CHAPTER 2

Theoretical Foundation and Literature Review

2.1 Definitions

Fractal is a geometric shape that is self-similar and has fractional dimensions.

Fractal dimension is characterized as a measure of the space-filling capacity of a pattern that tells how a fractal scales differently from the space it is embedded in.

2.2 Literature review

Cromwell et al. (2000) tested the Hurst exponent of commodity price by using fractal analysis. This study evaluated how the apparent random fluctuation related to short run behavior can last when testing long run behavior. Then, they used the Hurst exponent to test the color of commodity price by categorizing in black, white, pink and brown noise.

Weron and Weron (2000) used a fractal approach to analyze financial time series by scaling different degrees of time intervals. They introduced the heterogenous market hypothesis that different market participants will analyze the past cases and news in accordance with different time horizons. Meanwhile, this study gave a new model for asset returns with fractal market hypothesis. It showed that the exponential dependence (CED) property in the capital market systems can be attached to investors who get involved into the market.

In order to estimate the Hurst exponent, Mandelbrot–Levy characteristic exponent, and fractal dimension, **Mulligan (2003)** tested technology equity price series by five self-affine fractal analysis techniques, such as rescaled-range analysis, the variogram or structure function method, power-spectral density analysis, roughness-length analysis, and wavelet analysis, The results strongly support the there is anti-persistence in many technology equities, suggesting markets cannot price all technology securities efficiently. **Mulligan (2010)** tested the fractal characters and stochastic dependence of Austrian business cycle with a big bunch of macro-monetary data by using rescaled

range and power spectral density analysis. Both of Austrian business cycle theory and real business cycle theory are evaluated by fractal statistics. The results support the anti-persistent stochastic dependence exists between transactions money and components of the monetary aggregates and most of the components directly affect the transactions, which suggest an active monetary policy. Saving shows a persistent long memory. However, the virtual economic behaviors show the anti-persistence, which strong violates the real business cycle theory.

Serletis and Andreadis (2004) tested the dynamical systems theory to approve random fractals for North American energy markets, by using daily crude oil data of Chicago and Henry Hub natural gas prices at Los Angeles on West Texas Intermediate (WTI). The results provide the strong evidence for WTI crude oil prices has a random multifractal structure, while Henry Hub natural gas prices only correspond with a random fractal model.

Tabak and Cajueiro (2007) tested the time-varying long-range dependence in crude oil price and volatility by using the rescaled range analysis. The study adopted the data (crude oil prices) form WTI and Brent. As results, the crude oil price in 1980s were much more inefficient than the 1990s, the H values are on the average 0.624 and 0.572 for Brent and WTI for the 1980s. Moreover, Hurst exponents are higher for the volatility of WTI crude oil prices if compared to Brent. Finally, the WTI crude oil prices are weaker form efficient than Brent prices.

Yao and Han (2008) employed the rescaled range analysis to test the fractality of China's real estate market. The author collected the daily closing price form 22 August 2001 to 11 May 2010 of the stock market Real Estate Index in China, and then they got the Hurst exponent H= 0.5810, which means that the time series is persistent, that is : there is fractal characteristic in the China real estate market.

Li and Lu (2011) used the Multi-fractal Detrended Analysis and multi-fractal spectrum analysis to test the multi-fractal characteristics of a volatility time series in China' agricultural commodity future price. They found that the multi-fractal properties existed obviously in China's main agricultural commodity future market.

Fernandez-Martinez and Sanchez-Granero (2012) provide a new model for computing the fractal dimension of a subset belongs to a generalized fractal space.

They thought that the fractal structure is a perfect place to get fractal dimension, and they employed discretization of the Hausdorff theory of fractal dimension. Hence, they calculated the fractal dimension of strict self-similar sets, which are not necessary for identifying the open set condition.

Ihlen (2012) found the fractal structures exist in biomedical time series of most physiological phenomena with a wide range. He estimated the multi-fractal spectrum of biomedical time series with multi-fractal detrended fluctuation analysis by suing the software Matlab.

Yin et al. (2013) studies the gold market with fractal analysis in China. This paper investigated the fluctuation features of China's gold market based on fractals. They found that China's gold market has fractal properties and determines the length of a non-periodic circulation, by using R/S analysis and fractal dimension analysis. Moreover, this paper employed the MF-DFA method confirming the factors that affect the gold market and multi-fractal characteristics. Therefore, this paper gave a predictable idea for gold investors and managers in accordance with price movement.

Table 2.1: Summery of literature review

Authors	Topic	Data	Method(s) of Analysis	Results
Cromwell, et al. (2000)	What color are commodity prices? A fractal analysis.	Fifteen international commodity spot spice in monthly (January 1960-June 1994) : bananas, beef, cocoa, coffee, copper, lead, rice, rubber, soybeans, sugar, tea, tin, wheat, wool, and zinc.	R/S analysis	Most of the agricultural commodities are black noises, and obtain the Hurst exponent value greater than 0.5 with long term memory, except bananas is pink noise.

Table 2.1: Summery of literature review (continued)

Authors	Topic	Data	Method(s) of Analysis	Results
Dong et al.(2009)	Multi-fractal Analysis of World Crude Oil Prices	Daily crude oil price on WTI (02 January 1986—01 July 2008) and Brent (20 May 1987—01 July 2008)	R/S analysis; Multi-fractal analysis.	WTI crude oil prices have persistence; both of them are persistent process with 12 days memories. Hurst exponent shows the non-liner characteristic of time series; Brent prices are more stable.
Li Z. & Lu X. (2011)	Multi-fractal analysis of China's agricultural commodity futures markets	Daily closing price for Hard Winter Wheat futures contract (28 March 2003—12 November 2010); Strong Gluten Wheat futures contract (01 November 1999—12 November 2010) form Zhengzhou Commodity Exchange.	Multifactal Detrended Fluctuation Analysis (MDFA)	Multi-fractality exists in China's agricultural commodity futures markets; Hard Winter

Table 2.1: Summery of literature review (continued)

Authors	Topic	Data	Method(s) of Analysis	Results
		Soy bean futures contracts (15 July 2002—12 November 2010) and corn futures contracts (22 September (2004—12 November 2010) from Dalian Commodity Exchange.		Wheat futures market has richer multi-fractality and wider singularity spectrum.
Mulligan (2003)	Fractal analysis of highly volatile markets	Daily closing prices data ,54 technology firms' traded equity (December 31, 1993 -- June 18, 2001)	R/S analysis, the variogram or structure function method, power-spectral density analysis, roughness-length analysis, and wavelet analysis.	There is anti-persistence in many technology equities.

Table 2.1: Summery of literature review (continued)

Authors	Topic	Data	Method(s) of Analysis	Results
Mulligan (2010)	A fractal comparison of real and Austrian business cycle models	The data are monthly and quarterly observed monetary aggregates, aggregate expenditures, and price and output indices, and productivity ratios of Austria .(01 April 1953 – 29 August 2008)	R/S analysis	Anti-persistent stochastic dependence exists between transactions money and components of the monetary aggregates and most of the components directly affect the transactions; Saving shows a persistent long memory.
Serletis & Andreadis (2004)	Random fractal structures in North American energy markets	Daily crude oil price on WTI (02 January 1990—28 February 2001); daily natural gas price on Henry Hub (24 January 1991—28 February 2001)	R/S analysis; The power spectrum;	WTI shows multi-fractal properties. Henry Hub natural gas prices are only consistent with a random fractal model.

Table 2.1: Summery of literature review (continued)

Authors	Topic	Data	Method(s) of Analysis	Results
Tabak and Cajueiro (2007)	Are the crude oil market becoming weakly efficient over time? A test for time-varying long-range dependence in prices and volatility	Daily closing prices of crude oil form 16 May 1983 to 28 July 2004 for WTI; from 16 May 1983 to 29 July 2004 for Brent.	R/S analysis	1980s were much more inefficient than the 1990s, H are on average 0.624 and 0.572 for Brent and WTI in the 1980s. Hurst exponents are higher for the volatility of WTI crude oil prices compared to Brent. The WTI crude oil prices are weaker form efficient than Brent prices.
Weron & Weron (2000)	Fractal market hypothesis and two power- laws	Standard & Poor 500 composite index on the New York Stock Exchange (02 July 1962 – 31 December 1991); NASDAQ index (14 December 1972 – 31 December	Conditionally exponential dependence (CED); Power laws.	S&P500 , NASDAQ , exchange rates show negative returns.

Table 2.1: Summery of literature review (continued)

Authors	Topic	Data	Method(s) of Analysis	Results
		1991); USD/SFR exchange rate (20 May 1985 – 12 April 1991)		
Yao & Han (2008)	The Empirical Study on Fractal Characteristics of Real Estate Market	The daily closing price of the stock market Real Estate Index in China. (22 August 2001—11 May 2010)	R/S analysis	H = 0.581, time series is persistent, there is fractal characteristic in the China real estate market.
Yin et al. (2013)	Fractal analysis of the gold market in China.	This paper selects the AU99.99 closing price data of the Shanghai gold exchange between October 30, 2002 and November 30, 2012 (2324 days in total).	R/S analysis	Long-term memory about 112 days is got in China's gold market; the gold price shows multi-fractalities.

2.3 Theorem

2.3.1 Fractal Theory

Fractals can be applied to the objects in fluctuations in time or in space, which has property of self-similarity and cannot be explained within a single absolute scale of measurement. Fractals means irregular segments of the whole one, which is like so many branches of a broccoli, one of the fragments is similar to a shifting or stretching copy when compared with the whole. There is a novel idea being provided

by fractal geometry, which is how both nonlinear and linear systems is due to the relevance of self-similar fluctuations over many time in orders, and how both systems are employed to the complex patterns over multiple levels of space.

There is still not a clearly definition of 'fractals', but Mandelbrot gave a loose definition of fractal in 1987, which is a structure, each of consisting of parts is a reduced size copy of the whole.

In 1960s, Mandelbrot tested the US agricultural commodity spot prices by employing the fractal geometry, which demonstrates that those open market prices do not comply with the " random walk" hypothesis, however, they shows different attributes like long-run correlation or memory, chaos, fractal.

The most important feature of fractal objectives is self-similarity. It is the degree of the fragmentariness and irregularities at different scales. When changed the scale, the observation has the picture entirely identical or similar to the initial one. Like when we use the camera, selecting different zooms to get the picture we want, sometimes we need the whole scenery, but sometimes we want the close-up. Zoom here is just like a scale. We employ them to describe the different sceneries belong to the whole one in accordance with our demands.

Hausdorff dimension is a vital characteristic of fractal, which can be fractional, but opposed to a topological dimension. For price movement, the Hausdorff dimension is tightly linked with the Hurst coefficient (or the Hurst exponent). Hausdorff dimension is unique and certain; it is defined as mono-fractals. However, most fractals have different fractal dimensions that dependent on their scales in the real nature. They are called multifractals.

Some instinct properties in the complex systems can lead to dynamic consequences, such as symmetry and invariance. Many physical systems occupy the features of scale variance, and most of them possess general power law scaling, hence, the quantities can be defined as a function of power law scaling, the function depends on scaling invariant ratios of the parameters.

However, the noise exists in real environment and realistic system can destroy the symmetry of a system. The serious question is that how much isomorphism between the real system and its fractal model.

2.3.2 Classification of Color noises

According to spectral activity, commodity prices can be associated with different colors at different time, such as white noise or Brownian motion. Due to statistical approaches that aimed at analyzing the underlying economic behavior, which whether can be definite as random, but the economic behaviors are not always fully random, because demand and supply have strong force to change the situation in short time, which would suggest that price behavior has more regular form. Hence, those behaviors can be explained spectrally. It can be classified with different colors to indicate the potential economic meanings. The economic behavior cannot be predicted for white noise, and it cannot be predicted in increments if it was brown noise, however, the black noise indicates that there are long term persistence and long term memory, which can be used for price forecasting.

Table 2.2: Classification of noises by power law exponent

Noise Classification	**Power Law** $f^{-\alpha}$
White	0
Pink	1
Brown	2
Black	$\lvert\alpha\rvert > 2$

Source: Schroeder, M. R. (1991), *Fractal, Chaos, Power Law*. New York: W.H. Freeman.

The noise process is divided into different color based on its each power law exponent α . The color of noise divides into four colors such as white, pink, brown, and black. Schroeder (1991) classifies the color of noise like in Table 2.1.

When α=0, the economic behavior is defined as white noise. Each value of the process is totally independent of its past values. In other words, this process cannot be predictable completely. Schroeder (1991) defined pink noise, when a situation happen between white and brown noise, as α=1, and is associated with hyperbolic behavior, as found in nonlinear models of chaotic behavior. Brown noise or Brownian motion is defined when α=2, where the increments are independent of past, given by Mandelbrot and Van Ness, 1968. These processes are corresponding to the idea of

economic market efficiency and prediction. At last, when α >2, black noise is occurred, and it is related to gigantic disasters such as droughts and hurricanes. Because the divergence of an independent increments series is dependent on the square root of time, these processes denote a high level of persistence. Black noise process demonstrates the strong economic growth or decline during a period of time, or the price falls or increases of all a sudden.

Hurst (1951) applied the method of rescaled range analysis (R/S analysis) by using the single fractal feature as the first test based on fractal theory. The R/S analysis yields the so-called Hurst exponent *H*. Let α is the power law exponent. The Hurst exponent is related to the power law exponent α as:

$$H(\alpha) = (\alpha-1)/2 \quad (1)$$

Since the Hurst exponent *H* is related to the power law exponent α, the dynamic characteristics of time series can be known when the fractal dimension *H* is determined. Knowing the dynamics characteristics of time series, suitable mathematical models can be rationally selected.