	0.0507	17,641	55.86	20.54	20.59
24	1,188,729	0.0502	19,173	56.12	20.58	20.63
25	1,100,433	0.0523	27,163	54.44	20.89	20.94
26	1,805,585	0.0484	26,768	57.26	20.83	20.88
27	1,807,158	0.0477	28,475	57.46	20.85	20.90
28	1,814,071	0.0475	31,563	56.29	20.91	20.96
29	2,227,125	0.0468	34,114	54.64	20.95	21.00
30	1,837,872	0.0474	40,498	47.21	21.09	21.14
31	3,407,578	0.0455	41,516	39.35	21.08	21.13
32	4,157,133	0.0434	41,343	-	21.04	21.09

Table 3.7: Estimation results for the Logistic function using OLS and quadratic interpolation and floating y-intercept

No. of Obs	Mstar	beta	SSE (Billion)	MAPE	AIC	BIC
3	14,206,418	0.1533	128,742	4,090.07	32.06	31.76
4	17,506,575	0.0768	271,120	8,567.81	32.35	32.19
5	17,474,348	0.0810	321,718	7,912.65	32.20	32.12
6	17,467,623	0.0521	398,455	10,010.27	32.16	32.13
7	17,498,133	-0.0273	577,603	18,651.75	32.33	32.32
8	17,487,017	-0.0028	613,993	15,285.95	32.22	32.23
9	14,146,111	0.0366	384,687	8,245.69	31.61	31.63
10	17,448,404	0.0129	713,254	12,454.42	32.10	32.13
11	17,445,629	0.0063	805,493	12,746.84	32.11	32.14
12	17,430,938	0.0147	834,249	11,324.17	32.04	32.08
13	17,432,294	0.0070	940,762	11,624.54	32.07	32.11
14	14,088,671	0.0325	554,070	6,779.44	31.45	31.50
15	14,047,217	0.0405	547,368	5,924.48	31.36	31.41
16	14,026,344	0.0405	569,575	5,721.50	31.33	31.38
17	14,012,036	0.0387	600,164	5,652.03	31.31	31.36
18	14,021,505	0.0275	688,650	5,993.83	31.39	31.44
19	13,977,607	0.0356	659,804	5,198.35	31.28	31.33
20	13,932,661	0.0406	643,819	4,696.98	31.20	31.25
21	13,917,851	0.0392	670,183	4,601.10	31.19	31.24
22	13,871,753	0.0429	656,660	4,270.34	31.12	31.17
23	13,833,293	0.0446	654,673	4,097.41	31.07	31.12
24	13,811,502	0.0439	670,643	4,062.48	31.04	31.09
25	13,741,956	0.0476	646,863	3,892.09	30.96	31.01
26	13,752,916	0.0423	702,751	3,928.40	31.00	31.05
27	13,734,692	0.0413	721,721	3,927.28	30.99	31.04
28	13,707,001	0.0413	729,553	3,979.00	30.96	31.01
29	13,685,166	0.0407	743,460	4,114.18	30.94	30.99
30	13,642,973	0.0417	737,834	4,657.77	30.90	30.95
31	13,636,511	0.0397	769,074	5,512.14	30.91	30.95
32	13,642,970	0.0361	822,552	0.00	30.94	30.99

In Table 3.8 it is clear that the MAPE of the model with fixed y-intercept is much lower than that of the floating one. Moreover, the AIC and BIC of the model with fixed y-intercept are also lower.

These differences of the means between both models are statistically significant. In Table 3.9 the t-tests show that the means are significantly different at the 99% confidence level. Therefore, the model with fixed y-intercept can be considered better than the floating y-intercept.

Table 3.8: Descriptive statistics of MAPE, AIC and BIC from the estimation methods with fixed and floating y-intercept

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	FIXMAPE	50.722	29	4.850	.901
	FLOATMAP	7169.757	29	3905.065	725.152
Pair 2	FIXAIC	20.154	30	.587	.107
	FLOATAIC	31.486	30	.520	.095
Pair 3	FIXBIC	20.172	30	.611	.112
	FLOATBIC	31.504	30	.481	.088

Source: Calculation using SPSS version 11.0.

Table 3.9: Comparison of MAPE, AIC and BIC for the estimation methods with fixed and floating y-intercept

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	FIXMAPE FLOATMAP	-7119.036	3908.36	725.765	-8605.698	-5632.373	-9.81	28	.000
Pair 2	FIXAIC FLOATAIC	-11.333	1.06	.194	-11.731	-10.935	-58.27	29	.000
Pair 3	FIXBIC FLOATBIC	-11.332	1.06	.194	-11.729	-10.934	-58.28	29	.000

Source: Calculation using SPSS version 11.0.

Analysis 3: Sufficient number of observation

1) Sales forecasts with least squares using quadratic interpolation algorithm

When using the least squares method with quadratic interpolation algorithm to estimate parameter M*, the MAPE drops sharply when 7 observations are present in the model. After that it gradually rises after the 27th observation.

Table 3.10: Least squares with quadratic interpolation and fixed intercept At Vo

N	M*	Beta	Out-sample test (MAPE)
3	1.80E+06	0.2028	936.1434
4	1.48E+06	0.1343	371.2554
5	1.48E+06	0.1248	321.8111
6	1.80E+06	9.80E-02	185.2782
7	1.15E+06	0.042	26.9965
8	1.15E+06	0.0477	3.02E+01
9	1.17E+06	0.0657	60.8896
10	1.15E+06	0.0486	30.1602
11	1.15E+06	0.0414	27.0777
12	1.15E+06	0.0432	28.0434
13	1.15E+06	0.0365	2.98E+01
14	1.15E+06	4.92E-02	30.4141
15	1.16E+06	0.0527	36.0846
16	1.16E+06	0.052	36.7475
17	1.15E+06	0.0503	35.8103
18	1.47E+06	0.0431	25.4413
19	1.48E+06	0.0468	27.5098
20	1.16E+06	0.0496	31.3205
21	1.48E+06	0.048	30.9512
22	1.17E+06	0.0501	34.8615
23	1.25E+06	0.0507	39.3213
24	1.19E+06	0.0502	41.5403
25	1.10E+06	0.0523	48.9638
26	1.81E+06	0.0484	34.4178
27	1.81E+06	0.0477	35.6350
28	1.81E+06	0.0475	42.9912
29	2.23E+06	0.0468	51.6853
30	1.84E+06	0.0474	74.1752
31	3.41E+06	0.0455	95.4737
32	4.16E+06	0.0434	

Source: Own calculation

2) Sales forecasts with least squares using quasi-Newton algorithm

When using the least squares method with Quasi-Newton algorithm to estimate parameter M*, we observe the same effect as previously, i.e., the MAPE drops sharply from 7 observations and then gradually rises after the 24th observation.

Table 3.11: Least squares with Quasi-Newton and fixed intercept at Vo

N	M*	Beta	SSE	Out-sample test (MAPE)
3	1.77E+06	0.2006	3.14E+12	9.09E+02
4	1.77E+06	0.1325	3.14E+12	3.81E+02
5	1.78E+06	0.1227	3.16E+12	3.25E+02
6	1.77E+06	0.0968	3.16E+12	1.79E+02
7	1.78E+06	0.0408	3.17E+12	2.71E+01
8	1.78E+06	0.0464	3.17E+12	2.98E+01
9	1.82E+06	0.0637	3.31E+12	59.2146
10	1.78E+06	0.0473	3.18E+12	2.97E+01
11	1.77E+06	0.0403	3.15E+12	27.2792
12	1.77E+06	0.0421	3.16E+12	2.80E+01
13	1.77E+06	0.0357	3.14E+12	2.99E+01
14	1.76E+06	0.048	3.13E+12	2.99E+01
15	1.79E+06	0.051	3.20E+12	3.47E+01
16	1.79E+06	0.0503	3.20E+12	35.6878
17	1.78E+06	0.0489	3.18E+12	3.50E+01
18	1.75E+06	0.0429	3.13E+12	2.55E+01
19	1.77E+06	0.046	3.15E+12	2.68E+01
20	1.79E+06	0.0479	3.22E+12	3.00E+01
21	1.78E+06	0.0469	3.19E+12	2.98E+01
22	1.82E+06	0.0483	3.31E+12	33.0224
23	1.90E+06	0.0486	3.63E+12	36.3320
24	1.85E+06	0.0483	3.43E+12	39.0104
25	1.68E+06	0.0504	2.81E+12	4.60E+01
26	1.79E+06	0.0476	3.22E+12	3.22E+01
27	1.79E+06	0.047	3.23E+12	33.7473
28	1.80E+06	0.0467	3.26E+12	4.09E+01
29	1.80E+06	0.0462	3.28E+12	49.5553
30	1.83E+06	0.0463	3.38E+12	6.87E+01
31	1.81E+06	0.0454	3.31E+12	9.05E+01
32	1.78E+06	0.044	3.24E+12	

Source: Own calculation

3) Sales forecasts with maximum likelihood using quadratic interpolation algorithm

When using the maximum likelihood method with quadratic interpolation algorithm to estimate parameter M*, we observe again the same phenomenon as with the previous two methods, i.e., a drop of MAPE drops from 7 observations and a gradual increase after the 24th observation.

Table 3.12: Maximum likelihood with quadratic interpolation and fixed intercept at Vo

N	M*	Beta	Out-sample test (MAPE)
3	1.15E+06	0.2049	710.7455
4	1.15E+06	0.1352	341.249
5	1.15E+06	0.1257	299.5281
6	1.15E+06	0.099	172.7165
7	1.15E+06	0.042	26.9965
8	1.15E+06	4.77E-02	30.1830
9	1.17E+06	0.0657	60.8877
10	1.15E+06	0.0486	30.1596
11	1.15E+06	4.14E-02	27.0777
12	1.15E+06	0.0432	28.0433
13	1.15E+06	3.65E-02	29.7533
14	2.69E+06	0.0482	30.7983
15	2.20E+06	0.0517	36.7008
16	2.20E+06	0.0511	37.3895
17	2.19E+06	0.0495	36.282
18	4.05E+06	0.0423	25.6119
19	5.00E+06	0.0458	27.9018
20	4.11E+06	0.0482	31.9772
21	5.03E+06	0.047	31.4440
22	3.40E+06	0.0488	35.5675
23	1.94E+06	0.05	39.7354
24	5.25E+06	0.0487	42.5036
25	3.12E+06	0.0508	50.3024
26	1.40E+07	0.0473	35.0448
27	2.10E+07	0.0465	36.2831
28	2.10E+07	0.0463	43.6599
29	2.57E+07	0.0458	52.2262
30	2.14E+07	0.0462	75.3084
31	4.67E+07	0.0449	96.1237
32	6.60E+07	0.0429	

Source: Own calculation

4) Sales forecasts with maximum likelihood using quasi-Newton algorithm

When using the maximum likelihood method with quasi-Newton algorithm to estimate parameter M*, the MAPE again drops sharply when 7 observations presents in the model and then gradually rises after the 24th observation..

Table 3.13: Maximum likelihood with Quasi-Newton and fixed intercept at Vo

N	M*	Beta	Likelihood	Out-sample test (MAPE)
3	1.88E+06	0.2123	2.28E-15	1.04E+03
4	2.11E+06	0.1573	2.96E-20	622.3783
5	1.80E+06	0.1244	3.93E-25	338.1848
6	1.90E+06	0.1033	5.06E-30	215.7572
7	1.80E+06	0.0413	5.45E-35	27.0290
8	1.79E+06	4.68E-02	7.22E-40	30.1028
9	1.82E+06	0.0638	0.0638	59.5629
10	1.79E+06	0.0475	9.03E-50	29.9117
11	1.79E+06	0.0405	1.13E-54	27.1930
12	1.79E+06	0.0422	1.50E-59	28.0026
13	1.79E+06	3.58E-02	1.83E-64	29.8949
14	1.77E+06	0.0478	8.92E-70	29.7504
15	1.79E+06	0.051	1.15E-74	34.8033
16	1.79E+06	0.0504	1.50E-79	35.7265
17	1.78E+06	0.0488	1.88E-84	34.9676
18	1.78E+06	0.0424	1.48E-89	25.3802
19	1.78E+06	0.0458	1.40E-94	26.6273
20	1.79E+06	0.0479	1.62E-99	30.0215
21	1.78E+06	0.0469	1.96E-104	29.6755
22	1.82E+06	0.0483	2.28E-109	33.0818
23	1.90E+06	0.0487	3.00E-114	36.4223
24	1.85E+06	0.0483	3.77E-119	39.1461
25	1.68E+06	0.0504	2.90E-124	46.0518
26	1.78E+06	0.0475	1.23E-129	31.8652
27	1.78E+06	0.0468	1.42E-134	33.4098
28	1.78E+06	0.0467	1.86E-139	40.7572
29	1.78E+06	0.0462	2.26E-144	49.6259
30	1.81E+06	4.64E-02	2.86E-149	6.93E+01
31	1.78E+06	0.0455	2.09E-154	91.3084
32	1.74E+06	0.044	7.32E-160	

Source: Own calculation

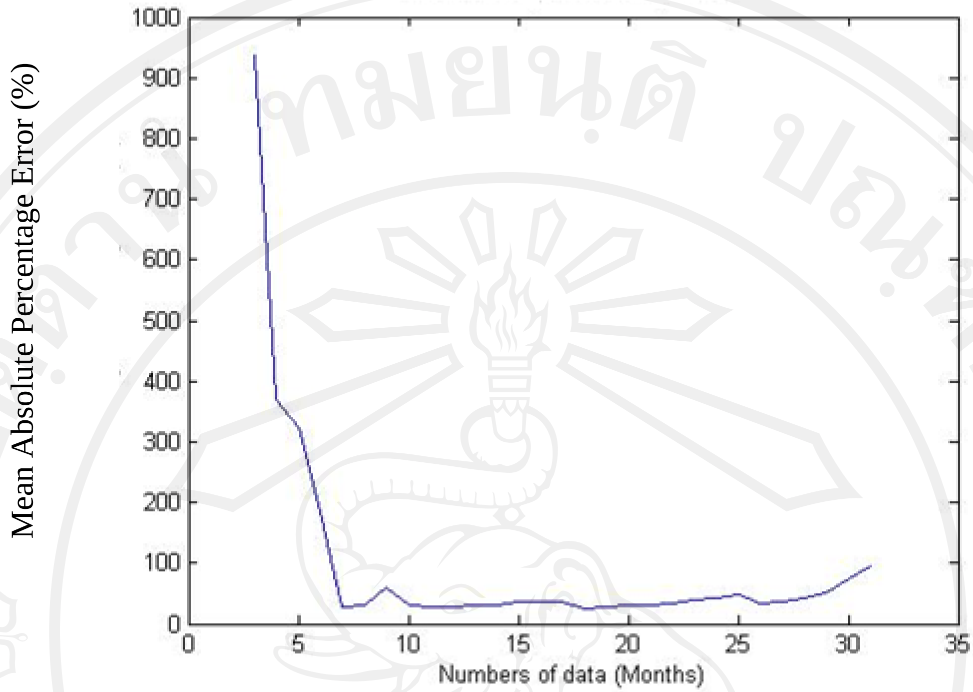

Figure 3.6: Mean Absolute Percentage Error of the Least squares with quadratic interpolation and fixed intercept At Vo

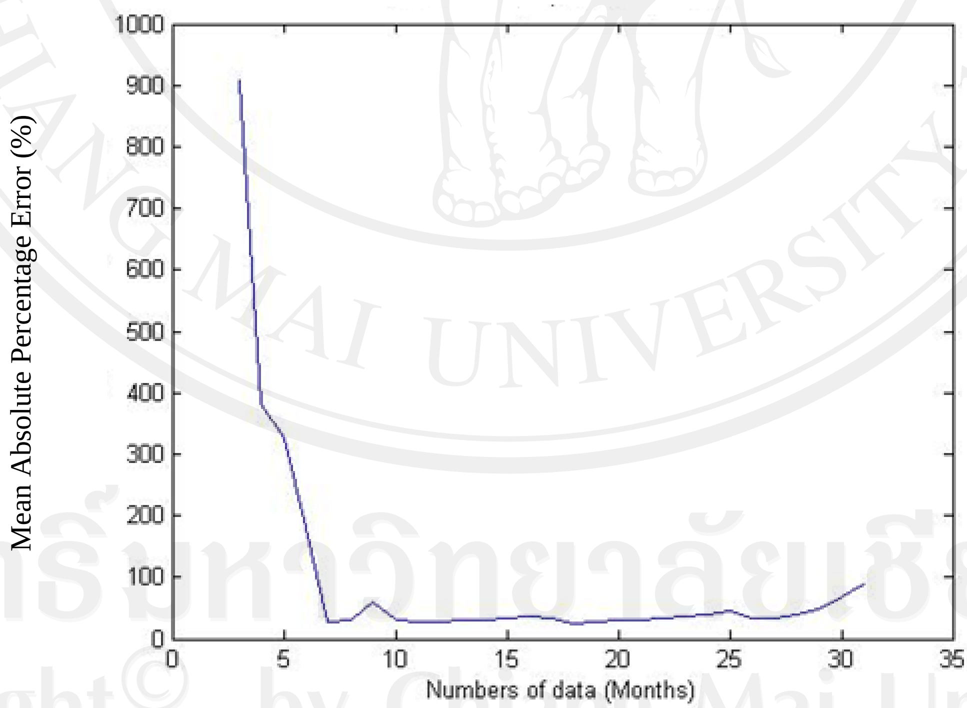

Figure 3.7: Mean Absolute Percentage Error of the Least squares with Quasi-Newton and fixed intercept at Vo

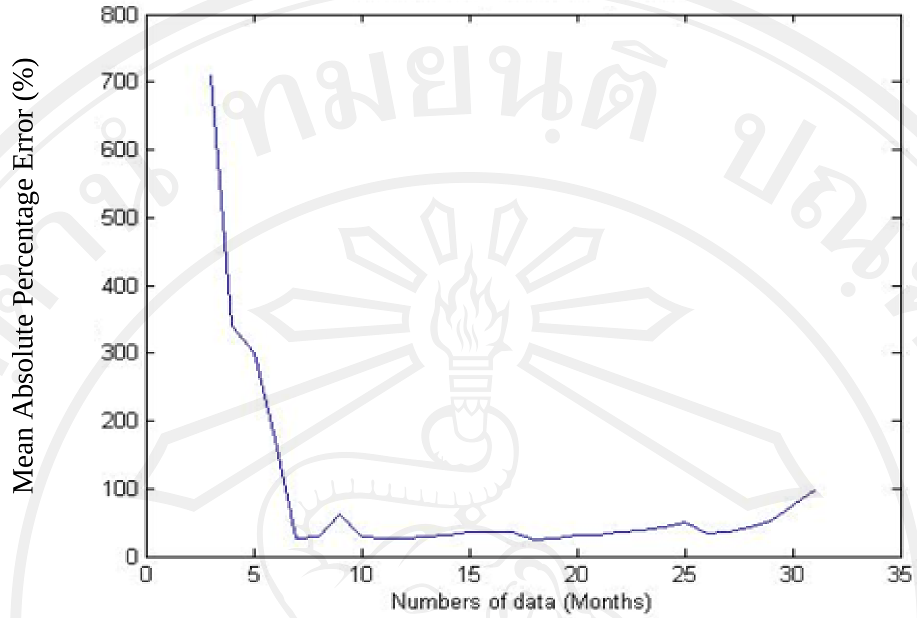

Figure 3.8: Mean Absolute Percentage Error of the Maximum Likelihood with quadratic interpolation and fixed intercept at Vo

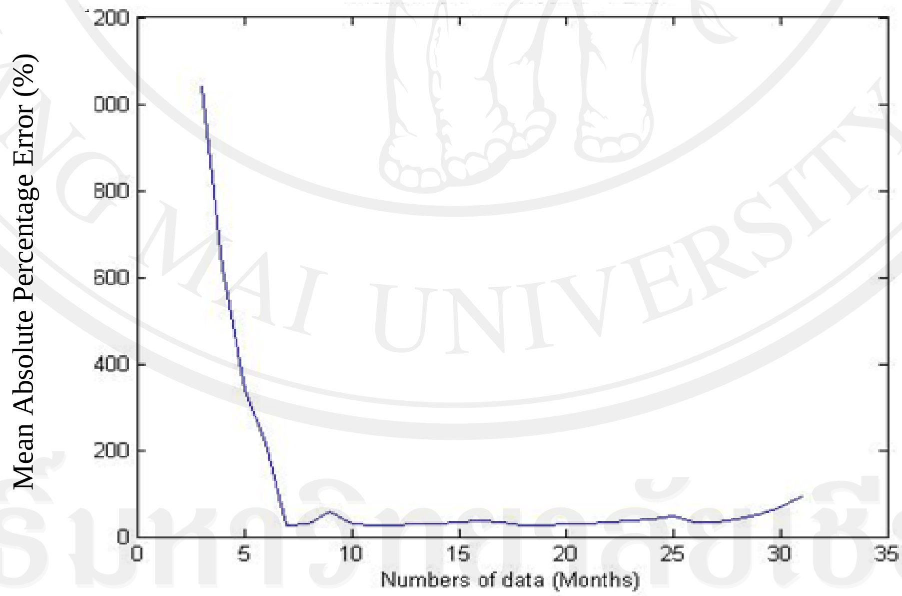

Figure 3.9: Mean Absolute Percentage Error of the maximum likelihood with Quasi-Newton and fixed intercept at Vo

With all the tested estimation methods, we find that the sufficient numbers of observations for sales forecasts of an innovative agro-industrial product, the feta cheese from buffalo milk, is between 7 to 24 months. The Mean Absolute Percentage Errors from the out-of-sample test drop sharply in this range. Therefore, practitioners can forecast the sales of the new product after half a year after the product launch with high accuracy.

Analysis 4: Comparison between Bass model and Logistic function

The results show the comparison between the Bass model and the logistic function for the whole dataset (32 observations from January 2010 to August 2012) and for the selected observations (from observation 7 to 24).

1) Logistic function

1.1) Logistic 1

The estimation result of the Logistic function using maximum likelihood with quadratic interpolation (to search for M) and fixed intercept at Vo (Logistic 1) is presented in Table 3.14.

Table 3.14: Estimation result of the Logistic function using maximum likelihood with quadratic interpolation (to search for M) and fixed intercept at Vo

N	M*	Beta	Out-sample test (MAPE)
3	1.15E+06	0.2049	710.7455
4	1.15E+06	0.1352	341.249
5	1.15E+06	0.1257	299.5281
6	1.15E+06	0.099	172.7165
7	1.15E+06	0.042	26.9965
8	1.15E+06	4.77E-02	30.1830
9	1.17E+06	0.0657	60.8877
10	1.15E+06	0.0486	30.1596
11	1.15E+06	4.14E-02	27.0777
12	1.15E+06	0.0432	28.0433
13	1.15E+06	3.65E-02	29.7533
14	2.69E+06	0.0482	30.7983
15	2.20E+06	0.0517	36.7008
16	2.20E+06	0.0511	37.3895
17	2.19E+06	0.0495	36.282
18	4.05E+06	0.0423	25.6119
19	5.00E+06	0.0458	27.9018
20	4.11E+06	0.0482	31.9772
21	5.03E+06	0.047	31.4440
22	3.40E+06	0.0488	35.5675
23	1.94E+06	0.05	39.7354
24	5.25E+06	0.0487	42.5036
25	3.12E+06	0.0508	50.3024
26	1.40E+07	0.0473	35.0448
27	2.10E+07	0.0465	36.2831
28	2.10E+07	0.0463	43.6599
29	2.57E+07	0.0458	52.2262
30	2.14E+07	0.0462	75.3084
31	4.67E+07	0.0449	96.1237
32	6.60E+07	0.0429	-

Source: Own calculation

1.2) Logistic 2

The estimation result of Logistic function using least squares with quadratic interpolation and fixed intercept at Vo (Logistic 2) is presented in Table 3.15.

Table 3.15: Estimation result of Logistic function using least squares with quadratic interpolation (to search for M) and fixed intercept at Vo

N	M*	Beta	Out-sample test (MAPE)
3	1.80E+06	0.2028	936.1434
4	1.48E+06	0.1343	371.2554
5	1.48E+06	0.1248	321.8111
6	1.80E+06	9.80E-02	185.2782
7	1.15E+06	0.042	26.9965
8	1.15E+06	0.0477	3.02E+01
9	1.17E+06	0.0657	60.8896
10	1.15E+06	0.0486	30.1602
11	1.15E+06	0.0414	27.0777
12	1.15E+06	0.0432	28.0434
13	1.15E+06	0.0365	2.98E+01
14	1.15E+06	4.92E-02	30.4141
15	1.16E+06	0.0527	36.0846
16	1.16E+06	0.052	36.7475
17	1.15E+06	0.0503	35.8103
18	1.47E+06	0.0431	25.4413
19	1.48E+06	0.0468	27.5098
20	1.16E+06	0.0496	31.3205
21	1.48E+06	0.048	30.9512
22	1.17E+06	0.0501	34.8615
23	1.25E+06	0.0507	39.3213
24	1.19E+06	0.0502	41.5403
25	1.10E+06	0.0523	48.9638
26	1.81E+06	0.0484	34.4178
27	1.81E+06	0.0477	35.6350
28	1.81E+06	0.0475	42.9912
29	2.23E+06	0.0468	51.6853
30	1.84E+06	0.0474	74.1752
31	3.41E+06	0.0455	95.4737
32	4.16E+06	0.0434	-

Source: Own calculation

1.3) Logistic 3

The estimation result of Logistic function using maximum likelihood with Quasi-Newton (to search for M and Beta) and fixed intercept at Vo (Logistic 3) is presented in Table 3.16.

Table 3.16: Estimation result of Logistic function using maximum likelihood with Quasi-Newton (to search for M and Beta) and fixed intercept at Vo

N	M*	Beta	Likelihood	Out-sample test (MAPE)
3	1.88E+06	0.2123	2.28E-15	1.04E+03
4	2.11E+06	0.1573	2.96E-20	622.3783
5	1.80E+06	0.1244	3.93E-25	338.1848
6	1.90E+06	0.1033	5.06E-30	215.7572
7	1.80E+06	0.0413	5.45E-35	27.0290
8	1.79E+06	4.68E-02	7.22E-40	30.1028
9	1.82E+06	0.0638	0.0638	59.5629
10	1.79E+06	0.0475	9.03E-50	29.9117
11	1.79E+06	0.0405	1.13E-54	27.1930
12	1.79E+06	0.0422	1.50E-59	28.0026
13	1.79E+06	3.58E-02	1.83E-64	29.8949
14	1.77E+06	0.0478	8.92E-70	29.7504
15	1.79E+06	0.051	1.15E-74	34.8033
16	1.79E+06	0.0504	1.50E-79	35.7265
17	1.78E+06	0.0488	1.88E-84	34.9676
18	1.78E+06	0.0424	1.48E-89	25.3802
19	1.78E+06	0.0458	1.40E-94	26.6273
20	1.79E+06	0.0479	1.62E-99	30.0215
21	1.78E+06	0.0469	1.96E-104	29.6755
22	1.82E+06	0.0483	2.28E-109	33.0818
23	1.90E+06	0.0487	3.00E-114	36.4223
24	1.85E+06	0.0483	3.77E-119	39.1461
25	1.68E+06	0.0504	2.90E-124	46.0518
26	1.78E+06	0.0475	1.23E-129	31.8652
27	1.78E+06	0.0468	1.42E-134	33.4098
28	1.78E+06	0.0467	1.86E-139	40.7572
29	1.78E+06	0.0462	2.26E-144	49.6259
30	1.81E+06	4.64E-02	2.86E-149	6.93E+01
31	1.78E+06	0.0455	2.09E-154	91.3084
32	1.74E+06	0.044	7.32E-160	-

Source: Own calculation

1.4) Logistic 4

The estimation result of Logistic function using least squares with Quasi-Newton (to search for M and Beta) and fixed intercept at Vo (Logistic 4) is presented in Table 3.17.

Table 3.17: Estimation result of Logistic function using least squares with Quasi-Newton (to search for M and Beta) and fixed intercept at Vo

N	M*	Beta	SSE	Out-sample test (MAPE)
3	1.77E+06	0.2006	3.14E+12	9.09E+02
4	1.77E+06	0.1325	3.14E+12	3.81E+02
5	1.78E+06	0.1227	3.16E+12	3.25E+02
6	1.77E+06	0.0968	3.16E+12	1.79E+02
7	1.78E+06	0.0408	3.17E+12	2.71E+01
8	1.78E+06	0.0464	3.17E+12	2.98E+01
9	1.82E+06	0.0637	3.31E+12	59.2146
10	1.78E+06	0.0473	3.18E+12	2.97E+01
11	1.77E+06	0.0403	3.15E+12	27.2792
12	1.77E+06	0.0421	3.16E+12	2.80E+01
13	1.77E+06	0.0357	3.14E+12	2.99E+01
14	1.76E+06	0.048	3.13E+12	2.99E+01
15	1.79E+06	0.051	3.20E+12	3.47E+01
16	1.79E+06	0.0503	3.20E+12	35.6878
17	1.78E+06	0.0489	3.18E+12	3.50E+01
18	1.75E+06	0.0429	3.13E+12	2.55E+01
19	1.77E+06	0.046	3.15E+12	2.68E+01
20	1.79E+06	0.0479	3.22E+12	3.00E+01
21	1.78E+06	0.0469	3.19E+12	2.98E+01
22	1.82E+06	0.0483	3.31E+12	33.0224
23	1.90E+06	0.0486	3.63E+12	36.3320
24	1.85E+06	0.0483	3.43E+12	39.0104
25	1.68E+06	0.0504	2.81E+12	4.60E+01
26	1.79E+06	0.0476	3.22E+12	3.22E+01
27	1.79E+06	0.047	3.23E+12	33.7473
28	1.80E+06	0.0467	3.26E+12	4.09E+01
29	1.80E+06	0.0462	3.28E+12	49.5553
30	1.83E+06	0.0463	3.38E+12	6.87E+01
31	1.81E+06	0.0454	3.31E+12	9.05E+01
32	1.78E+06	0.044	3.24E+12	-

Source: Own calculation

1.5) Bass 1

The estimation result of the Bass model using least squares and searching for only M (fixed p and fixed q) with quadratic interpolation (Bass 1) is presented in Table 3.18.

Table 3.18: Estimation result of the Bass model using least squares and searching for only M (fixed p and fixed q) with quadratic interpolation

N	M*	p*	q*	SSE	MAPE
3	7.92E+04	0.03	0.38	9.62E+08	56.6884
4	6.37E+04	0.03	0.38	1.15E+09	4.22E+01
5	5.92E+04	0.03	0.38	1.20E+09	40.2503
6	5.39E+04	0.03	0.38	1.34E+09	38.4486
7	4.57E+04	0.03	0.38	1.94E+09	36.4375
8	4.50E+04	0.03	0.38	1.95E+09	38.0884
9	4.84E+04	0.03	0.38	2.20E+09	36.9074
10	4.53E+04	0.03	0.38	2.50E+09	37.5299
11	4.36E+04	0.03	0.38	2.61E+09	39.2315
12	4.39E+04	0.03	0.38	2.62E+09	40.6573
13	4.26E+04	0.03	0.38	2.75E+09	42.2694
14	4.75E+04	0.03	0.38	4.91E+09	36.1523
15	4.93E+04	0.03	0.38	5.25E+09	34.6348
16	4.98E+04	0.03	0.38	5.27E+09	35.6607
17	4.99E+04	0.03	0.38	5.28E+09	37.5858
18	4.86E+04	0.03	0.38	5.62E+09	37.5954
19	5.11E+04	0.03	0.38	7.00E+09	34.1257
20	5.33E+04	0.03	0.38	8.09E+09	31.5217
21	5.37E+04	0.03	0.38	8.13E+09	33.0588
22	5.58E+04	0.03	0.38	9.53E+09	30.4537
23	5.75E+04	0.03	0.38	1.05E+10	29.04
24	5.84E+04	0.03	0.38	1.08E+10	29.4457
25	6.15E+04	0.03	0.38	1.47E+10	25.301
26	6.09E+04	0.03	0.38	1.48E+10	25.9967
27	6.17E+04	0.03	0.38	1.51E+10	26.5434
28	6.29E+04	0.03	0.38	1.58E+10	24.7814
29	6.38E+04	0.03	0.38	1.64E+10	23.8492
30	6.57E+04	0.03	0.38	1.87E+10	14.5013
31	6.60E+04	0.03	0.38	1.87E+10	19.2648
32	6.56E+04	0.03	0.38	1.88E+10	-

Source: Own calculation

1.6) Bass 2

The estimation result of the Bass model using least squares to search for M and q (fixed p) with Quasi-Newton (Bass 2) is presented in Table 3.19.

Table 3.19: Estimation result of the Bass model using least squares searching for M and q (fixed p) with Quasi-Newton

N	M*	p*	q*	SSE	MAPE
3	1.65E+05	0.03	0.1613	1.75E+10	170
4	1.32E+05	0.03	1.46E-01	3.34E+10	1.15E+02
5	1.11E+05	0.03	0.1335	4.57E+10	79.8921
6	9.10E+04	0.03	0.1234	5.64E+10	49.7268
7	7.28E+04	0.03	0.1098	6.28E+10	31.8864
8	6.27E+04	0.03	0.104	6.38E+10	3.04E+01
9	5.94E+04	0.03	0.1032	5.97E+10	30.2363
10	5.24E+04	0.03	9.77E-02	5.66E+10	33.2553
11	4.42E+04	0.03	9.74E-02	5.68E+10	4.15E+01
12	4.20E+04	0.03	0.0983	5.16E+10	44.0503
13	3.92E+04	0.03	9.80E-02	4.72E+10	4.91E+01
14	4.20E+04	0.03	0.1068	4.32E+10	44.7081
15	4.27E+04	0.03	0.1119	3.89E+10	4.37E+01
16	4.45E+04	0.03	0.1095	3.28E+10	42.7652
17	4.24E+04	0.03	0.1189	3.15E+10	45.8242
18	4.10E+04	0.03	1.20E-01	2.92E+10	4.88E+01
19	4.30E+04	0.03	0.1266	2.72E+10	45.5426
20	4.48E+04	0.03	0.1319	2.53E+10	42.6395
21	4.51E+04	0.03	0.135	2.32E+10	43.4889
22	4.69E+04	0.03	0.1396	2.21E+10	40.1709
23	4.83E+04	0.03	0.1435	2.10E+10	37.3793
24	4.92E+04	0.03	0.1461	1.96E+10	36.3505
25	5.43E+04	0.03	0.1285	1.91E+10	28.2265
26	4.85E+04	0.03	0.1982	2.39E+10	38.5536
27	4.82E+04	0.03	0.2294	2.41E+10	38.9902
28	5.03E+04	0.03	0.2098	2.26E+10	34.4547
29	5.12E+04	0.03	0.2152	2.21E+10	30.7847
30	5.28E+04	0.03	0.2213	2.34E+10	16.2434
31	5.32E+04	0.03	0.2258	2.28E+10	3.9243
32	5.30E+04	0.03	0.2297	2.23E+10	-

Source: Own calculation

1.7) Bass3

The estimation result of the Bass model using least squares to search for M, p and q with Quasi-Newton (Bass 3) is presented in Table 3.20.

Table 3.20: Estimation result of the Bass model using least squares to search for M, p and q with Quasi-Newton

N	M*	p*	q*	SSE	MAPE
3	8.42E+05	-5.40E-02	6.43E-01	3.39E+09	1.00E+02
4	4.50E+05	-1.49E-02	3.59E-01	4.93E+09	1.00E+02
5	5.98E+05	-3.42E-02	4.84E-01	6.60E+09	1.00E+02
6	4.06E+05	-1.57E-02	3.48E-01	8.13E+09	1.00E+02
7	-	-	-	8.58E+09	-
8	2.64E+06	-3.04E-01	2.90E+00	9.80E+09	1.00E+02
9	-6.66E+05	0.1408	-0.7207	1.21E+10	6.19E+09
10	9.15E+04	0.0166	0.0709	1.31E+11	3.12E+01
11	1.11E+06	-0.1585	1.5482	1.55E+10	100
12	7.51E+04	0.0363	0.0597	1.34E+11	27.6333
13	2.80E+05	-0.0235	0.3995	1.88E+10	100
14	8.18E+04	0.0305	0.0592	1.00E+11	24.9463
15	1.02E+05	0.0981	0.0201	3.05E+10	27.4695
16	6.48E+04	0.0071	0.0812	1.19E+11	30.9202
17	4.55E+04	0.0361	4.16E-02	1.56E+11	52.6333
18	8.41E+04	0.1067	0.0275	4.77E+10	31.1679
19	-	-	-	4.59E+10	-
20	1.02E+05	0.1795	-1.68E-02	1.03E+10	47.2168
21	4.27E+04	0.0227	0.0597	1.51E+11	52.368
22	4.00E+04	0.0215	0.0411	1.61E+11	59.9345
23	1.12E+05	0.2081	-0.0195	1.16E+10	44.1433
24	3.97E+04	0.0205	0.0553	1.63E+11	54.8113
25	4.21E+04	0.0226	0.0612	1.63E+11	47.9314
26	8.82E+04	0.188	0.0222	3.21E+10	27.9643
27	1.02E+05	0.1989	0.003	1.57E+10	32.8234
28	-1.24E+05	-0.4592	0.1857	7.17E+10	5.54E+05
29	6.39E+04	0.1379	0.0408	9.27E+10	28.3513
30	7.74E+04	0.1557	0.0363	6.05E+10	14.0515
31	7.67E+04	0.156	0.0367	6.10E+10	11.049
32	9.27E+04	0.0945	0.0275	2.64E+10	-

Source: Own calculation

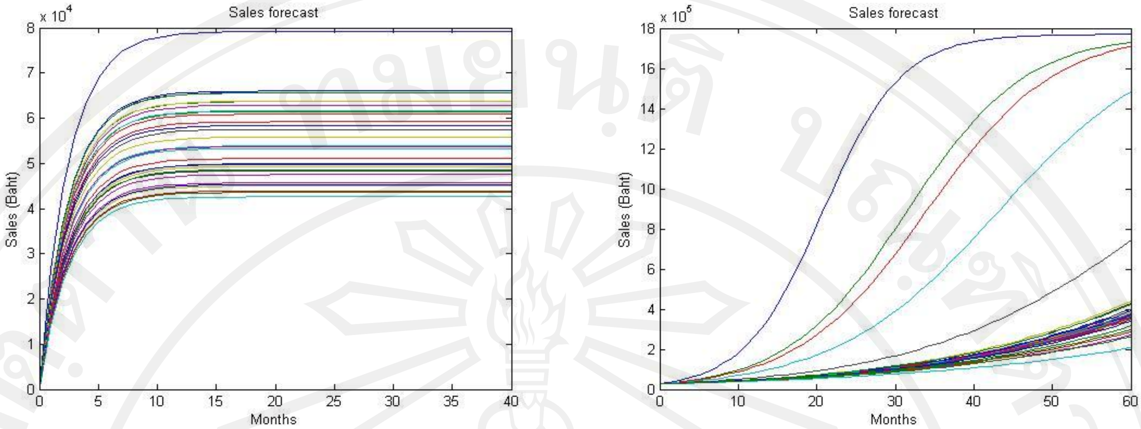

Figure 3.10: Forecasting results of Bass1 (the best with the Bass model—on the left) and Logistic 4 (the best with the Logistic function—on the right) show the maximum sales, growth of the sales and duration that the sales will reach the maturity period. Logistic function presents a clearer S-curve than Bass model

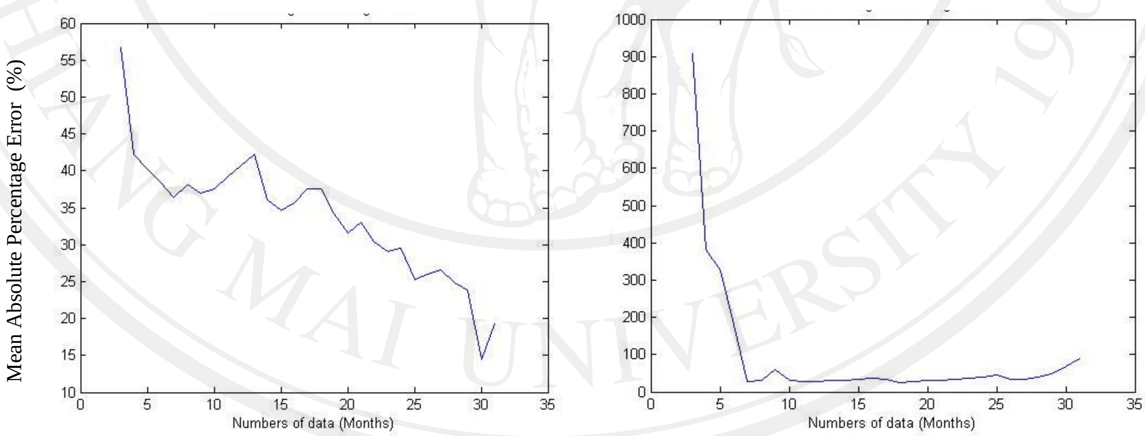

Figure 3.11: Mean Absolute Percentage Error (MAPE) of Bass1 (the best of Bass model on the left) and Logistic 4 (the best of Logistic function—on the right) at different numbers of observation. The MAPE of the Logistic function drops sharply at the 7^{th} month

2) Comparison between the Bass model and the logistic function

2.1) Comparison on the whole period

Table 3.21: Paired Samples Statistics using data from whole period

		Mean MAPE	N	Std. Deviation	Std. Error Mean
Pair 1	Logistic1	86.972	29	142.451	26.452
	BASS1	33.732	29	8.241	1.530
Pair 2	Logistic2	96.584	29	181.944	33.786
	BASS2	46.119	29	30.385	5.642
Pair 3	Logistic3	109.148	29	217.521	40.393
	BASS2	46.119	29	30.385	5.642
Pair 4	Logistic3	120.453	25	232.822	46.564
	BASS3	53.865	25	31.671	6.33
Pair 5	Logistic4	94.557	29	178.353	33.119
	BASS2	46.119	29	30.385	5.642
Pair 6	Logistic4	103.525	25	190.984	38.197
	BASS3	53.865	25	31.671	6.334

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95%Confidence Interval of the Difference	
					Lower	Upper
Pair 1	Logistic1-BASS1	53.241	138.050	25.635	.729	105.752
Pair 2	Logistic2-BASS2	50.465	154.908	28.766	-8.459	109.388
Pair 3	Logistic3-BASS2	63.029	189.668	35.220	-9.117	135.175
Pair 4	Logistic3-BASS3	66.589	218.134	43.627	-23.453	156.630
Pair 5	Logistic4-BASS2	48.437	151.150	28.068	-9.057	105.932
Pair 6	Logistic4-BASS3	49.661	177.400	35.480	-23.566	122.888

Table 3.21: (Continued)

		t	df	Sig. (2-tailed)
Pair 1	Logistic1-BASS1	2.077	28	.047
Pair 2	Logistic2- BASS2	1.754	28	.090
Pair 3	Logistic3- BASS2	1.790	28	.084
Pair 4	Logistic3- BASS3	1.526	24	.140
Pair 5	Logistic4- BASS2	1.726	28	.095
Pair 6	Logistic4- BASS3	1.400	24	.174

Source: Own calculation using SPSS

When using all the observations, the Bass model is superior to the logistic function. In the next section, we will compare the two models considering just the selected period (7 to 24 months) for which the MAPE of the logistic function improves sharply.

2.2) Comparison for the selected period

We compare the MAPE of Bass model and logistic function just for the range of 7 to 24 months. The results are as follows:

Table 3.22: Paired Samples Statistics using data from 7th to 24th month

		Mean MAPE	N	Std. Deviation	Std. Error Mean
Pair 1	Logistic1	33.834	18	8.269	1.949
	BASS1	35.578	18	3.737	.888
Pair 2	Logistic2	33.506	18	8.204	1.934
	BASS2	40.657	18	6.028	1.421
Pair 3	Logistic3	32.6277	18	7.739	1.824
	BASS2	40.657	18	6.028	1.421
Pair 4	Logistic3	31.605	15	3.87010	.999
	BASS3	52.296	15	27.131	7.005
Pair 5	Logistic4	32.605	18	7.639	1.801
	BASS2	40.657	18	6.028	1.421
Pair 6	Logistic4	31.582	15	3.825	.988
	BASS3	52.2960	15	27.131	7.005

Table 3.22: (Continued)

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference	
					Lower	Upper
Pair 1	Logistic1 BASS1	-1.744	10.069	2.373	-6.751	3.264
Pair 2	Logistic2 BASS2	-7.151	12.142	2.862	-13.189	-1.113
Pair 3	Logistic3 BASS2	-8.029	11.763	2.773	-13.879	-2.180
Pair 4	Logistic3 BASS3	-20.691	27.984	7.226	-36.188	-5.194
Pair 5	Logistic4 BASS2	-8.052	11.637	2.743	-13.839	-2.265
Pair 6	Logistic4 BASS3	-20.714	27.998	7.229	-36.219	-5.209

		t	df	Sig. (2-tailed)
Pair 1	Logistic1-BASS1	-.735	17	.473
Pair 2	Logistic2- BASS2	-2.499	17	.023
Pair 3	Logistic3- BASS2	-2.896	17	.010
Pair 4	Logistic3- BASS3	-2.864	14	.013
Pair 5	Logistic4- BASS2	-2.935	17	.009
Pair 6	Logistic4- BASS3	-2.865	14	.012

Source: Own calculation using SPSS

When considering only the selected period (7 to 24 months), the logistic function is superior to the Bass model. In 5 pairs out of 6, the MAPE of logistic function is significantly smaller than that of the Bass model.

2.3) Comparison between the best of Bass model and logistic function

In this section, the best Bass model which is BASS1 and the best logistic function which is Logistic4 will be compared. The results are shown in Table 3.23

Table 3.23: Paired samples statistics between the best logistic model and the best Bass model

		Mean MAPE	N	Std. Deviation	Std. Error Mean
Pair 1	Logistic4	32.605	18	7.639	1.801
	BASS1	35.578	18	3.737	.881

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference	
					Lower	Upper
Pair 1	Logistic4 BASS1	-2.973	9.143	2.155	-7.519	1.574

		t	df	Sig. (2-tailed)
Pair 1	Logistic4 BASS1	-1.379	17	.186

Source: Own calculation using SPSS

The MAPE of the logistic function is slightly lower than that of Bass model. However, the difference is not statistically significant. The logistic function is thus superior to the Bass model when the model uses the data between 7 to 24 months, where the MAPE of the Logistic function is low. However, the best Logistic function is not significantly superior to the best Bass model. Therefore, it can be said that the Logistic function yield at least as good performance as Bass model.

<u>Analysis 5</u>: Comparison between the method of rolling windows and cumulative observations

In this section, we will first present the results from the method of rolling windows. The width of the windows was varied from 3 to 19. Due to the large size of the results, they are presented in the appendix section of this chapter. However, two of the best models with width of 15 and 16 observations, which demonstrate the lowest average MAPE in this class, are displayed here.

Table 3.24: Estimation results for the Logistic function using OLS and quadratic interpolation with the method of rolling windows when the width of window is 15

Repeat	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
1	1,156,423	0.0527	6,000	47.22	19.94	19.99
2	1,482,125	0.0136	17,700	32.89	21.02	21.07
3	1,806,054	0.0213	11,800	34.63	20.61	20.66
4	2,205,003	0.0182	11,100	30.35	20.56	20.61
5	2,729,756	0.0093	21,700	18.97	21.23	21.27
6	3,357,948	0.0231	11,600	27.40	20.60	20.65

Table 3.25: Estimation results for the Logistic function using OLS and quadratic interpolation with the method of rolling windows when the width of window is 16

Repeat	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
1	1,156,741	0.0520	6,250	49.93	19.91	19.96
2	1,483,627	0.0142	17,900	32.13	20.96	21.01
3	1,802,822	0.0162	13,500	28.85	20.68	20.73
4	2,206,022	0.0232	11,300	32.03	20.50	20.55
5	2,731,507	0.0134	20,500	21.81	21.10	21.15

The study will find the best model with rolling windows using t-test. The results are shown in Tables 3.26 and 3.27.

Table 3.26: Descriptive statistics of MAPE, AIC and BIC from the estimation methods of rolling windows with widths15 and 16

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	MAPE15	32.810	5	10.107	4.520
	MAPE16	32.948	5	10.376	4.640
Pair 2	AIC15	20.673	5	.496	.222
	AIC16	20.629	5	.467	.209
Pair 3	BIC15	20.720	5	.496	.222
	BIC16	20.677	5	.467	.209

Source: Calculation using SPSS version 11.0.

The MAPE of the model with a width of 15 observations is slightly lower than that with 16 observations. The AIC and BIC of the former model are higher than those of the latter model. However, as shown in Table 3.26, these differences are not statistically significant. Therefore, the best model is the one with 15 observations because it uses fewer observations.

Table 3.27: Comparison of MAPE, AIC and BIC for the estimation methods of rolling windows with widths 15 and 16

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	MAPE15 MAPE16	-.138	3.611	1.615	-4.622	4.346	-.085	4	.936
Pair 2	AIC15 AIC16	.044	.071	.032	-.044	.131	1.390	4	.237
Pair 3	BIC15 BIC16	.043	.071	.032	-.045	.130	1.355	4	.247

Source: Calculation using SPSS version 11.0.

In the next step, this study will compare the performances of the best model from the method of rolling windows to the best model with the method of cumulative observations. The results are shown in Tables 3.28 and 3.29.

Table 3.28: Descriptive statistics of MAPE, AIC and BIC from the estimation methods of rolling windows with width 15 and the model with the method of cumulative observations (OLS with quadratic interpolation)

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	MAPE15	31.909	6	9.306	3.799
	CUMMAPE	46.542	6	2.004	.818
Pair 2	AIC15	20.661	6	.445	.181
	CUMAIC	19.690	6	.260	.106
Pair 3	BIC15	20.708	6	.445	.181
	CUMBIC	19.597	6	.190	.077

Source: Calculation using SPSS version 11.0

Table 3.29: Comparison of MAPE, AIC and BIC between the estimation methods of rolling window with width 15 and the model with the method of cumulative observations (OLS with quadratic interpolation)

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	MAPE15 CUMMAPE	-14.633	8.419	3.437	-23.468	-5.798	-4.26	5	.008
Pair 2	AIC15 CUMAIC	.971	.572	.234	.370	1.571	4.15	5	.009
Pair 3	BIC15 CUMBIC	1.111	.478	.195	.609	1.613	5.69	5	.002

Source: Calculation using SPSS version 11.0.

The results show that the MAPE of the model with rolling windows is much lower than that of the model with cumulative observations. However, the AIC and BIC of the model with rolling windows are higher than those of the model with cumulative observations. These results are clearly significant at the confidence level of 99%.

To judge which one is better, the purpose of the prediction should be the first priority. The model with smaller MAPE calculated from the out-of-sample test should be considered better. Therefore, the model using rolling windows should be preferred to the model using cumulative observations.

<u>Analysis 6</u>: Comparison between Quasi-Newton, Gauss-Newton and Newton Raphson algorithms

The final part of this chapter is a comparison of forecasting accuracy when using the Logistic function with three estimation methods, namely: Quasi-Newton, Gauss-Newton, and Newton Raphson, by pairing one another in order to ascertain the statistical results.

Table 3.30: Estimation results for the Logistic function using the Quasi-Newton method

No. of Obs	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
3	1,769,174	0.2006	1,379	44.35	21.28	20.68
4	1,769,195	0.1325	1,694	46.06	20.86	20.56
5	1,775,316	0.1227	1,773	47.54	20.49	20.33
6	1,773,386	0.0968	2,009	48.55	20.30	20.23
7	1,779,552	0.0408	3,597	41.94	20.63	20.61
8	1,778,500	0.0464	3,599	43.17	20.42	20.44
9	1,818,692	0.0637	3,425	45.88	20.20	20.25
10	1,780,237	0.0473	4,121	43.08	20.24	20.30
11	1,773,006	0.0403	4,566	41.68	20.21	20.28
12	1,774,508	0.0421	4,515	43.83	20.08	20.16
13	1,766,806	0.0357	5,022	41.63	20.08	20.17
14	1,761,750	0.0480	6,577	44.78	20.25	20.34
15	1,786,617	0.0510	7,087	46.01	20.24	20.33
16	1,787,332	0.0503	7,218	47.46	20.18	20.27
17	1,778,910	0.0488	7,287	49.21	20.11	20.21
18	1,749,170	0.0429	7,342	48.68	20.05	20.15
19	1,766,718	0.0460	9,373	49.19	20.23	20.33
20	1,790,914	0.0479	11,344	49.42	20.36	20.46
21	1,780,803	0.0469	11,640	50.88	20.32	20.42
22	1,817,839	0.0483	14,368	50.65	20.48	20.58
23	1,904,091	0.0486	16,649	50.34	20.57	20.67
24	1,849,916	0.0483	17,864	50.65	20.59	20.69
25	1,678,951	0.0504	25,060	48.75	20.89	20.98
26	1,786,158	0.0476	24,873	52.54	20.83	20.93
27	1,786,979	0.0470	26,312	52.83	20.85	20.94
28	1,797,910	0.0467	28,963	51.46	20.90	21.00
29	1,803,229	0.0462	31,217	49.74	20.93	21.03
30	1,833,973	0.0462	36,800	41.12	21.06	21.15
31	1,810,095	0.0453	37,967	33.08	21.06	21.15
32	1,775,842	0.0440	38,196	-	21.03	21.12

Table 3.31: Estimation results of the Logistic function using the Gauss-Newton method

No.of obs	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
3	-	-	-	-	-	-
4	-	-	-	-	-	-
5	798,825	0.1262	852	48.23	19.75	19.60
6	-	-	-	-	-	-
7	841,842	0.0412	2,446	42.66	20.24	20.23
8	797,660	0.0433	1,841	45.13	19.75	19.77
9	-	-	-	-	-	-
10	-	-	-	-	-	-
11	-	-	-	-	-	-
12	785,322	0.0509	2,354	49.52	19.43	19.51
13	818,453	0.0295	3,253	44.19	19.65	19.73
14	-	-	-	-	-	-
15	941,751	0.0359	6,970	42.54	20.22	20.32
16	-	-	-	-	-	-
17	809,849	0.0499	6,343	53.15	19.97	20.07
18	806,331	0.0438	6,281	53.92	19.89	19.99
19	806,590	0.0480	8,767	54.85	20.16	20.26
20	801,428	0.0451	10,943	54.28	20.32	20.42
21	795,767	0.0520	11,943	57.71	20.35	20.45
22	796,400	0.0568	15,589	58.24	20.56	20.66
23	802,636	0.0536	18,024	56.92	20.65	20.75
24	-	-	-	-	-	-
25	809,129	0.0502	26,834	53.95	20.95	21.05
26	803,964	0.0463	26,608	57.17	20.90	21.00
27	800,562	0.0516	29,480	58.91	20.96	21.06
28	797,427	0.0513	32,804	57.92	21.02	21.12
29	803,710	0.0440	33,806	54.39	21.01	21.11
30	802,982	0.0488	41,234	48.18	21.17	21.27
31	-	-	-	-	-	-
32	797,590	0.0433	42,138	0	21.12	21.22

As seen from Table 3.31 the Gauss-Newton method failed for some observations. This is a drawback of Gauss-Newton that should not be ignored.

Table 3.32: Estimation results of the Logistic function using the Newton-Raphson method

No. of obs	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
3	808,619	0.2097	451	49.89	20.16	19.56
4	803,758	0.1375	501	49.40	19.65	19.34
5	817,901	0.1302	527	50.50	19.27	19.12
6	807,550	0.1012	551	50.72	19.00	18.93
7	925,510	0.0493	1,130	45.72	19.47	19.46
8	897,251	0.0542	1,089	48.04	19.23	19.25
9	938,099	0.0783	1,511	50.25	19.38	19.43
10	933,702	0.0576	1,635	49.51	19.31	19.37
11	913,564	0.0480	1,719	49.75	19.23	19.30
12	897,359	0.0492	1,761	51.65	19.14	19.22
13	888,280	0.0411	1,867	51.68	19.09	19.18
14	990,619	0.0620	4,903	55.05	19.96	20.05
15	964,321	0.0646	5,994	55.80	20.07	20.17
16	933,076	0.0617	6,369	56.50	20.05	20.15
17	908,432	0.0581	6,602	57.43	20.01	20.11
18	903,282	0.0499	6,354	59.33	19.90	20.00
19	919,693	0.0553	9,399	60.31	20.23	20.33
20	919,387	0.058	12,339	60.72	20.44	20.54
21	900,180	0.0555	12,925	61.56	20.43	20.53
22	901,694	0.0576	16,746	61.56	20.63	20.73
23	895,218	0.058	19,962	61.34	20.76	20.85
24	882,434	0.0566	21,744	61.39	20.79	20.89
25	894,059	0.0596	30,600	59.98	21.09	21.18
26	879,593	0.0552	30,341	62.58	21.03	21.13
27	868,891	0.0537	32,316	62.64	21.05	21.15
28	863,153	0.0531	35,828	61.55	21.11	21.21
29	856,184	0.0522	38,805	60.10	21.15	21.25
30	858,009	0.0528	45,815	53.50	21.28	21.37
31	848,169	0.0507	47,226	46.70	21.27	21.37
32	839,224	0.0479	47,135	-	21.24	21.33

The following part was carried out in order to conduct a comparison and a statistical analysis using the t-test where incomplete data were omitted prior to the comparison.

Table 3.33: Data for the comparison between the Quasi-Newton and Gauss-Newton method

No. of obs	Quasi-Newton					
	Mstar	beta	SSE	MAPE	AIC	BIC
5	1,775,316	0.1227	1,773	47.54	20.49	20.33
7	1,779,552	0.0408	3,597	41.94	20.63	20.61
8	1,778,500	0.0464	3,599	43.17	20.42	20.44
12	1,774,508	0.0421	4,515	43.83	20.08	20.16
13	1,766,806	0.0357	5,022	41.63	20.08	20.17
15	1,786,617	0.0510	7,087	46.01	20.24	20.33
17	1,778,910	0.0488	7,287	49.21	20.11	20.21
18	1,749,170	0.0429	7,342	48.68	20.05	20.15
19	1,766,718	0.0460	9,373	49.19	20.23	20.33
20	1,790,914	0.0479	11,344	49.42	20.36	20.46
21	1,780,803	0.0469	11,640	50.88	20.32	20.42
22	1,817,839	0.0483	14,368	50.65	20.48	20.58
23	1,904,091	0.0486	16,649	50.34	20.57	20.67
25	1,678,951	0.0504	25,060	48.75	20.89	20.98
26	1,786,158	0.0476	24,873	52.54	20.83	20.93
27	1,786,979	0.0470	26,312	52.83	20.85	20.94
28	1,797,910	0.0467	28,963	51.46	20.90	21.00
29	1,803,229	0.0462	31,217	49.74	20.93	21.03
30	1,833,973	0.0462	36,800	41.12	21.06	21.15
32	1,775,842	0.0440	38,196	-	21.03	21.12

No. of obs	Gauss-Newton					
	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
5	798,825	0.1262	852	48.23	19.75	19.60
7	841,842	0.0412	2,446	42.66	20.24	20.23
8	797,660	0.0433	1,841	45.13	19.75	19.77
12	785,322	0.0509	2,354	49.52	19.43	19.51
13	818,453	0.0295	3,253	44.19	19.65	19.73
15	941,751	0.0359	6,970	42.54	20.22	20.32
17	809,849	0.0499	6,343	53.15	19.97	20.07
18	806,331	0.0438	6,281	53.92	19.89	19.99
19	806,590	0.0480	8,767	54.85	20.16	20.26
20	801,428	0.0451	10,943	54.28	20.32	20.42
21	795,767	0.0520	11,943	57.71	20.35	20.45
22	796,400	0.0568	15,589	58.24	20.56	20.66
23	802,636	0.0536	18,024	56.92	20.65	20.75
25	809,129	0.0502	26,834	53.95	20.95	21.05
26	803,964	0.0463	26,608	57.17	20.90	21.00
27	800,562	0.0516	29,480	58.91	20.96	21.06
28	797,427	0.0513	32,804	57.92	21.02	21.12
29	803,710	0.0440	33,806	54.39	21.01	21.11
30	802,982	0.0488	41,234	48.18	21.17	21.27

Table 3.33: (Continued)

No.of obs	Gauss-Newton					
	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
32	797,590	0.0433	42,138	-	21.12	21.22

Table 3.34: Descriptive statistics of MAPE, AIC and BIC from the estimations using the Quasi-Newton and Gauss-Newton methods

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	QNMAPE	45.4462	20	11.30581	2.52806
	GUSMAPE	49.5930	20	12.85781	2.87509
Pair 2	QNAIC	20.5273	20	.34291	.07668
	GUSAIC	20.4050	20	.55077	.12315
Pair 3	QNBIC	20.6002	20	.34993	.07825
	GUSBIC	20.4789	20	.57704	.12903

Source: Calculation using SPSS version 11.0.

Table 3.35: Comparison of MAPE, AIC and BIC from the estimation using the Quasi-Newton and Gauss-Newton methods

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	QNMAPE GUSMAPE	-4.147	2.872	.642	-5.491	-2.803	-6.46	19	.000
Pair 2	QNAIC GUSAIC	.122	.289	.065	-.013	.258	1.89	19	.074
Pair 3	QNBIC GUSBIC	.121	.288	.064	-.014	.256	1.88	19	.075

Source: Calculation using SPSS version 11.0.

Table 3.34 shows that the Quasi Newton method yields smaller MAPE than that obtained with the Gauss-Newton method, with a very high confidence level. In contrast, the Gauss-Newton method provides significantly smaller average values of AIC and BIC at a confidence level of 90%.

Table 3.36: Descriptive statistics of MAPE, AIC and BIC for the estimation using the Quasi-Newton and Newton-Raphson methods

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	QNMAPE	45.150	30	9.545	1.743
	NRMAPE	53.504	30	11.488	2.097
Pair 2	QNAIC	20.524	30	.359	.066
	NRAIC	20.181	30	.7739	.141
Pair 3	QNBIC	20.557	30	.3339	.061
	NRBIC	20.217	30	.8269	.151

Source: Calculation using SPSS version 11.0.

Table 3.37: Comparison of MAPE, AIC and BIC for the estimation methods using the Quasi-Newton and Newton-Raphson methods

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	QNMAPE NRMAPE	-8.355	3.438	.628	-9.639	-7.071	-13.31	29	.000
Pair 2	QNAIC NRAIC	.343	.591	.108	.122	.563	3.18	29	.004
Pair 3	QNBIC NRBIC	.342	.590	.108	.122	.563	3.17	29	.004

Source: Calculation using SPSS version 11.0.

According to Table 3.36, the average value of MAPE using the Quasi-Newton method is significantly smaller than that using Newton-Raphson method at the 99% confidence level of 99%. However, the Newton-Raphson method provides significantly smaller average values of AIC and BIC at the confidence level of 99%.

Table 3.38: Data for the comparison between Gauss-Newton and Newton-Raphson

No. of obs	Gauss-Newton					
	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
5	798,825	0.1262	852	48.23	19.75	19.60
7	841,842	0.0412	2,446	42.66	20.24	20.23
8	797,660	0.0433	1,841	45.13	19.75	19.77
12	785,322	0.0509	2,354	49.52	19.43	19.51
13	818,453	0.0295	3,253	44.19	19.65	19.73
15	941,751	0.0359	6,970	42.54	20.22	20.32
17	809,849	0.0499	6,343	53.15	19.97	20.07
18	806,331	0.0438	6,281	53.92	19.89	19.99
19	806,590	0.0480	8,767	54.85	20.16	20.26
20	801,428	0.0451	10,943	54.28	20.32	20.42
21	795,767	0.0520	11,943	57.71	20.35	20.45
22	796,400	0.0568	15,589	58.24	20.56	20.66
23	802,636	0.0536	18,024	56.92	20.65	20.75
25	809,129	0.0502	26,834	53.95	20.95	21.05
26	803,964	0.0463	26,608	57.17	20.90	21.00
27	800,562	0.0516	29,480	58.91	20.96	21.06
28	797,427	0.0513	32,804	57.92	21.02	21.12
29	803,710	0.0440	33,806	54.39	21.01	21.11
30	802,982	0.0488	41,234	48.18	21.17	21.27
32	797,590	0.0433	42,138	-	21.12	21.22

No. of obs	Newton-Raphson					
	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
5	817,901	0.1302	527	50.50	19.27	19.12
7	925,510	0.0493	1,130	45.72	19.47	19.46
8	897,251	0.0542	1,089	48.04	19.23	19.25
12	897,359	0.0492	1,761	51.65	19.14	19.22
13	888,280	0.0411	1,867	51.68	19.09	19.18
15	964,321	0.0646	5,994	55.80	20.07	20.17
17	908,432	0.0581	6,602	57.43	20.01	20.11
18	903,282	0.0499	6,354	59.33	19.90	20.00
19	919,693	0.0553	9,399	60.31	20.23	20.33
20	919,387	0.0580	12,339	60.72	20.44	20.54
21	900,180	0.0555	12,925	61.56	20.43	20.53
22	901,694	0.0576	16,746	61.56	20.63	20.73
23	895,218	0.0580	19,962	61.34	20.76	20.85
25	894,059	0.0596	30,600	59.98	21.09	21.18
26	879,593	0.0552	30,341	62.58	21.03	21.13
27	868,891	0.0537	32,316	62.64	21.05	21.15
28	863,153	0.0531	35,828	61.55	21.11	21.21
29	856,184	0.0522	38,805	60.10	21.15	21.25

Table 3.38: (Continued)

No. of obs	Newton-Raphson					
	Mstar	beta	SSE (Million)	MAPE	AIC	BIC
30	858,009	0.0528	45,815	53.50	21.28	21.37
32	839,224	0.0479	47,135	-	21.24	21.33

Table 3.39: Descriptive statistics of MAPE, AIC and BIC for the estimation using the Gauss-Newton and Newton-Raphson methods

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	GUSMAPE	49.593	20	12.858	2.875
	NRMAPE	54.230	20	13.796	3.085
Pair 2	GUSAIC	20.405	20	.551	.123
	NRAIC	20.331	20	.770	.172
Pair 3	GUSBIC	20.479	20	.577	.129
	NRBIC	20.406	20	.803	.180

Source: Calculation using SPSS version 11.0.

Table 3.40: Comparison of MAPE, AIC and BIC for the estimation using the Gauss-Newton and Newton-Raphson methods

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
					Lower	Upper			
Pair 1	GUSMAPE NRMAPE	-4.707	2.659	.595	-5.951	-3.462	-7.92	19	.000
Pair 2	GUSAIC NRAIC	.0739	.285	.0637	-.059	.207	1.16	19	.260
Pair 3	GUSBIC NRBIC	.0734	.284	.0635	-.060	.206	1.16	19	.262

Source: Calculation using SPSS version 11.0.

From a paired comparison between the results obtained with the Gauss-Newton and Newton-Raphson methods, it is clear that the average values of MAPE from the Gauss-Newton method are much lower, with statistical significance at a confidence level of 99%, whereas the differences between the average values of AIC and BIC are not statistically significant.

However, this study covers only one product, so that the comparisons and the results just reflect this particular case. Therefore, it should be noted that our conclusions regarding the superiority of an algorithm or another are valid only for this case and should not be blindly generalized.

3.7 Conclusions

In this chapter, we used limited data to forecast the sales of the feta cheese by estimating the S-curves following the theory of product life cycle. First of all, we figured out whether the OLS or EGLS is better to be used in the estimation process. we also tried to find the sufficient number of observations that yields the most accurate forecasts due to the limitation of number of observations that usually occurs in the case of innovative agro-industrial products. Moreover, two functional forms of the model were compared: the traditional Bass model and the Logistic function. The methods of cumulative observations vs. rolling windows and the methods of fixed y-intercept vs. floating y-intercept were also compared. Finally, we studied the suitability of three estimation algorithms for sales forecasting: Quasi-Newton, Gauss-Newton and Newton-Raphson.

Our major results lead to the following conclusions:

1. OLS is better than EGLS for parameter estimation in the Logistic transformation process.
2. The sufficient number of observations is at least 7 months. The data should not exceed 24 months in order to make the forecasts accurate.
3. The Logistic function is superior to the Bass model in terms of forecasting performance. It also shows a clearer S-curve pattern..
4. The method of rolling windows out perfoms the method of cumulative observations. The optimal window width is 15 months.
5. The model with fixed y-intercept is much better than the with floating y-intercept when the intercept is pegged at the first deseasonalized value of the sales series.
6. Quasi-Newton seems to yield better forecasts than Gauss-Newton and Newton-Raphson when the accuracy is measured by MAPE from the out-

of-sample test. However, the Gauss-Newton and Newton-Raphson algorithms yield smaller AIC and BIC from the in-sample measurement of the goodness of fit. This may be considered another critical finding of this study that a model with higher AIC and BIC will yield a better predictive performance.