CHAPTER 4

Results and discussion

This chapter reports the results of 3-D eye gaze tracking by using a three dimensional eye model with a single camera. Experiments were tested on 50 participants by setting a distance between the participant's head and the screen approximately between 60 and 80 cm.

The experimental results are divided into six sections. Section 4.1 describes the results of the eye gaze distance estimation obtained by three proposed methods: using the principle of gray-level intensity of image patches; using the eigenvalues of the iris; and using the iris area for eye gaze distance equation modeling.

The results of 3-D eye gaze tracking system by using the principle of the summation of gray-level intensity of image patches in eye gaze distance estimation are described in section 4.2. The 3-D eye gaze on the screen estimation was tested under three conditions of the participant’s head movements: first, the participant’s head was located at the center direction of the screen; second, the participant’s head was tilted to the right; and third, the participant's head was tilted to the left of the screen.

The results of 3-D eye gaze tracking on the computer screen obtained by using the iris area for eye gaze distance computation are described in section 4.3. For each set of results obtained from each proposed method, the 3-D eye gaze tracking system efficiency was evaluated by the confidence interval of the test point on the computer screen computation shown in section 4.4. The results of confidence interval for eye gaze distance estimation are shown in section 4.5, and in section 4.6 a summary and discussion are given.

4.1 Experimental results of eye gaze distance estimation

The eye gaze distance between the participant's head and the screen is important for eye gaze tracking system because head movements can cause errors in estimating point of gaze on the screen estimation. Therefore, various methods for estimating the eye gaze distance were proposed: the first method using the summation of gray-level intensity of image patches; the second one using the eigenvalues; and the last one using the iris area. The efficiency of each proposed method was tested with different regression methods. The results consist of the accuracy, the mean average error, and the standard deviation of distance estimation.

4.1.1 Results of eye gaze distance estimation based on gray-level intensity of image patches

An equation of eye gaze distance estimation on the principle of the summation of gray-level intensity of image patches was constructed and tested for the efficiency of the proposed method. There are three models of the eye gaze distance estimation which are the linear least squares, the least squares second order polynomial, and power regression. Third models were compared with two-eye distance method and the pixels count method. The results of distance accuracy estimation are shown in Table 4.1. The results of the average distance estimation were performed with five regression methods as shown in Table 4.2.

Table 4.1 Results of eye gaze distance estimation accuracy based on gray-level intensity of image patches.

Regression method	Estimated distances accuracy					
	60 cm	65 cm	70 cm	75 cm	80 cm	Average (%)
Linear least squares	93.75	95.33	95.94	96.20	96.97	95.64
Least squares second order polynomial	97.21	96.92	97.24	95.11	96.24	96.54
Power	96.38	96.20	96.90	95.20	96.74	96.29
Two-eye distance	90.66	92.55	92.95	94.82	94.97	93.19
Pixels count	90.56	88.70	91.34	95.54	95.40	92.31

Table 4.2 Results of eye gaze distance estimation based on gray-level intensity of image patches.

Regression method	Estimated distances				
	60 cm	65 cm	70 cm	75 cm	80 cm
Linear least squares	59.14	65.92	72.43	74.94	80.08
Least squares second order polynomial	61.44	65.20	70.56	73.28	79.40
Power	61.56	66.72	70.66	71.94	79.20
Two-eye distance	65.02	69.30	73.95	77.52	82.21
Pixels count	58.80	70.13	74.98	76.84	81.24

In addition, the mean average errors of the results of the proposed method are shown in Table 4.3.

Table 4.3 Results of the MAE of the eye gaze distance estimation based on gray-level intensity of image patches.

Regression method	Mean absolute errors (MAE)					
	60 cm	65 cm	70 cm	75 cm	80 cm	Average (cm)
Linear least squares	3.74	3.03	2.83	2.84	2.41	2.97
Least squares second order polynomial	1.72	2.00	1.98	3.63	3.05	2.47
Power	2.17	2.47	2.17	3.60	2.61	2.60
Two-eye distance	5.60	4.83	4.93	3.88	4.01	4.65
Pixels count	5.66	7.34	6.05	3.4	3.67	5.21

From Table 4.1, it can be seen that the value obtained by using the least squares second order polynomial regression method gave the best result of the maximum average accuracy which is 96.54%. The pixel count method showed 92.31% accuracy which is lower than those of other regression methods.

4.1.2 Results of eye gaze distance estimation based on eigenvalues

The eye gaze distance estimation can be derived by using the principle of the eigenvalues of the participant's iris images. The equation of the eye gaze distance estimation was constructed by using the participant's iris images which were taken from a distance of 60 to 80 cm. Tests for the eye gaze distance equation performances were given to 30 participants. The performance of eye gaze distance estimations consists of the accuracy, the mean average distance, the mean absolute error, and the standard deviation. The results of eye gaze distance estimation obtained by the exponential, linear, logarithmic, and power regression methods are shown in Table 4.4 to Table 4.7.

Table 4.4 Results of eye gaze distance estimation based on eigenvalues by using the exponential regression method.

	Estimated distances					
	60 cm	65 cm	70 cm	75 cm	80 cm	Average
Accuracy (%)	96.10	96.55	96.68	97.08	97.04	96.69
Mean absolute error (cm)	2.40	2.26	2.35	2.24	2.39	2.33
Standard deviation (cm)	2.89	2.80	2.83	2.75	2.85	2.82

	Estimated distances				
	60 cm	65 cm	70 cm	75 cm	80 cm
Mean average distance (cm)	59.96	64.89	70.60	75.00	79.15

Table 4.5 Results of eye gaze distance estimation based on eigenvalues by using linear regression method.

	Estimated distances					
	60 cm	65 cm	70 cm	75 cm	80 cm	Average
Accuracy (%)	94.08	95.64	96.61	96.95	96.62	95.98
Mean absolute error (cm)	3.57	2.89	2.46	2.32	2.68	2.78
Standard deviation (cm)	4.11	3.44	2.92	2.83	3.14	3.29

	Estimated distances				
	60 cm	65 cm	70 cm	75 cm	80 cm
Mean average distance (cm)	57.54	63.43	69.56	73.96	77.83

Table 4.6 Results of eye gaze distance estimation based on eigenvalues by using logarithmic regression method.

	Estimated distances					
	60 cm	65 cm	70 cm	75 cm	80 cm	Average
Accuracy (%)	96.43	96.30	96.47	96.34	96.08	95.33
Mean absolute error (cm)	2.96	2.65	2.42	2.53	2.85	2.68
Standard deviation (cm)	2.55	2.85	2.93	3.34	3.72	3.08

	Estimated distances				
	60 cm	65 cm	70 cm	75 cm	80 cm
Mean average distance (cm)	59.08	63.48	68.95	73.72	78.63

Table 4.7 Results of eye gaze distance estimation based on eigenvalues by using power regression method.

	Estimated distances					
	60 cm	65 cm	70 cm	75 cm	80 cm	Average
Accuracy (%)	97.03	96.53	96.36	96.12	95.79	96.37
Mean absolute error (cm)	2.01	2.33	2.49	2.85	3.26	2.59
Standard deviation (cm)	2.20	2.68	3.04	3.58	4.05	3.11

	Estimated distances				
	60 cm	65 cm	70 cm	75 cm	80 cm
Mean average distance (cm)	59.82	63.82	69.00	73.82	79.17

The test results of the exponential regression method yielded 96.69% accuracy which is higher than those of the linear, logarithmic, and power regression methods. The test result of the exponential regression method showed 2.33 cm mean absolute error and 2.83 cm standard deviation.

4.1.3 The results of eye gaze distance estimation based on iris area

The eye gaze distance estimation by using the principle of iris area was also constructed from the participant's iris image. Both eyes images (the right and left eyes) were used to deduce the equation. In addition, the average of both irises area was used for modeling eye gaze distance equation. Tests evaluating the efficiencies of the eye gaze distance estimation equation were performed on 30 participants. In this case, the equation was used for determining the distance between the participant's head and the screen under conditions of head movements. The test results were obtained by using the exponential, linear, logarithmic, and power regression methods. Evaluation of the accuracy, the average distance, and the standard deviation were then performed. The results of the average accuracy are represented in Table 4.8. The results of the average distance and standard deviation are shown in Table 4.9 and Table 4.10, respectively.

Table 4.8 The results of eye gaze distance measurement accuracy based on iris area.

Regression methods	Accuracy (%)					
	60 cm	65 cm	70 cm	75 cm	80 cm	Average
Exponential	97.00	97.14	97.39	97.77	97.86	97.43
Linear	95.69	95.94	95.97	96.32	96.30	96.04
Logarithmic	97.36	96.27	95.73	95.91	96.18	96.29
Power	97.86	96.12	94.98	95.11	95.78	95.97

Table 4.9 The results of mean average distance computation based on iris area.

Regression methods	Estimated distances				
	60 cm	65 cm	70 cm	75 cm	80 cm
Exponential	60.42	65.22	69.95	74.80	79.09
Linear	56.81	62.59	67.96	73.10	77.38
Logarithmic	59.03	62.88	67.12	72.14	77.46
Power	59.29	62.67	66.63	71.59	77.22

Table 4.10 The results of standard deviation computation based on iris area.

Regression methods	Standard deviation (cm)					
	60 cm	65 cm	70 cm	75 cm	80 cm	Average
Exponential	1.93	1.93	1.63	1.51	1.51	1.70
Linear	2.43	2.27	1.80	1.55	1.47	1.90
Logarithmic	1.47	1.62	1.55	1.69	2.04	1.67
Power	1.27	1.48	1.48	1.72	2.23	1.64

From the results shown in Table 4.8, the accuracy of the exponential method, which is 97.43%, is the best among results obtained from all methods. The average distance estimation of the exponential regression method is better than those of the linear, logarithmic, and the power regression methods. Although the standard deviation of the power regression showed the lowest value, the exponential method shows higher values in the accuracy and the average distance estimation than those of the power method. Thus, the exponential regression method was selected to determine the eye gaze distance estimation and used in the experiment.

4.2 Results of eye gaze tracking system using gray-level intensity of image patches for eye gaze distance estimation.

4.2.1 Head was placed at the center direction of the screen

The first experiment used two near-infrared light sources for illuminating the participant's cornea. Meanwhile, the distance between the participant and the screen was derived by using the principle of the summation of gray-level intensity of image patches from each eye of the participant. The experiment was tested when the participant's head was placed at the center direction of the screen. Two near-infrared light sources were attached at the bottom left and right of the screen frame. A single camera was located at the center bottom of the screen with resolution of 1920 × 800 pixels and the size of 17 inches (34 cm × 27 cm). A C-mount lens with a focal length of 25 mm was used. The distance between the participant and the screen was approximately 60 to 80 cm. Then, the performance of the proposed method of the eye gaze tracking system was evaluated. After that, the root mean square errors (*RMSE*) of the measured positions were computed by comparing to the ground truth positions. The *RMSE* values of the point of gaze on the computer screen positions can be calculated as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2\right)}{N}} \tag{4.1}$$

where (x_0, y_0) is the defined positions on the screen, and (x_i, y_i) is the experimental results of the eye gaze on the screen position, and N is the number of positions on the screen.

Fifty one participants were used to verify the proposed method. For each participant, tests were done by setting the distance between the participant's head and the screen as 60 to 80 cm, with the increment of 5 cm. The screen was divided into six rows and ten columns. Thus, each distance has 60 test points for the experiment. Therefore, 300 test points were provided to each participant in the experiment. The experimental results include the *RMSE*, and the standard deviation (SD) of the point of gaze estimation. The results of the eye gaze on the screen estimation when the participant's head was placed in the center direction of the screen are shown in Table 4.11.

Table 4.11 The *RMSE* of eye gaze on the computer screen estimation when the participant's head was placed in the center direction of the screen.

Distance (cm)	*RMSE* (cm)	SD (cm)	*RMSE* min.	*RMSE* max.
60	2.59	0.34	1.91	3.77
65	2.49	0.29	1.86	3.21
70	2.52	0.30	1.91	3.30
75	2.59	0.28	2.17	3.68
80	2.55	0.24	1.91	3.15
Average	2.55	0.29	1.95	3.42

Table 4.11 showed the eye gaze system performance which has 2.55 cm *RMSE*, 0.29 cm standard deviation, 1.95 cm minimum *RMSE*, and 3.42 cm maximum *RMSE*. In addition, the results of eye gaze distance estimation and the iris radius estimation are represented in Table 4.12 to Table 4.13.

Table 4.12 Experimental results of distance estimation when the participant's head was placed in the center direction of the screen.

Distance (cm)	Estimated distances (cm)	SD (cm)	Minimum distances (cm)	Maximum distances (cm)	Distances accuracy (%)
60	62.89	2.09	61.43	68.42	95.19
65	64.42	2.12	63.41	69.15	97.11
70	70.80	2.19	66.16	75.86	97.51
75	74.34	2.40	74.09	80.89	97.53
80	79.54	2.77	82.14	86.64	97.05
Average		2.31			96.88

Table 4.13 Experimental results of iris radius estimation when the participant's head was placed in the center direction of the screen.

Distance (cm)	Estimated iris radii (cm)	SD (cm)	Min. Estimated iris radii (cm)	Max. Estimated iris radii (cm)
60	1.12	0.05	1.00	1.27
65	1.11	0.05	1.03	1.24
70	1.14	0.06	0.99	1.29
75	1.15	0.07	1.02	1.37
80	1.17	0.07	1.02	1.32
Average	1.14	0.06	1.01	1.30

4.2.2 The participant's head tilted to the right and the left of the screen

The second experiment was performed when the participant's head was tilted to the right and left of the screen. The experiment system setup was identical to the previous system setting. Fifty participants were used for testing the performance of the proposed method. The results of *RMSE*, the average distance estimation, and the average iris radius estimation for the participant's head movements are shown in Table 4.14 to Table 4.16.

Table 4.14 The *RMSE* of eye gaze on the computer screen estimation when the participant's head was tilted to the right of the screen.

Distance (cm)	*RMSE* (cm)	SD (cm)	*RMSE* min.	*RMSE* max.
60	2.72	0.32	2.14	3.47
65	2.72	0.30	2.18	3.45
70	2.68	0.20	2.14	3.13
75	2.74	0.25	2.24	3.58
80	2.75	0.30	2.19	3.59
Average	2.72	0.27	2.21	3.44

Table 4.15 Experimental results of distance estimation when the participant's head was tilted to the right of the screen.

Distance (cm)	Estimated distances (cm)	SD (cm)	Minimum distances (cm)	Maximum distances (cm)	Distances accuracy (%)
60	63.27	1.52	60.29	66.81	94.55
65	64.75	1.99	60.68	68.62	97.56
70	70.32	2.26	63.74	75.62	97.40
75	75.21	2.19	71.17	79.54	97.71
80	80.12	2.20	75.79	85.02	97.68
Average		2.03			96.98

Table 4.16 Experimental results of iris radius estimation when the participant's head was tilted to the right of the screen.

Distance (cm)	Estimated iris radii (cm)	SD (cm)	Min. Estimated iris radii (cm)	Max. Estimated iris radii (cm)
60	1.11	0.05	1.00	1.22
65	1.10	0.06	0.94	1.22
70	1.11	0.05	1.00	1.27
75	1.11	0.06	0.96	1.28
80	1.14	0.07	1.00	1.28
Average	1.11	0.06	0.98	1.25

The result of eye gaze on the screen estimation when the participant's head was tilted to the right of the screen achieved 2.72 cm *RMSE* which is greater than the result obtained when the participant's head was placed in the center direction of the screen. In the same manner, tests were performed on fifty participants under the condition that the participant's head was tilted to the left of the screen. The results which include the *RMSE*, the distance estimation, and the iris radii estimation are represented in Table 4.17 to Table 4.19.

Table 4.17 The *RMSE* of eye gaze on the computer screen estimation when the participant's head was tilted to the left of the screen.

Distance (cm)	*RMSE* (cm)	SD (cm)	*RMSE* min.	*RMSE* max.
60	2.81	0.32	2.17	3.63
65	2.79	0.32	2.11	3.79
70	2.78	0.27	1.98	3.38
75	2.82	0.33	1.95	4.20
80	2.83	0.29	2.12	3.60
Average	2.81	0.31	2.06	3.72

Table 4.18 Experimental results of distance estimation when the participant's head was tilted to the left of the screen.

Distance (cm)	Estimated distances (cm)	SD (cm)	Minimum distances (cm)	Maximum Distances (cm)	Distances Accuracy (%)
60	63.39	1.76	60.58	68.45	94.35
65	65.81	2.07	61.20	71.58	97.34
70	70.80	1.94	66.82	76.41	97.61
75	75.27	2.23	71.30	80.26	97.55
80	79.86	2.70	73.39	85.08	97.33
Average		2.14			96.84

Table 4.19 Experimental results of iris radius estimation when the participant's head was tilted to the left of the screen.

Distance (cm)	Estimated iris radii (cm)	SD (cm)	Min. Estimated iris radii (cm)	Max. Estimated iris radii (cm)
60	1.10	0.04	1.02	1.18
65	1.10	0.05	1.00	1.22
70	1.12	0.05	1.02	1.20
75	1.12	0.06	0.94	1.26
80	1.13	0.06	1.00	1.29
Average	1.11	0.05	1.00	1.23

The best results of eye gaze tracking on the computer screen estimation is 2.55 cm *RMSE* when the participant's head was placed at the center direction of the screen. Meanwhile, the results of eye gaze tracking on the computer screen estimation with the participant's head movements show 2.72 and 2.81 cm *RMSE* for the head being tilted to the right and left of the screen, respectively. The result of the standard deviation of the *RMSE* computation for different conditions such as when the head was placed at the center direction, when the head was tilted to the right, and when the head was tilted to the left are 0.29, 0.27, and 0.31 cm, respectively.

The eye gaze distance estimation for head movements which uses the principle of the summation of the gray-level intensity of the image patches gives 96.91% average accuracy. The average of estimated iris radius is 1.11 cm.

4.3 Results of eye gaze tracking system using the iris area for eye gaze distance estimation

4.3.1 The participant's head was placed in the center direction of the screen

This experiment was tested with fifty participants under three conditions: when the participant's head is located in the center direction of the screen; when the participant's head was tilted to the right; and when the participant's head was tilted to the left of the screen. The difference in this experiment was that the eye gaze distance estimation was computed by using the iris area. The experimental results were obtained when the participant's head was placed in the center of the screen direction. The experimental results which used the principle of iris radius include the *RMSE*, the eye gaze distance estimation, and the iris estimation as represented in Table 4.20 to Table 4.22.

Table 4.20 The *RMSE* of eye gaze on the computer screen estimation when the participant's head was placed at the center direction of the screen.

Distance (cm)	*RMSE* (cm)	SD(cm)	*RMSE* min.	*RMSE* max.
60	2.42	0.34	1.74	3.75
65	2.38	0.28	1.86	3.05
70	2.43	0.31	1.87	3.49
75	2.49	0.30	1.97	3.02
80	2.49	0.28	1.82	3.13
Average	2.44	0.30	1.85	3.29

Table 4.21 Experimental results of distance estimation when the participant's head was placed at the center direction of the screen.

Distance (cm)	Estimated distances (cm)	SD (cm)	Minimum distances (cm)	Maximum distances (cm)	Distances accuracy (%)
60	61.35	2.26	56.33	65.68	96.51
65	62.65	1.33	58.58	65.21	96.37
70	70.29	1.41	66.01	73.16	98.43
75	73.46	1.36	70.21	76.70	97.68
80	78.51	1.18	76.28	81.11	97.99
Average		1.51			97.39

Table 4.22 Experimental results of iris radius estimation when the participant's head was placed at the center direction of the screen.

Distance (cm)	Estimated iris radii (cm)	SD (cm)	Min. Estimated iris radii (cm)	Max. Estimated iris radii (cm)
60	1.14	0.07	1.00	1.31
65	1.11	0.05	0.98	1.20
70	1.12	0.07	0.97	1.26
75	1.13	0.07	0.98	1.26
80	1.12	0.07	0.96	1.27
Average	1.12	0.07	0.98	1.26

The results in which the participant's head was located at the center direction of the screen show 2.44 cm *RMSE*, and 0.30 cm standard deviation. The result of distance estimation shows 97.39% accuracy and 1.51 cm standard deviation. The average iris radius estimated is 1.13 cm and the standard deviation is 0.07 cm.

Then, the eye gaze on the screen estimation using the principle of iris area for distance estimation was tested with fifty participants under the conditions that the participant's head was tilted to the right of the screen. The experimental results are shown in Table 4.23 to Table 4.25.

Table 4.23 The *RMSE* of eye gaze on the computer screen estimation when the participant's head was tilted to the right of the screen.

Distance (cm)	*RMSE* (cm)	SD (cm)	*RMSE* min.	*RMSE* max.
60	2.52	0.30	1.89	3.47
65	2.44	0.28	2.07	3.03
70	2.50	0.27	1.95	3.02
75	2.51	0.24	2.04	3.05
80	2.55	0.26	2.03	3.10
Average	2.50	0.27	2.02	3.05

Table 4.24 Experimental results of distance estimation when the participant's head was tilted to the right of the screen.

Distance (cm)	Estimated distances (cm)	SD (cm)	Minimum distances (cm)	Maximum distances (cm)	Distances accuracy (%)
60	60.26	1.89	58.05	64.56	96.75
65	64.92	2.23	62.01	69.38	95.40
70	69.51	1.83	69.81	72.86	99.73
75	75.61	1.45	75.78	78.23	98.96
80	80.02	0.93	78.26	82.00	97.83
Average		1.66			97.73

Table 4.25 Experimental results of iris radius estimation when the participant's head was tilted to the right of the screen.

Distance (cm)	Estimated iris radii (cm)	SD (cm)	Min. Estimated iris radii (cm)	Max. Estimated iris radii (cm)
60	1.10	0.07	0.95	1.26
65	1.13	0.07	0.95	1.29
70	1.13	0.07	0.97	1.29
75	1.16	0.07	1.05	1.33
80	1.16	0.07	1.00	1.34
Average	1.14	0.07	0.98	1.30

The experimental results in which the participant's head was tilted to the right of the screen show 2.50 cm *RMSE* with 0.27 cm standard deviation. The eye gaze distance estimation was validated for the performance of the proposed method which gave 97.73% accuracy and 1.66 cm standard deviation. The estimated iris radius is approximately 1.14 cm. The standard deviation of the iris radius estimation is 0.07 cm. The maximum of the *RMSE* is 3.47 cm at the distance of 60 cm.

In the same manner, the proposed method was tested with fifty participants under the conditions that the participant's head was tilted to the left of the screen. The experimental results are shown in Table 4.26 to Table 4.28.

Table 4.26 The *RMSE* of eye gaze on the computer screen estimation when the participant's head was tilted to the left of the screen.

Distance (cm)	*RMSE* (cm)	SD (cm)	*RMSE* min.	*RMSE* max.
60	2.50	0.28	1.71	2.98
65	2.48	0.29	1.76	3.14
70	2.53	0.31	1.94	3.47
75	2.54	0.26	2.06	3.59
80	2.59	0.28	1.95	3.12
Average	2.53	0.28	1.88	3.26

Table 4.27 Experimental results of distance estimation when the participant's head was tilted to the left of the screen.

Distance (cm)	Estimated distances (cm)	SD (cm)	Minimum distances (cm)	Maximum distances (cm)	Distances accuracy (%)
60	60.95	1.91	56.98	65.46	97.15
65	64.38	1.56	63.11	67.51	98.01
70	70.89	1.64	67.38	74.13	97.75
75	73.69	1.55	74.46	76.48	97.82
80	78.42	2.05	80.06	82.13	97.56
Average		1.74			97.66

Table 4.28 Experimental results of iris radius estimation when the participant's head was tilted to the left of the screen.

Distance (cm)	Estimated iris radii (cm)	SD (cm)	Min. Estimated iris radii (cm)	Max. Estimated iris radii (cm)
60	1.16	0.08	1.00	1.33
65	1.16	0.07	1.02	1.28
70	1.15	0.07	1.01	1.32
75	1.14	0.06	1.02	1.28
80	1.13	0.07	1.00	1.29
Average	1.15	0.07	1.01	1.30

The results of 3-D eye gaze tracking on the screen estimation by using the principle of the iris area for determining the eye gaze distance show 2.44 cm *RMSE* in the case where the participant's head was placed at the center direction of the screen. For the situation where the participant's head was tilted to the right and left on the screen, the results of *RMSE* obtained are 2.50 and 2.53 cm, respectively.

In summary, the 3-D eye gaze tracking on the screen estimations obtained by using the principle of the summation of gray-level intensity of the image patch and by using the principle of the iris area for eye gaze distance estimation were compared and shown in Table 4.29 to Table 4.31.

Table 4.29 Comparisons on 3-D eye gaze tracking on the screen when the participant's head was placed at the center direction of the screen.

Methods for distance estimation	RMSE (cm)	SD (cm)	Distances accuracy	SD (cm)	Iris radii (cm)	SD (cm)
Gray-level intensity of image patches	2.55	0.29	96.88	2.31	1.14	0.06
Iris area	2.44	0.30	97.39	1.51	1.13	0.07

Table 4.30 Comparisons on 3-D eye gaze tracking on the screen when the participant's head was tilted to the right of the screen.

Methods for distance estimation	*RMSE* (cm)	SD (cm)	Distances Accuracy	SD (cm)	Iris radii (cm)	SD (cm)
Gray-level intensity of image patches	2.72	0.27	96.98	2.03	1.11	0.06
Iris area	2.50	0.27	97.73	1.66	1.14	0.07

Table 4.31 Comparisons on 3-D eye gaze tracking on the screen when the participant's head was tilted to the left of the screen.

Methods for distance estimation	*RMSE* (cm)	SD (cm)	Distances Accuracy	SD (cm)	Iris radii (cm)	SD (cm)
Gray-level intensity of image patches	2.81	0.31	96.84	2.14	1.11	0.05
Iris area	2.53	0.28	97.66	1.74	1.15	0.07

As shown in the results represented in Table 4.29 to Table 4.31, the best *RMSE* obtained is 2.44 cm. The high accuracy of the 3-D eye gaze tracking on the screen system was achieved by using the high accuracy of eye gaze distance estimation method. In order to evaluate the 3-D eye gaze tracking on the screen, according to the results, it can be concluded that the method using the eye gaze distance estimation based on the iris area is more accurate than the method using the principle of gray-level intensity of image patches.

4.4 Results of confidence interval of eye gaze tracking on the screen computation

The confidence interval is a method which was used to measure the efficiency of the result of eye gaze points tracking on the screen obtained by the proposed method. The experimental results of 3-D eye gaze tracking on the screen estimation obtained by using the principle the summation of gray-level intensity of the image patch and the principle of the iris area were validated by confidence interval.

For each distance, the sixty points of gaze on the screen were computed. Fifty participants were set as the number of the samples in order to compute the confidence interval. The confidence levels for γ were chosen as 95%, and 99% in computation.

The k value used to define the confidence interval was needed to be computed. The confidence interval could be computed by $k = cs\sqrt{n}$. The confidence interval for the mean μ of a normal distribution with unknown variance σ^2 was used.

The results obtained by using the principle of gray-level intensity of the image patch in the case where the participant's head was placed at the center direction of the screen are shown in Table 4.32 to Table 4.34.

Table 4.32 Confidence interval of the eye gaze on the screen using gray-level intensity of image patch when the participant's head was placed at the center direction of the screen.

Distance (cm)	Mean average errors (cm)	k	$CONF_{0.95}$	k	$CONF_{0.99}$
		γ=95%, n=50, c=2.01		γ=99%, n=50, c=2.68	
60	1.67	0.80	[0.87, 2.47]	1.07	[0.60, 2.74]
65	1.60	0.78	[0.82, 2.38]	1.04	[0.56, 2.64]
70	1.63	0.80	[0.83, 2.43]	1.06	[0.57, 2.69]
75	1.66	0.81	[0.85, 2.47]	1.07	[0.59, 2.73]
80	1.65	0.79	[0.86, 2.44]	1.06	[0.59, 2.71]
Average	1.64				

The results of the eye gaze on the screen estimation using the principle of gray-level intensity of the image patches showed the mean average error 1.64 cm when the participant's head was located in the center direction of the screen. The confidence interval computation was to measure the margin of error within the boundary of the estimated test points. When the confidence level of 95% was selected, as shown in Table 4.32, the accuracy of the estimated eye gaze point on the screen was located within a range of the test point distance of 65 cm. In addition, when the high confidence level of 99% was chosen, the confidence interval of the estimated eye gaze points on the screen had the margin of error minimum at the distance of 65 cm.

The results of confidence interval computation under the conditions that the head was tilted to the right and left of the screen are represented in Table 4.33 and Table 4.34.

Table 4.33 Confidence interval of the eye gaze on the screen by using gray-level intensity of image patch when the participant's head was tilted to the right of the screen.

Distance (cm)	Mean average errors (cm)	k	$CONF_{0.95}$	k	$CONF_{0.99}$
		γ=95%, n=50, c=2.01		γ=99%, n=50, c=2.68	
60	1.76	0.84	[0.92, 2.60]	1.12	[0.64, 2.88]
65	1.77	0.84	[0.93, 2.61]	1.11	[0.66, 2.87]
70	1.76	0.83	[0.93, 2.59]	1.11	[0.65, 2.87]
75	1.77	0.84	[0.92, 2.61]	1.13	[0.93, 2.90]
80	1.78	0.85	[0.93, 2.63]	1.14	[0.93, 2.92]
Average	1.77				

Table 4.34 Confidence interval of the eye gaze on the screen by using gray-level intensity of image patch when the participant's head was tilted to the left of the screen.

Distance (cm)	Mean average errors (cm)	k	$CONF_{0.95}$	k	$CONF_{0.99}$
		γ=95%, n=50, c=2.01		γ=99%, n=50, c=2.68	
60	1.82	0.87	[0.95, 2.69]	1.16	[0.66, 2.98]
65	1.80	0.86	[0.94, 2.66]	1.14	[0.66, 2.94]
70	1.79	0.86	[0.93, 2.65]	1.14	[0.65, 2.93]
75	1.83	0.89	[0.94, 2.72]	1.18	[0.65, 3.01]
80	1.82	0.97	[0.85, 2.79]	1.28	[0.54, 3.10]
Average	1.81				

The results of confidence interval computation when the participant's head was tilted to the right of the screen showed the best accuracy was on the distance of 70 cm. In addition, for the case when the participant's head was tilted to the left of the screen, the best result was on the distance of 65 cm and 70 cm.

In summary, the confidence interval for eye gaze point on the screen estimation which used the principle of the summation of gray-level intensity of image patches for distance estimation showed the best result when the participant's head was placed at the middle of the screen.

The results obtained by using the iris area in eye gaze distance computation are shown in Table 4.35 to Table 4.37.

Table 4.35 Confidence interval of the eye gaze on the screen by using iris area when the participant’s head was placed center direction of the screen.

Distance (cm)	Mean average errors (cm)	k γ=95%, n=50, c=2.01	$CONF_{0.95}$	k γ=99%, n=50, c=2.68	$CONF_{0.99}$
60	1.55	0.77	[0.78, 2.32]	1.02	[0.53, 2.57]
65	1.55	0.75	[0.80, 2.30]	0.99	[0.56, 2.54]
70	1.56	0.81	[0.75, 2.37]	1.07	[0.49, 2.63]
75	1.62	0.77	[0.85, 2.39]	1.03	[0.59, 2.65]
80	1.60	0.90	[0.70, 2.50]	1.20	[0.40, 2.80]
Average	1.58				

Table 4.36 Confidence interval of the eye gaze on the screen by using the iris area when the participant’s head was tilted to the right of the screen.

Distance (cm)	Mean average errors (cm)	k γ=95%, n=50, c=2.01	$CONF_{0.95}$	k γ=99%, n=50, c=2.68	$CONF_{0.99}$
60	1.63	0.79	[0.84, 2.42]	1.05	[0.58, 2.68]
65	1.59	0.79	[0.80, 2.38]	1.04	[0.55, 2.63]
70	1.63	0.78	[0.85, 2.41]	1.04	[0.59, 2.67]
75	1.62	0.78	[0.84, 2.40]	1.03	[0.59, 2.65]
80	1.65	0.80	[0.85, 2.45]	1.07	[0.58, 2.72]
Average	1.63				

Table 4.37 Confidence interval of the eye gaze on the screen by using the iris area when the participant’s head was tilted to the left of the screen.

Distance (cm)	Mean average errors (cm)	k γ=95%, n=50, c=2.01	$CONF_{0.95}$	k γ=99%, n=50, c=2.68	$CONF_{0.99}$
60	1.62	0.79	[0.83, 2.41]	1.04	[0.58, 2.66]
65	1.60	0.80	[0.80, 2.40]	1.06	[0.54, 2.66]
70	1.63	0.82	[0.81, 2.45]	1.09	[0.54, 2.72]
75	1.65	0.80	[0.85, 2.45]	1.06	[0.59, 2.71]
80	1.69	0.89	[0.80, 2.58]	1.18	[0.51, 2.87]
Average	1.64				

The results for two cases when the participant's head was tilted to the right and left of the screen are shown in Table 4.36 and Table 4.37. From the tables, it can be seen that the results gathered by using the principle of iris area gave the lower margin of errors than those gathered by using the principle of gray-level intensity of image patch. These results implied that the principle of iris area was more accurate than the principle of gray-level intensity of image patches in order to be used for the eye gaze tracking system.

In conclusion, the confidence interval computation can be used for measuring the margin of errors surrounding the tested points. In this experiment, all test points on the screen (i.e. 300 test points for each participant) were computed. From the results, it can be seen that the low confidence level (95%) gives more accurate results than the high confidence level (99%) for eye gaze point estimation.

The results obtained by using the principle of iris area showed low margin of errors than those obtained by using the principle of gray-level intensity of image patch. These results for eye gaze distance estimation confirmed that the method which used the principle of iris area provided higher accuracy for eye gaze on the screen estimation than the one which used the principle of gray-level intensity of image patch for the case of head movements.

4.5 Results of confidence interval of distance estimation

The confidence interval of the eye gaze distance estimation by using the principle of the gray-level intensity of the image patches and by using the principle of the iris area were computed. Also, the confidence levels for γ were chosen as 95%, and 99% in computation. The results obtained by the principle of gray-level intensity of the image patches under three conditions of head movements are represented in Table 4.38 to Table 4.40.

Table 4.38 Confidence interval of the eye gaze distance estimation by using gray-level intensity of image patch when the participant's head was placed at the center direction of the screen.

Distance (cm)	Estimated distances (cm)	k γ=95%, n=50, c=2.01	$CONF_{0.95}$	k γ=99%, n=50, c=2.68	$CONF_{0.99}$
60	62.89	0.59	[62.30, 63.48]	0.79	[62.10, 63.68]
65	64.42	0.60	[63.92, 65.02]	0.80	[63.62, 65.22]
70	70.80	0.62	[70.18, 71.42]	0.83	[69.97, 71.63]
75	74.34	0.68	[73.66, 75.02]	0.91	[73.43, 75.25]
80	79.54	0.79	[78.75, 80.33]	1.05	[78.49, 80.59]
Average		0.65		0.87	

Table 4.39 Confidence interval of the eye gaze distance estimation by using gray-level intensity of image patch when the participant's head was tilted to the right of the screen.

Distance (cm)	Estimated distances (cm)	k γ=95%, n=50, c=2.01	$CONF_{0.95}$	k γ=99%, n=50, c=2.68	$CONF_{0.99}$
60	63.27	0.43	[62.84, 63.70]	0.57	[62.69, 63.84]
65	64.75	0.57	[64.18, 65.31]	0.75	[63.99, 65.50]
70	70.32	0.64	[69.68, 70.97]	0.86	[69.47, 71.18]
75	75.21	0.62	[74.59, 75.84]	0.83	[74.38, 76.04]
80	80.12	0.62	[79.50, 80.74]	0.83	[79.29, 80.95]
Average		0.58		0.77	

Table 4.40 Confidence interval of the eye gaze distance estimation by using gray-level intensity of image patch when the participant's head was tilted to the left of the screen.

Distance (cm)	Estimated distances (cm)	k γ=95%, n=50, c=2.01	$CONF_{0.95}$	k γ=99%, n=50, c=2.68	$CONF_{0.99}$
60	63.39	0.50	[62.89, 63.88]	0.66	[62.73, 64.05]
65	65.81	0.58	[65.23, 66.39]	0.78	[65.03, 66.59]
70	70.80	0.58	[70.22, 71.38]	0.77	[70.03, 71.57]
75	75.27	0.64	[74.63,75.91]	0.85	[74.42, 76.12]
80	79.86	0.77	[79.09, 80.62]	1.02	[78.83, 80.88]
Average		0.61		0.82	

The results of confidence interval computation for distance estimation by using the principle of the iris area showed in Table 4.41 to Table 4.43.

Table 4.41 Confidence interval of the eye gaze distance estimation by using iris area when the participant's head was placed center direction of the screen.

Distance (cm)	Estimated distances (cm)	*k* γ=95%, *n*=50, *c*=2.01	$CONF_{0.95}$	*k* γ=99%, *n*=50, *c*=2.68	$CONF_{0.99}$
60	61.35	0.64	[60.71, 61.99]	0.86	[60.49, 62.21]
65	62.65	0.38	[62.27, 63.03]	0.51	[62.14, 63.15]
70	70.29	0.40	[69.88, 70.69]	0.54	[69.75, 70.82]
75	73.46	0.39	[73.08, 73.85]	0.52	[72.95, 73.98]
80	78.51	0.34	[78.17, 78.84]	0.45	[78.06, 78.95]
Average		0.43		0.57	

Table 4.42 Confidence interval of the eye gaze distance estimation by using the iris area when the participant's head was tilted to the right of the screen.

Distance (cm)	Estimated distances (cm)	*k* γ=95%, *n*=50, *c*=2.01	$CONF_{0.95}$	*k* γ=99%, *n*=50, *c*=2.68	$CONF_{0.99}$
60	60.26	0.54	[59.72, 60.80]	0.72	[59.54, 60.98]
65	64.92	0.63	[64.29, 65.56]	0.85	[64.08, 65.77]
70	69.51	0.52	[68.99, 70.03]	0.69	[68.82, 70.21]
75	75.61	0.41	[75.20, 76.02]	0.55	[75.06, 76.16]
80	80.02	0.26	[79.75, 80.28]	0.35	[79.66, 80.37]
Average		0.47		0.63	

Table 4.43 Confidence interval of the eye gaze distance estimation by using the iris area when the participant's head was tilted to the left of the screen.

Distance (cm)	Estimated distances (cm)	*k* γ=95%, *n*=50, *c*=2.01	$CONF_{0.95}$	*k* γ=99%, *n*=50, *c*=2.68	$CONF_{0.99}$
60	60.96	0.54	[60.41, 61.50]	0.72	[60.23, 61.68]
65	64.38	0.44	[63.94, 64.82]	0.59	[63.79, 64.97]
70	70.89	0.47	[70.43, 71.36]	0.62	[70.27, 71.51]
75	73.69	0.44	[73.25, 74.12]	0.59	[73.10, 74.27]
80	78.42	0.58	[77.84, 79.00]	0.78	[77.64, 79.19]
Average		0.49		0.66	

The results of confidence interval computation showed that the gray-level intensity of the image patches method gave higher confidence interval value than the iris area method did. Therefore, the principle of the iris area is more accurate than the principle of the gray-level intensity in estimating the eye gaze distance.

4.6 Summary and discussion

The aim of this thesis is to solve the problem of eye gaze tracking on the screen with low resolution. Typically, the accuracy of the eye gaze tracking system decreases with the participant's head movements. The objective of the proposed method is to introduce the novel of eye gaze tracking system which can be used with the participant's head movements. The proposed method performed the new eye gaze distance estimation to construct a three-dimensional eye model which would be applied in eye gaze mapping to the screen positions.

The eye gaze distance estimation function was derived by using 3 new methods: the principle of the summation of gray-level intensity of the image patches; the principle of the eigenvalues; and the principle of the iris area. Three proposed methods of the eye gaze distance estimation equation were tested for the performance. The eye gaze distance estimation which applied the principle of gray-level intensity of image patches by using the least squares second order polynomial gave 96.54% accuracy. On the other hand, the eye gaze distance estimation which applied the principle of the eigenvalues by using the exponential regression method gave 96.69% accuracy. And the eye gaze distance estimation which applied the principle of the iris area by using the exponential regression method gave 97.43% accuracy.

In this thesis, two methods which used the principle of the gray-level intensity of image patches and the principle of the iris area were selected to be tested in the experiment.

After the eye gaze distance had been computed, the real-world 3-D eye structure was able to be modelled by computing the eyeball radius, which is the twice of the iris radius obtained from the Hough ellipse transform. The visual angle of eye gaze on the screen could be calculated and the distance could be achieved by the proposed method.

Then, the eye gaze points on the screen were obtained by using the inverse perspective transformation.

The eye gaze on the screen position were tested on 51 participants under different conditions of the participant's head movements: when the participant's head was located at the center direction of the screen; when the participant's head was tilted to the right of the screen; and when the participant's head was tilted to the left of the screen. In order to confirm the suggested methods in eye gaze tracking on the screen, methods of using the principle of the gray-level intensity of the image patches and the principle of the iris area were chosen. Then the accuracy values obtained from two approaches were compared with each other.

The results of eye gaze distance estimation obtained by the principle of the summation of gray-level intensity of image patches showed 2.55 cm *RMSE* when the head was located at the middle of the screen, 2.72 cm *RMSE* when the head was tilted to the right of the screen, and 2.81 cm *RMSE* when the head was tilted to the left of the screen. Therefore, the eye gaze distance estimation showed 96.88%, 96.98% and 96.84% accuracy. The estimated iris radiuses were 1.14, 1.11, and 1.11 cm.

On the other hand, the results of eye gaze point on the screen estimation with the principle of the iris area under the same head conditions showed 2.44, 2.50, and 2.53 cm *RMSE*, respectively. The eye gaze distance estimation gave 97.39%, 97.73% and 97.66% accuracy. These results prove that the more accurate the eye gaze distance estimation is, the more accurate the computation of the eye gaze point on the screen becomes. Therefore, it confirms that the proposed method using the principle of the iris area can provide the high accuracy of eye gaze tracking on the computer screen with a single camera under the condition of the participant's head movements.

In addition, the efficiency of the proposed method for 3-D eye gaze tracking by using the principle of gray-level intensity of image patches and by using the iris area were determined by computing the confidence interval of all the gaze point on the screen from the experimental outcomes. The confidence level 95% and 99% were used for computing the confidence interval. We used the method of a confidence interval for the mean μ of a normal distribution with unknown variance σ^2 for confidence interval computation with 50 samples of *n*. The *c* values of the confidence level γ=0.95 was 2.01, and the confidence level γ=0.99 was 2.68.

The results of confidence interval computation when the participant's head was placed at the middle of the screen obtained from both approaches did not show much difference in accuracy values. Under the condition of the head movements, the iris area method is better than gray-level intensity method since the former method yielded lower confidence interval. These results showed that the proposed method can be applied in the eye gaze tracking system which uses the eye model obtained from a single camera under the condition of the participant's head movement.