Chapter 5
Conclusion

In this work, firstly, we prove fixed point theorems for two classes of nonself multivalued mappings. Secondly, we derive common fixed point theorems for a commuting family of nonexpansive mappings one of which is multivaled. The main results are summarized as follows:

5.1 Fixed Point Theorems via Technique of Ultra- asymptotic Centers

(1) Let E be a weakly compact convex subset of a Banach space X having property (D'). Assume that $T : E \to KC(X)$ is a multivalued mapping satisfying condition $(*)$. If T is continuous, then T has a fixed point.

(2) Let E be a weakly compact convex subset of a Banach space X having property (D'). Let $T : E \to KC(X)$ be a multivalued mapping satisfying condition $(**)$. If T is an upper semicontinuous mapping and E is T-invariant, then T has a fixed point.

5.2 Common Fixed Point Theorems via Technique of Nonexpansive Retracts

(1) Let E be a weakly compact convex subset of a Banach space X. Suppose E has (MFPP) and (CFPP). Let S be any commuting family of nonexpansive self-mappings of E. If $T : E \to KC(E)$ is a multivalued nonexpansive mapping that commutes with every member of S. Then, $F(S) \cap Fix(T) \neq \emptyset$.

(2) Let E be a weakly compact convex subset of a Banach space X satisfying the Kirk-Massa condition. Let S be any commuting family of nonexpansive self-mappings of E. Suppose $T : E \to KC(E)$ is a multivalued mapping satisfying condition (C_λ) for some $\lambda \in (0, 1)$ that commutes with every member of S. If T is upper semicontinuous, then $F(S) \cap Fix(T) \neq \emptyset$.

(3) Let E be a weakly compact convex subset of a Banach space X. Suppose E has (MFPP) and (CFPP). Let S be any commuting family of nonexpansive self-mappings of E. If $T : E \to KC(E)$ is a multivalued nonexpansive mapping that commutes with every member of S. Suppose in addition that T satisfies:

(i) there exists a nonexpansive mapping $s : E \to E$ such that $sx \in Tx$ for each $x \in E$,

(ii) Fix(T)$= \{x \in E : Tx = \{x\}\} \neq \emptyset$.

Then, $F(S) \cap Fix(T)$ is a nonempty nonexpansive retract of E.