CHAPTER 4

Determination of radial velocity curve

4.1 The Least-Squares Deconvolution method

The Least-Squares Deconvolution (LSD) is a tool for the extraction an average high-precision line profiles from stellar spectra. The technique is based on two fundamental assumptions: (i) all spectral lines in the stellar spectrum are similar in shape, can be represented by the same average profile scaled in depth by a certain factor and (ii) the intensities of overlapping spectral lines add up linearly according to $Donati\ et\ al.$ (1997)[2].

The basic idea of LSD technique is to compute a mean of all individual spectral lines in a particular wavelength range. The solution is formulated mathematically as the whole spectrum $Y(v)$ is described as a sum of scaled and shifted identical profiles $Z(v)$,

$$Y(v) = \sum_i w_i \delta(v - v_i) Z(v_i) \tag{4.1}$$

$Y(v)$ is a model residual intensity and v is represents the velocity coordinate $c\Delta\lambda/\lambda$ associated with a wavelength shift $\Delta\lambda$ from the line central wavelength λ. The position in velocity space and the central line depths of the individual lines are respectively given by v_i and w_i.

From the Eq. 4.1 can be expressed as a convolution of the line pattern function and the mean profile

$$Y = M * Z, \tag{4.2}$$

where $M(v) = \sum_i w_i \delta(v - v_i)$

Alternatively, the expression can be represented as a matrix multiplication

$$Y = M \cdot Z, \tag{4.3}$$

Where M is a line pattern matrix containing information on line position and their relative strengths and mean of individual line profile spanning a certain range in velocity is represented by Z. The equation (4.3) can be expressed as the inverse problem

$$Z = (M^T \cdot S^2 \cdot M)^{-1} \cdot (M^T \cdot S^2 \cdot Y_0) \tag{4.4}$$

Here S^2 is a diagonal weighting matrix based on the variance of the data points in the spectrum. which solving for the LSD profile Z.

4.2 Selection of spectral range

The spectra obtained from HERCULES spectrograph cover approximately 47 order spectrum over the wavelength range of 4600-7000 Angstrom. However, the whole spectral range cannot be used in the LSD method because of a contamination. The earth's atmosphere is not transparent. Atoms and molecules can interact with the starlight. The wavelength range of starlight are absorbed from this interaction. These are telluric lines. As in previous chapter, this should not include in the LSD method, because their radial velocities are those of the Earth's atmosphere not of the starlight.

To analyze HERCULES spectra in this research, the chosen wavelength range are between 4677-5777 Angstrom to avoid telluric lines, strong chromospheric emission lines, and some wavelength range that are low signal-to-noise ratio.

4.3 Determination of Least-Squares Deconvolution profiles

In this section, LSD profiles was calculated to study the mean profile. According to section 4.1, to prepare LSD technique, the line pattern also computed by using SynthV code and extracted from VALD database to obtain line position and their relative strength. The VALD assume that a star has the same atmospheric parameters as in the previous chapter and a particular wavelength 4677-5777 Angstrom are determined as observed spectrum.

After a line pattern was prepared, to compute LSD profiles, program would be installed to calculate these line position with individual line strength more than 0.1. The

LSD profiles is used to perform line strength corrections for the mask that was initially used for the calculation of the mean profile, and the procedure repeats until no significant changes in the profile.

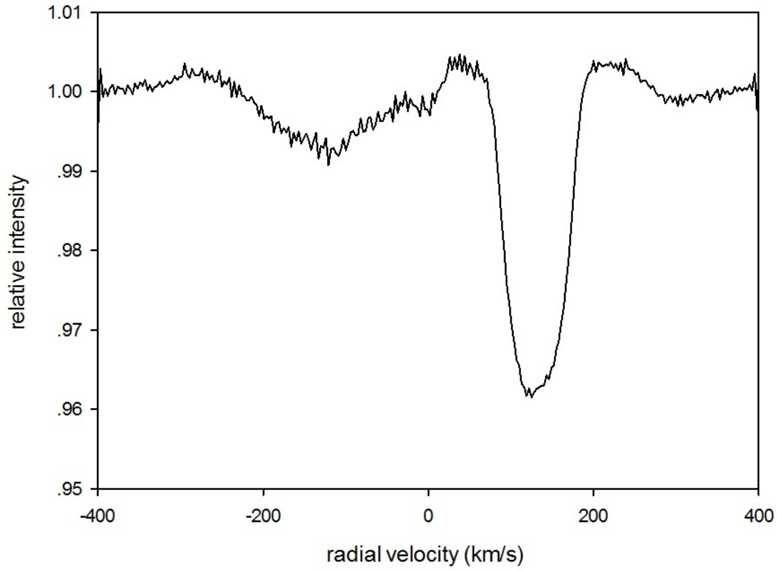

Figure 4.1: The example original LSD profile represent all individual spectral line.

After the LSD profiles were calculated, the mean profile will be appeared in velocity scale and be shifted according to whole line shifted in that spectrum which were expressed in figure 4.1. To measure the radial value, the LSD profile would be fitted with a polynomial function to find out a minimum value of the profile and then the radial velocity curve was obtained.

4.4 Improvement of a radial velocity

To improve the result, the LSD profile must be corrected from Barycentric velocity correction (BC) which each profile depend on the location of observatory and position of Earth around the Sun at that observed time. For the HERCULES, the Barycentric velocity correction value was already calculated and contained in the header file of each observed spectrum files, so it is instantly corrected. In table 4.1 was showed Barycentric velocity value and correct radial velocity in whole data.

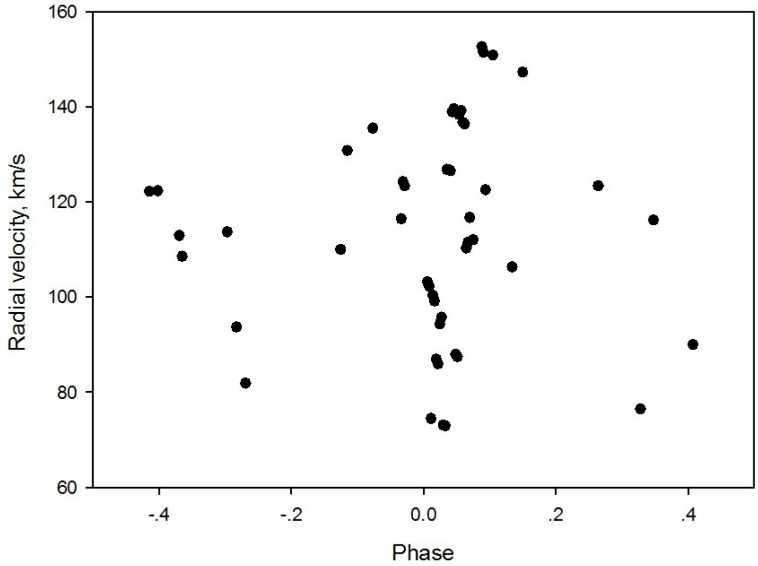

Figure 4.2: RV curve was obtained from each LSD profiles.

To check the validity of the result, the RV curve of main component was compared with a literature *Washuettl* (2009)[18] and found that a result of mean RV curve was different. The obtained mean RV curve was 101.902 km/s but from the literature it was 21.6 km/s which the different was 80.22 km/s. To check the different value, instrument correction was calculated by telluric absorption lines of observed spectrum in order to the validity of line position. The telluric line were shifted approximately 81.35 km/s. These LSD profiles and radial velocity values were subtracted by 81.35 km/s to correction. The result was shown in figure 4.3.

Since EI Eri is a single lined spectroscopic binary with only the RV of the primary component can be measured. No RV information is available for the secondary component as in figure 4.5 shows the RV curve of primary component.

4.5 Orbital analysis from radial velocities

Fundamental parameters of a system′s orbit can be derived from a set of radial velocities of any spectroscopic binary system. The analysis of the radial velocities is based

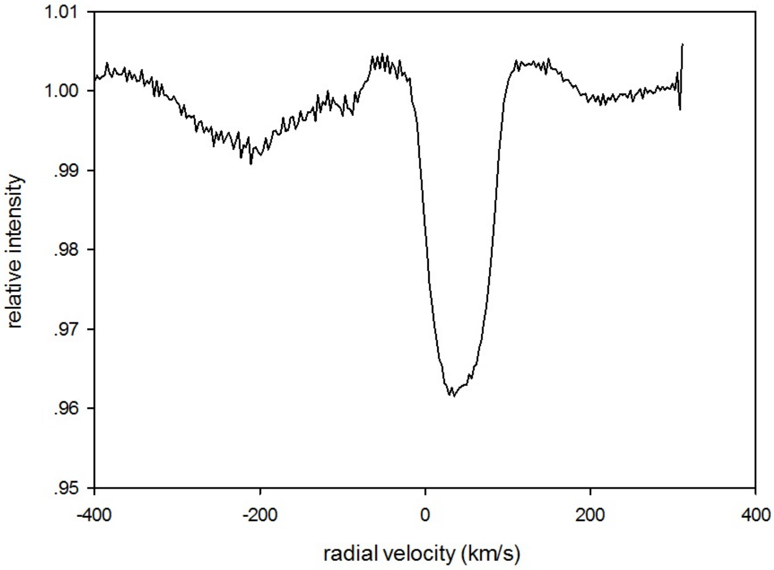

Figure 4.3: The expample LSD profile was adjusted Barycentric correction and substracted 81.35 km/s.

on the velocity equation

$$V_{red}(K, e, \omega, T, P, \gamma) = K[\cos(v + \omega) + e\cos\omega] + \gamma \tag{4.5}$$

where v is a function of T and P. This equation to determine these parameters was introduced by *Lehmann-Filhes* (1894). In present day, the analytical method and differential method are more generally used.

Least-squares differential correction

Six orbital elements of a particular binary system $K, e, \omega, T, P, \gamma$ can be determined from a set of *n* observations of radial velocity $V_{rad}(t)$. The best values of these elements can be found following an iterative least-squares analysis known as *differential corrections*

From equation (4.5), a small change in any of the orbital elements has an impact on the radial velocity. Thus, equation (4.5) can be expanded as a first-order Taylor series as.

$$\Delta V = \frac{\partial F}{\partial K}\Delta K + \frac{\partial F}{\partial e}\Delta e + \frac{\partial F}{\partial \omega}\Delta\omega + \frac{\partial F}{\partial T}\Delta T + \frac{\partial F}{\partial P}\Delta P + \frac{\partial F}{\partial \gamma}\Delta\gamma \tag{4.6}$$

The result of this partial derivative was first shown by $Lehmann - Filhes$ (1894). But Lehmann-Filhes's method will provide a reasonable result for a system with eccentricity e > 0.15. For a system with low eccentricity, such as a system that we study in this research, the solution become more unstable. This is because e approaches to zero, when the longitude of periastron ω and the time of periastron passage T are increasingly indeterminate.

To eliminate this problem, a new method of analysis was introduced by $Sterne$ (1941)[13].

$$\begin{aligned}
\Delta V \;=\; & [\cos(v+\omega)+e\cos\omega]\Delta K \\
& +K[\cos\omega-\frac{\sin(v+\omega)\sin v(2+e\cos v)}{1-e^2}]\Delta e \\
& -K[\sin(v+\omega)+e\cos\omega-\frac{\sin(v+\omega)(1+e\cos v)^2}{(1-e^2)^{3/2}}]\Delta\omega \\
& +K[\sin(v+\omega)(1+e\cos v)^2\frac{2\pi}{P(1-e^2)^{3/2}}]\Delta T \\
& +K[\sin(v+\omega)(1+e\cos v)^2\frac{2\pi(t-T)}{P^2(1-e^2)^{3/2}}]\Delta P \\
& +\Delta\gamma \qquad (4.7)
\end{aligned}$$

where T is replaced by the *time of zero mean longitude* T_0 in equation 4.7 which is the time of ascending-node passage. For a system with e approaching to zero, equation 4.7 becomes

$$\begin{aligned}
\Delta V \;=\; & \cos L\Delta K + Ke\cos\omega\cos 2L + Ke\sin\omega\sin 2L \\
& +\frac{2\pi K\sin L}{P}\Delta T_0+\frac{2\pi K(t-T_0)\sin L}{P^2}\Delta P+\Delta\gamma. \qquad (4.8)
\end{aligned}$$

For e < 0.03, The mean longitude (L) can be approximated as $L \approx v+\omega$.

Orbital solution of EI Eri

The radial velocity of this system was first found to be variable by Bidelman & MacConnell (1973)[1]. A. Washettl (2006) presented a new detailed determination of the parameters of the chromospherically active binary star EI Eri. Washettl investigated the orbit of this system from 354 radial velocity values from different observatories. In that analysis, the original orbital solution was recalculated orbital element for EI Eri. The original values are from Strassmeier (1990)[15] except T_0 which from Fekel et al. (1987) [5].

The orbital solution of this system as analysed by Washettl (2006) is a circular orbit with P = 1.947 days. The other parameters from their analysis can be found in table 4.2. These data was reanalyzed and it was found that the orbital solution was different from the published solution. This should be compared with the solution from HERCULES data.

In this research, there are 48 HERCULES spectra of EI Eri with the 41000 resolving power. It is found that the calculated eccentricity has a large error, e = 0.021±0.012. A circular orbit should be adopted from these data with a precision of 1.31 km/s. These two solution can be compared in table 4.3. The radial velocities and the residuals are shown in the plot of figure 4.6. The zero phase is at the time $T = T_0$ = 2453304.4522 ± 0.0052.

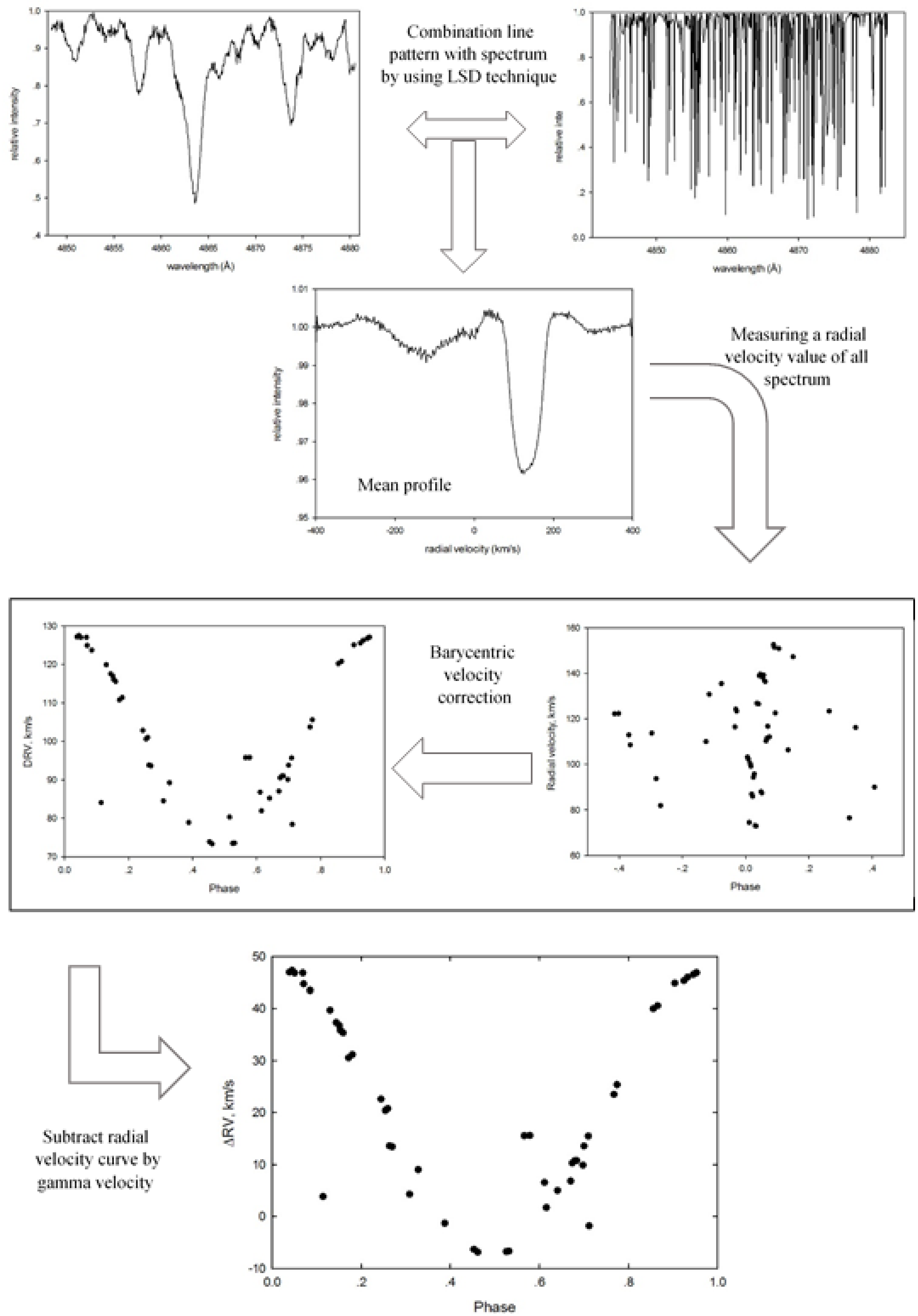

Figure 4.4: The Least-Squares Deconvolution method and improved of a radial velocities curve.

Table 4.1: The information of radial velocity value of each spectrum before and after correction.

Spectrum	JD	Phase	RV(original)	BC	RV(Cal)
3692043	53692.02689	0.0402	126.63	1.2892	46.8499
3692045	53692.03838	0.0461	126.588	1.2600	47.1747
3692047	53692.04951	0.0518	126.359	1.2316	46.6837
3402018	53401.94681	0.0697	152.435	-25.5682	46.7056
3404005	53403.89877	0.0721	151.235	-25.7469	44.5875
3404011	53403.92656	0.0864	150.665	-25.8010	43.3506
3406018	53405.93071	0.1156	106.174	-26.0468	3.7223
3406024	53405.96155	0.1315	147.077	-26.0970	39.5774
3659045	53659.12883	0.1454	102.966	15.2176	37.2212
3659047	53659.14197	0.1521	102.103	15.1853	36.6717
3741002	53740.93049	0.1546	136.54	-19.2946	35.7288
3741004	53740.94215	0.1606	136.224	-19.3223	35.2371
3663054	53663.07683	0.1729	100.204	13.8308	30.4383
3663056	53663.09392	0.1816	98.991	13.7895	31.0518
3377015	53376.97555	0.2457	123.211	-19.7922	22.501
3457019	53456.83214	0.2560	124.086	-21.1810	20.3026
3457021	53456.84189	0.2610	123.178	-21.1902	20.6787
3667071	53667.15057	0.2649	86.734	12.0649	13.4774
3667073	53667.16244	0.2710	85.801	12.0364	13.2898
3305034	53305.05306	0.3100	76.279	10.9504	4.1791
3424022	53423.87168	0.3292	116.024	-26.5757	8.8851
3350035	53349.99353	0.3891	89.825	-9.3432	-1.3995
3722027	53722.04135	0.4541	87.789	-12.3640	-6.4526
3722029	53722.05711	0.4622	87.291	-12.3984	-6.9756
3689059	53689.0797	0.5266	72.926	2.5427	-6.8511
3689061	53689.09107	0.5325	72.782	2.5143	-6.7814
3403008	53402.91479	0.5668	122.061	-25.6473	15.4347
3403014	53402.93952	0.5795	122.164	-25.6937	15.4653
3405062	53404.9502	0.6121	112.776	-25.9651	6.427
3403029	53403.01147	0.6165	108.387	-25.7971	1.6015
3779010	53778.87645	0.6417	111.888	-26.5388	4.8847
3660056	53660.15209	0.6709	74.306	14.7885	6.71
3742002	53741.94353	0.6748	110.137	-19.6488	10.185
3742004	53741.95441	0.6804	111.303	-19.6741	10.6418
3452010	53451.8247	0.6845	113.542	-22.4685	10.6594
3337024	53336.96585	0.6988	93.526	-3.3439	9.7473
3452018	53451.85694	0.7010	116.259	-22.5032	13.4509
3744010	53743.96015	0.7104	116.543	-20.3135	15.3562
3337028	53336.99263	0.7125	81.71	-3.4107	-1.8944
3668072	53668.13006	0.7680	94.154	11.7080	23.4055
3668074	53668.14388	0.7751	95.541	11.6740	25.2604

Spectrum	JD	Phase	RV(original)	BC	RV(Cal)
3304072	53304.16979	0.8564	109.83	11.0792	39.8767
3351026	53350.92286	0.8664	130.601	-9.6103	40.4575
3351036	53350.99799	0.9050	135.359	-9.7980	44.7282
3723014	53722.95886	0.9252	138.133	-12.5821	45.2051
3723016	53722.97576	0.9339	139.013	-12.6243	45.8984
3721034	53721.05345	0.9467	138.752	-11.9709	46.4065
3721036	53721.06649	0.9534	139.371	-11.9987	46.7394

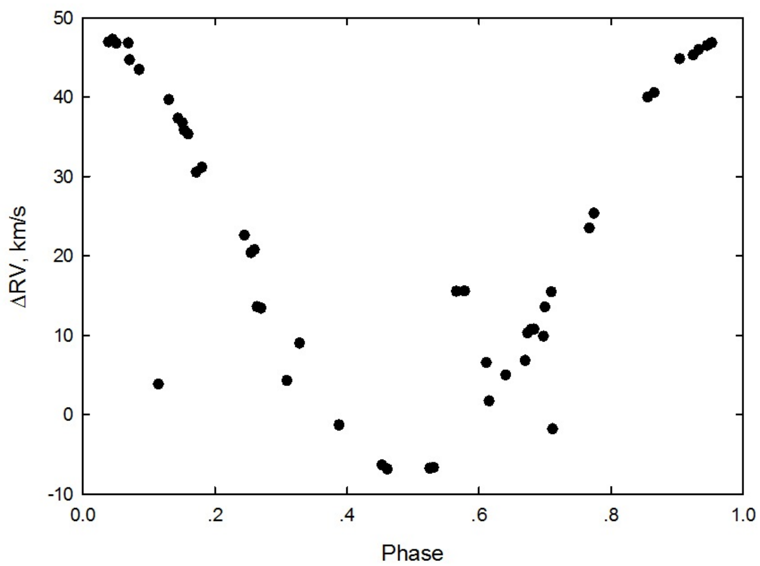

Figure 4.5: The RV curve was indicated total adjusted radial velocity of spectrum.

Table 4.2: Orbital parameters of HD 26337 as derived from the data collected by Washuttl (2006).

Parameter	Washuttl (2006)	
	published	recalculated
K (km/s)	27.4	26.83
e	0.0	0.0 (adopted)
ω	0.0	-
T_0 (HJD)	2446 074.384	2448 054.7109
P (days)	1.947 227	1.947 2324
γ (km/s)	17.6	21.64
T (HJD)	-	-
$a \sin i$ (km)	733 000	718 400
$f(M)$ ($M_\odot$)	0.004 15	0.003 91
$\sharp_{obj}$	-	-
$\sharp_{rej}$	-	-
σ (km/s)	-	3.2

Table 4.3: Orbital parameters of HD 26337 analyzed from HERCULES data.

Parameter	New value from this analysis	
	with $e = 0$	eccentric solution
K (km/s)	27.82 $\pm$ 0.25	27.62 $\pm$ 0.43
e	0.0	0.021 $\pm$ 0.012
ω	-	72 $\pm$ 50
T_0 (HJD)	2453 304.4522 $\pm$ 0.0052	2453 304.4380 $\pm$ 0.0074
P (days)	1.947 145 $\pm$ 0.000 037	1.947 219 $\pm$ 0.000 047
γ (km/s)	20.61 $\pm$ 0.19	20.60 $\pm$ 0.28
T (HJD)	2453 304.4522 $\pm$ 0.0052	2453 304.82 $\pm$ 0.27
$a \sin i$ (km)	744 900 $\pm$ 6900	740 000 $\pm$ 12 000
$f(M)$ ($M_\odot$)	0.004 34 $\pm$ 0.000 119	0.004 25 $\pm$ 0.000 202
$\sharp_{obj}$	42	44
$\sharp_{rej}$	6	4
σ (km/s)	1.31	1.88

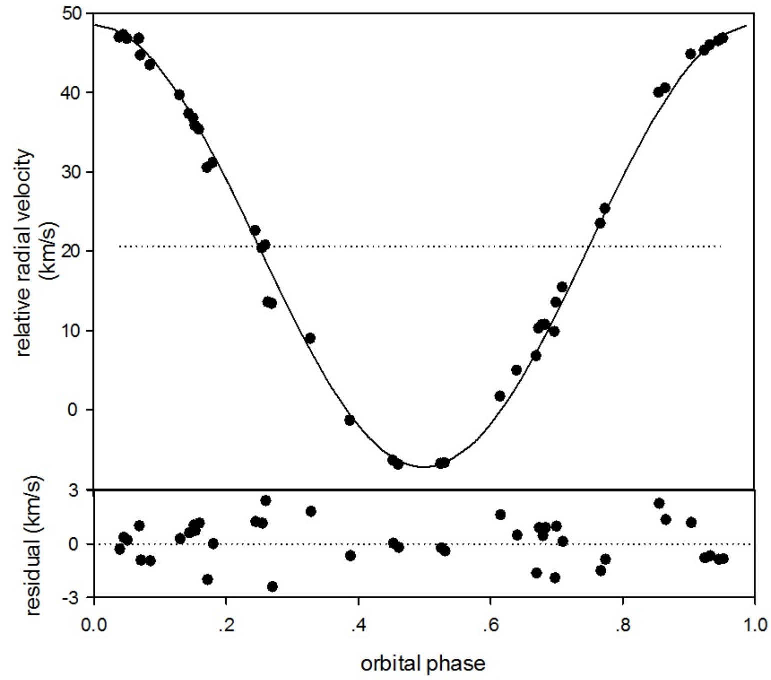

Figure 4.6: Phase plot of EI Eri radial velocities.