CHAPTER 6

Controller Design

The main objective for the controller design presented in this chapter is to achieve global stability of the nonlinear rotordynamic system and thereby eliminate the possibility of sustained (or unstable) vibration response modes involving rotor-stator contact. The performance of the controller will also be examined in terms of reductions in contact force level when rotor-stator interaction is unavoidable. The controller which is considered in this chapter is termed a dynamic force feedback controller. This control approach requires the direct measurement of the rotor-stator interaction force. The design is based on the dynamic model and identified system parameters for the test rig as described in Chapter 4.

6.1 Controller Formulation

Consider a state space model of the rotor system including of control forces **u** applied to the rotor through the magnetic bearing:

$$\begin{aligned} \dot{\mathbf{w}} &= \mathbf{A}\mathbf{w} + \mathbf{B}_f\mathbf{f} + \mathbf{B}_u\mathbf{u} \\ \mathbf{z} &= \mathbf{C}\mathbf{w}, \quad \mathbf{f} = \beta(\mathbf{z}) \end{aligned} \tag{6.1}$$

Here, the nonlinear interaction model presented in section 4.4 is embedded in the static mapping $\beta : \mathbb{R}^2 \rightarrow \mathbb{R}^2$. A controller will be considered having the following form

$$\begin{aligned} \dot{\mathbf{w}}_c &= (\mathbf{A} + \mathbf{B}_u\mathbf{K})\,\mathbf{w}_c + \mathbf{B}_f\mathbf{f} \\ \mathbf{u} &= \mathbf{K}\mathbf{w}_c \end{aligned} \tag{6.2}$$

A schematic diagram of the controlled system is shown in figure 6.1. Comparing equations (6.1) and (6.2), it can be seen that the controller states will be an estimate of the component of the system states associated with the interaction force **f**. As the response component in $\mathbf{w}(t)$ due to **f** is the source of instability, accounting for this component through the control action $\mathbf{u} = \mathbf{K}\mathbf{w}_c$ should allow the stability properties of the system to

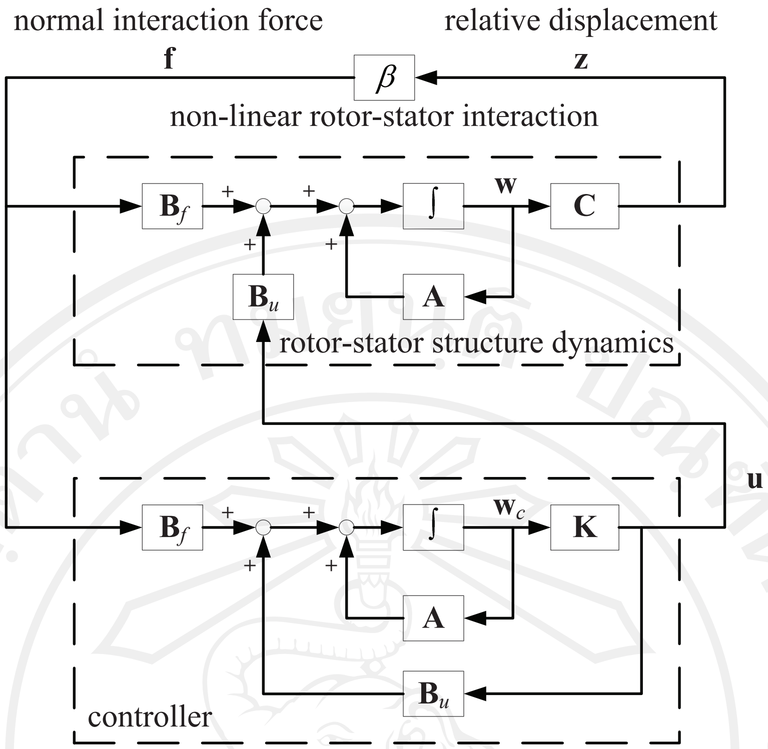

Figure 6.1: Control scheme based on feedback of rotor-stator interaction forces

be influenced. Note that implementation of this controller will require that direct measurement, or inference, of the rotor-stator interaction force is possible. This can be achieved by using signals from the force sensing device described in Chapter 3. The controlled system dynamics are given by

$$\begin{bmatrix} \dot{\mathbf{w}} \\ \dot{\mathbf{w}}_e \end{bmatrix} = \begin{bmatrix} \mathbf{A} + \mathbf{B}_u\mathbf{K} & -\mathbf{B}_u\mathbf{K} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{w}_e \end{bmatrix} + \begin{bmatrix} \mathbf{B}_f \\ \mathbf{0} \end{bmatrix} \mathbf{f} \tag{6.3}$$

where $\mathbf{w}_e = \mathbf{w} - \mathbf{w}_c$. It follows that if $\mathbf{A}$ is a stable matrix, which is implied by stable dynamics of the linear interaction-free system, $\mathbf{w}_e$ will converge to zero. Moreover, the dynamics of $\mathbf{w}$ and $\mathbf{w}_e$ are uncoupled and so a stability-performance analysis can be based on the following subsystem dynamics obtained by setting $\mathbf{w}_e = 0$ in (6.3):

$$\begin{aligned} \dot{\mathbf{w}} &= (\mathbf{A} + \mathbf{B}_u\mathbf{K})\,\mathbf{w} + \mathbf{B}_f\mathbf{f} \\ \mathbf{z} &= \mathbf{C}\mathbf{w}, \quad \mathbf{f} = \beta(\mathbf{z}) \\ \mathbf{y} &= \mathbf{C}_y\mathbf{w} \end{aligned} \tag{6.4}$$

The auxiliary output $\mathbf{y}$ is defined for the purpose of setting state-related specifications in the controller design. It is reasonable to include weighted components of both vibration states and control forces in the output vector according to

$$\mathbf{y} = \begin{bmatrix} \alpha\mathbf{z} \\ \mathbf{u} \end{bmatrix} \tag{6.5}$$

For this definition of $\mathbf{y}$ we have

$$\mathbf{C}_y = \begin{bmatrix} \alpha\mathbf{C} \\ \mathbf{K} \end{bmatrix} \tag{6.6}$$

where the scalar $\alpha \geq 0$ is a free parameter to be selected in the controller design process. The influence of this parameter will be examined in detail in Chapter 7.

The main objective for the controller design is to achieve global stability of the nonlinear system (6.4). Some attractive properties of the scheme described here are that:

1) The estimator-based formulation (6.2) leads to reduced-order stability analysis and controller synthesis problems.

2) The parameterization (and synthesis) of control solutions is made over the gain matrix $\mathbf{K}$.

3) The linear part of the system model (6.3) has a feed-forward structure and this has intrinsically robust stability properties.

4) A control action occurs only when limits of linear behaviour are exceeded i.e. when $\mathbf{f} \neq 0$. This means that an initial system/controller design can be made according to requirements of linear operation. The globally stabilizing controller can then be applied in parallel without affecting performance during linear operation.

6.2 Stability Conditions for Controlled System

The following subsections deal with how to obtain a suitable stabilizing gain matrix $\mathbf{K}$ for the proposed controller. Applying the Lyapunov theory developed in Section 5.2 and 5.3, subsection 6.2.1 develops mathematical conditions for global stability of a fixed equilibrium point for the controlled system (6.4). These conditions may be viewed as minimum requirements for stability. They do not, however, guarantee global stability of a forced response, which must be established if the possibility of a jump response is also to be eliminated. Therefore, subsection 6.2.2 extends the stability conditions to the case of global stabilization of a periodic forced response. These are the actual conditions used in the synthesis of the controller gain **K**.

6.2.1 Condition for global stability of a fixed equilibrium

Controller design should be made subject to minimization of a suitable cost function, considered here to be the generalized $\mathcal{H}_2$ norm of the nonlinear system (6.4). This system norm is evaluated as the worst-case $\boldsymbol{L}_2$ norm of the output $\mathbf{y}(t)$ over a specified set of initial values for the state vector (arising nominally due to the injection of impulse disturbances when $t = 0$). The norm-bound condition $\|\mathbf{y}\|_2 < \gamma$ holds if

$$\lim_{\tau\to\infty} \int_0^\tau \mathbf{y}^T\mathbf{y}\,dt < \gamma^2 \tag{6.7}$$

It is well known that this condition holds if there exists a Lyapunov function $V(\mathbf{w})$ satisfying

$$\dot{V}(\mathbf{w}) + \mathbf{y}^T\mathbf{y} \;<\; 0, \quad \forall \mathbf{w} \neq 0 \tag{6.8}$$

$$V(\mathbf{w}(0)) \;<\; \gamma^2 \tag{6.9}$$

A Lyapunov function may be adopted for system (6.4) that combines a quadratic function with a 2-dimensional Lur'e-type term, as defined in subsection 5.3.3

$$V(\mathbf{w}) = \mathbf{w}^T\mathbf{P}\mathbf{w} - 2\nu \int_0^{\mathbf{z}} \mathbf{f}^T\,d\mathbf{z} \tag{6.10}$$

where $\mathbf{P} = \mathbf{P}^T > 0$ and $\nu \geq 0$ must be determined such that (6.8) and (6.9) are satisfied. To account for the nonlinear relation between **f** and **z**, the constraint (5.20) may be augmented with the constraint (6.8) via the scalar $\mathcal{S}$-procedure. The resulting requirement is that

$\sigma > 0$ exists such that

$$\dot{V}(\mathbf{w}) - 2\sigma(k^{-1}\mathbf{f}^T\mathbf{f} + \mathbf{f}^T\mathbf{z}) + \mathbf{y}^T\mathbf{y} < 0, \quad \forall \begin{bmatrix} \mathbf{w} \\ \mathbf{f} \end{bmatrix} \neq 0 \tag{6.11}$$

where

$$\dot{V}(\mathbf{w}) = \dot{\mathbf{w}}^T\mathbf{P}\mathbf{w} + \mathbf{w}^T\mathbf{P}\dot{\mathbf{w}} - 2\nu\mathbf{f}^T\dot{\mathbf{z}} \tag{6.12}$$

With the system (6.4), $\dot{\mathbf{z}} = \mathbf{C}\dot{\mathbf{w}} = \mathbf{C}(\mathbf{A} + \mathbf{B}_u\mathbf{K})\mathbf{w} + \mathbf{C}\mathbf{B}_f\mathbf{f}$. When $\mathbf{C}\mathbf{B}_f = 0$ and $\mathbf{C}\mathbf{B}_u = 0$, equation (6.12) becomes

$$\dot{V}(\mathbf{w}) = \mathbf{w}^T[\mathbf{P}(\mathbf{A} + \mathbf{B}_u\mathbf{K}) + (\mathbf{A} + \mathbf{B}_u\mathbf{K})^T\mathbf{P}]\mathbf{w} + 2\mathbf{f}^T(\mathbf{B}^T\mathbf{P} - \nu\mathbf{C}\mathbf{A})\mathbf{w} \tag{6.13}$$

The stability condition (6.13) has the quadratic form

$$\begin{bmatrix} \mathbf{w} \\ \mathbf{f} \end{bmatrix}^T \mathbb{M}_0 \begin{bmatrix} \mathbf{w} \\ \mathbf{f} \end{bmatrix} < 0, \quad \forall \begin{bmatrix} \mathbf{w} \\ \mathbf{f} \end{bmatrix} \neq 0 \tag{6.14}$$

Or equivalently

$$\mathbb{M}_0 = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} < 0 \tag{6.15}$$

where

$$\begin{aligned} \mathbf{M}_{11} &= \mathbf{P}(\mathbf{A} + \mathbf{B}_u\mathbf{K}) + (\mathbf{A} + \mathbf{B}_u\mathbf{K})^T\mathbf{P} + \mathbf{K}^T\mathbf{K} + \alpha^2\mathbf{C}^T\mathbf{C}, \\ \mathbf{M}_{21} = \mathbf{M}_{12}^T &= \mathbf{B}_f^T\mathbf{P}^T - \nu\mathbf{C}\mathbf{A} - \sigma\mathbf{C}, \\ \mathbf{M}_{22} &= -2\sigma k^{-1}\mathbf{I} \end{aligned}$$

For a given initial condition $\mathbf{w}(0) = \mathbf{w}_0$, assumed to be interaction-free (i.e. $\mathbf{f}(0) = 0$), we have

$$V(\mathbf{w}_0) = \mathbf{w}_0^T\mathbf{P}\mathbf{w}_0 \tag{6.16}$$

Therefore, from (6.9), the $\mathcal{H}_2$ gain bound is satisfied if

$$\mathbf{w}_0^T\mathbf{P}\mathbf{w}_0 - \gamma^2\mathbf{I} < 0 \tag{6.17}$$

Although controller synthesis could be undertaken based on satisfying LMI equation (6.15) and (6.17), the analysis here does not take account of the efforts of disturbances. For a real rotor, the main source of disturbance is unbalance which will produce a non-zero vibration orbit. This will be considered in the next subsection.

6.2.2 Condition for global stability of a steady orbit

Consider the system model (6.1) with disturbances $\mathbf{d}(t)$ acting on the rotor (nominally unbalance force).

$$\dot{\mathbf{w}} = \mathbf{A}\mathbf{w} + \mathbf{B}_f\mathbf{f} + \mathbf{B}_u\mathbf{u} + \mathbf{d} \tag{6.18}$$

Suppose there is a solution to (6.18) given by $\mathbf{w}(t) = \mathbf{w}_d(t)$ for which $\mathbf{f}(t) = 0$, i.e. $\mathbf{w}_d(t)$ is a locally stable interaction free response. The objective for the control is to ensure that this solution is also globally stable, thereby eliminating the possibility of an alternative response for which $\mathbf{f}(t) \neq 0$. Defining $\mathbf{W}(t) = \mathbf{w}(t) - \mathbf{w}_d(t)$ and $\mathbf{W}_e(t) = \mathbf{w}_e(t) - \mathbf{w}_d(t)$, the requirement of $\mathbf{W}(t) \to 0$ when $t \to \infty$ for all $\mathbf{W}(0) = \mathbf{W}_0$ is needed to ensure that $\mathbf{w}_d(t)$ is a globally stable trajectory of the system. In this case, the dynamics of the controlled system may be derived from (6.18) and (6.2) as

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{W}} \\ \dot{\mathbf{W}}_e \end{bmatrix} &= \begin{bmatrix} \mathbf{A} + \mathbf{B}_u\mathbf{K} & -\mathbf{B}_u\mathbf{K} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{W} \\ \mathbf{W}_e \end{bmatrix} + \begin{bmatrix} \mathbf{B}_f \\ \mathbf{0} \end{bmatrix} \mathbf{f} \\ \mathbf{f} &= \beta(\mathbf{z}), \quad \mathbf{z} = \mathbf{C}\mathbf{W} + \mathbf{z}_d \end{aligned} \tag{6.19}$$

where $\mathbf{z}_d = \mathbf{C}\mathbf{w}_d(t)$. As long as $\mathbf{A}$ is stable matrix, $\mathbf{W}_e$ will approach to zero when time approaches to infinity. The dynamic stability of the controlled system can thus be analysed based on

$$\begin{aligned} \dot{\mathbf{W}} &= (\mathbf{A} + \mathbf{B}_u\mathbf{K})\,\mathbf{W} + \mathbf{B}_f\mathbf{f} \\ \mathbf{z} &= \mathbf{C}\mathbf{W} + \mathbf{z}_d, \quad \mathbf{f} = \beta(\mathbf{z}) \\ \mathbf{y} &= \mathbf{C}_y\mathbf{W} \end{aligned} \tag{6.20}$$

Stability now depends on the dynamics of $\mathbf{W}$ and so a Lypunov function may be considered having form

$$V(\mathbf{W}) = \mathbf{W}^T\mathbf{P}\mathbf{W} - 2\nu \int_{\mathbf{z}_d}^{\mathbf{z}} \mathbf{f}^T\, d\mathbf{z} \tag{6.21}$$

For this Lyapunov function

$$\dot{V}(\mathbf{W}) = \mathbf{W}^T\mathbf{P}\dot{\mathbf{W}} + \dot{\mathbf{W}}^T\mathbf{P}\mathbf{W} - 2\nu\mathbf{f}^T\dot{\mathbf{z}} \tag{6.22}$$

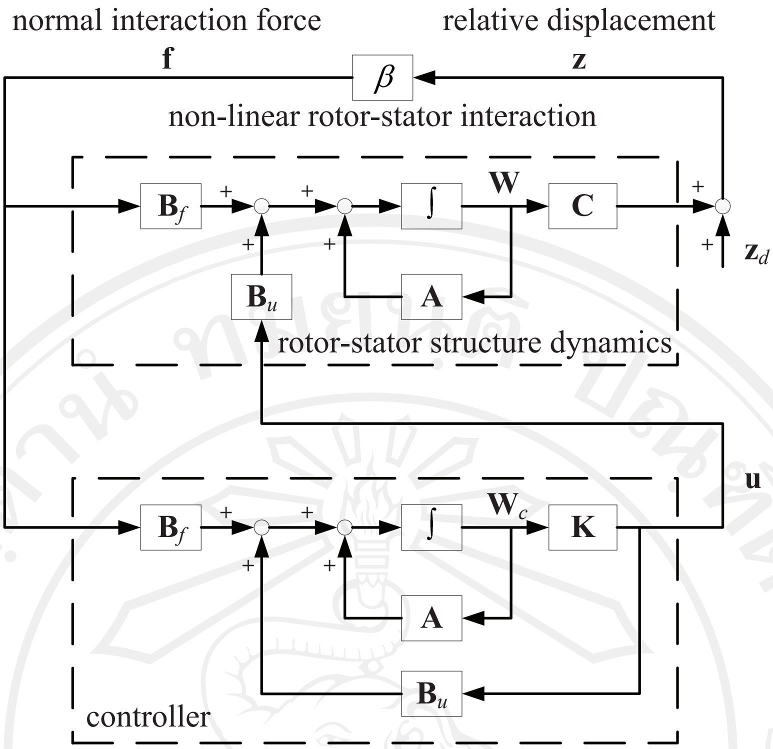

Figure 6.2: Control scheme based on feedback of rotor-stator interaction forces with a steady orbit

If $\mathbf{z}_d$ is a circular orbit then according to the analysis in subsection 5.3.4 we have

$$\mathbf{f}^T\dot{\mathbf{z}} = \mathbf{f}^T(\mathbf{C}\dot{\mathbf{W}} + \omega\Pi\mathbf{C}\mathbf{W})$$

where ω is the angular speed of the orbital motion (which typically would match the rotational speed of the rotor) and

$$\Pi = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Following the procedure in previous section, the basic stability condition $\dot{V}(\mathbf{W}) < 0$ can be combined with the constraint (5.20) and the $\mathcal{H}_2$ norm-bound (6.7) to give the condition

$$\begin{bmatrix} \mathbf{W} \\ \mathbf{f} \end{bmatrix}^T \mathbb{N}_0 \begin{bmatrix} \mathbf{W} \\ \mathbf{f} \end{bmatrix} < 0, \quad \forall \begin{bmatrix} \mathbf{W} \\ \mathbf{f} \end{bmatrix} \neq 0 \tag{6.23}$$

Or equivalently

$$\mathbb{N}_0 = \begin{bmatrix} \mathbf{N}_{11} & \mathbf{N}_{12} \\ \mathbf{N}_{21} & \mathbf{N}_{22} \end{bmatrix} < 0 \tag{6.24}$$

where

$$\begin{aligned}
\mathbf{N}_{11} &= \mathbf{P}(\mathbf{A}+\mathbf{B}_u\mathbf{K})+(\mathbf{A}+\mathbf{B}_u\mathbf{K})^T\mathbf{P}+\mathbf{K}^T\mathbf{K}+\alpha^2\mathbf{C}^T\mathbf{C},\\
\mathbf{N}_{21}=\mathbf{N}_{12}^T &= \mathbf{B}_f^T\mathbf{P}^T-\nu(\mathbf{CA}+\omega\Pi\mathbf{C})-\sigma\mathbf{C},\\
\mathbf{N}_{22} &= -2\sigma k^{-1}\mathbf{I}
\end{aligned}$$

6.2.3 Controller synthesis to stabilize a contact-free orbit

To allow a controller solution to be obtained using standard LMI solvers, bilinear terms can first be eliminated from (6.24) by using the following lemma.

Lemma 1. $\mathbf{N}<0 \Leftrightarrow \mathbf{RNR}^T<0$ *where* $\mathbf{R}$ *nonsingular matrix of compatible dimensions.*

Suppose that $\mathbf{R}=\begin{bmatrix}\mathbf{P}^{-1} & \mathbf{0}\\ \mathbf{0} & \sigma^{-1}\mathbf{I}\end{bmatrix}$ and applying lemma 1 with (6.24) gives

$$\mathbf{Q}(\omega)=\mathbf{R}^T\mathbb{N}_0(\omega)\mathbf{R}<0 \qquad (6.25)$$

Using substitutions $\mathbf{S}=\mathbf{P}^{-1}$, $\mathbf{K}=\mathbf{LS}^{-1}$, $\zeta=\sigma^{-1}$ and $\eta=\nu\sigma^{-1}$ then the inequality matrix can be written as

$$\mathbf{Q}(\omega)=\begin{bmatrix}\mathbf{Q}_{11} & \mathbf{Q}_{12}\\ \mathbf{Q}_{21} & \mathbf{Q}_{22}\end{bmatrix}<0 \qquad (6.26)$$

where

$$\begin{aligned}
\mathbf{Q}_{11} &= \mathbf{AS}+\mathbf{SA}^T+\mathbf{B}_u\mathbf{L}+\mathbf{L}^T\mathbf{B}_u^T+\mathbf{L}^T\mathbf{L}+\alpha^2\mathbf{SC}^T\mathbf{CS},\\
\mathbf{Q}_{21}=\mathbf{Q}_{12}^T &= \zeta\mathbf{B}_f^T-\eta(\mathbf{CA}+\omega\Pi\mathbf{C})\mathbf{S}-\mathbf{CS},\\
\mathbf{Q}_{22} &= -2\zeta k^{-1}\mathbf{I}.
\end{aligned}$$

Considering the dependency of $\mathbf{Q}_{11}$ on $\mathbf{L}$, by a completion of squares argument, the optimal value of $\mathbf{L}$ is $\mathbf{L}=-\mathbf{B}_u^T$, which gives

$$\mathbf{Q}_{11}=\mathbf{AS}+\mathbf{SA}^T-\mathbf{B}_u\mathbf{B}_u^T+\alpha^2\mathbf{SC}^T\mathbf{CS}$$

By Schur complements, the stability condition is obtained as

$$\tilde{\mathbb{N}}_0(\omega) = \begin{bmatrix} \tilde{\mathbf{N}}_{11} & \tilde{\mathbf{N}}_{12} & \tilde{\mathbf{N}}_{13} \\ \tilde{\mathbf{N}}_{21} & \tilde{\mathbf{N}}_{22} & \tilde{\mathbf{N}}_{23} \\ \tilde{\mathbf{N}}_{31} & \tilde{\mathbf{N}}_{32} & \tilde{\mathbf{N}}_{33} \end{bmatrix} < 0 \tag{6.27}$$

where

$$\begin{aligned} \tilde{\mathbf{N}}_{11} &= \mathbf{AS} + \mathbf{SA}^T + \mathbf{B}_u \mathbf{B}_u^T, \\ \tilde{\mathbf{N}}_{21} = \tilde{\mathbf{N}}_{12}^T &= \zeta \mathbf{B}_f^T - \eta(\mathbf{CA} + \omega \Pi \mathbf{C})\mathbf{S} - \mathbf{CS}, \\ \tilde{\mathbf{N}}_{22} &= -2\zeta k^{-1}\mathbf{I}, \\ \tilde{\mathbf{N}}_{31} = \tilde{\mathbf{N}}_{13}^T &= \mathbf{CS}, \\ \tilde{\mathbf{N}}_{33} &= -\alpha^{-2}\mathbf{I}. \end{aligned}$$

For a nominal rotor/orbit angular speed ω, the optimal gain matrix $\mathbf{K}^*$ can be found by solving the following LMI optimization problem:

Controller synthesis problem. Minimize γ over $\mathbf{S}$, ζ and η subject to $\mathbf{S} = \mathbf{S}^T > 0$, $\zeta > 0$, $\eta > 0$, $\tilde{\mathbb{N}}_0(\omega) < 0$ and

$$\begin{bmatrix} -\gamma^2 \mathbf{I} & \mathbf{W}_0^T \\ \mathbf{W}_0 & -\mathbf{S} \end{bmatrix}$$

From the solution to this problem we obtain $\mathbf{K}^* = -\mathbf{B}_u^T \mathbf{S}^{-1}$ and the state space controller $(\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c)$ follows as

$$\begin{aligned} \mathbf{A}_c &= \mathbf{A} - \mathbf{B}_u \mathbf{B}_u^T \mathbf{S}^{-1}, \\ \mathbf{B}_c &= \mathbf{B}_f, \\ \mathbf{C}_c &= -\mathbf{B}_u^T \mathbf{S}^{-1}. \end{aligned}$$

6.3 Controller Synthesis to Stabilize Contact-Free Whirl in the Presence of Friction

A modified version of the controller synthesis in Section 6.2 will now be described that accounts for the possibility of friction at the rotor-stator interface . In this section, a state space model for lateral vibration of a rotor-stator structure is considered based on equation 5.31

$$\begin{aligned} \dot{\mathbf{W}} &= \mathbf{AW} + \mathbf{B}_u \mathbf{u} + \mathbf{B}_f \mathbf{T}_\mu \mathbf{f} \\ \mathbf{z} &= \mathbf{CW} + \mathbf{z}_d, \quad \mathbf{f} = \beta(\mathbf{z}) \end{aligned} \tag{6.28}$$

where the matrix $\mathbf{T}_\mu$ is a constant matrix that models the effects of friction at the contact surface, as defined in Section 5.3.

The control scheme for this case is shown in figure 6.3 with controller defined by

$$\begin{aligned} \dot{\mathbf{W}}_c &= \mathbf{A}\mathbf{W}_c + \mathbf{B}_u\mathbf{u} + \mathbf{B}_f\mathbf{T}_\mu\mathbf{f} \\ \mathbf{u} &= \mathbf{K}\mathbf{W}_c \end{aligned} \tag{6.29}$$

Note that $\mathbf{f}$ is the normal component of the interaction force, while the measured force is the total interaction force $\mathbf{T}_\mu\mathbf{f}$. The state vector $\mathbf{W}_c$ is an estimate of the components of $\mathbf{W}$ due to the total interaction force $\mathbf{T}_\mu\mathbf{f}$. The controller gain $\mathbf{K}$ must be chosen to ensure stability of the controlled system dynamics as given by

$$\begin{aligned} \begin{bmatrix} \dot{\mathbf{W}} \\ \dot{\mathbf{W}}_e \end{bmatrix} &= \begin{bmatrix} \mathbf{A} + \mathbf{B}_u\mathbf{K} & -\mathbf{B}_u\mathbf{K} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{W} \\ \mathbf{W}_e \end{bmatrix} + \begin{bmatrix} \mathbf{B}_f\mathbf{T}_\mu \\ \mathbf{0} \end{bmatrix} \mathbf{f} \\ \mathbf{z} &= \mathbf{C}\mathbf{W} + \mathbf{z}_d, \quad \mathbf{f} = \beta(\mathbf{z}) \end{aligned} \tag{6.30}$$

where $\mathbf{W}_e = \mathbf{W} - \mathbf{W}_c$. As in the previous cases, the state vector $\mathbf{W}_e$ is independent of $\mathbf{f}$ and will therefore converge to zero as long as the matrix $\mathbf{A}$ is a stable matrix. A stability-performance analysis can be based on

$$\begin{aligned} \dot{\mathbf{w}} &= (\mathbf{A} + \mathbf{B}_u\mathbf{K})\mathbf{x} + \mathbf{B}_f\mathbf{T}_\mu\mathbf{f} \\ \mathbf{z} &= \mathbf{C}\mathbf{w}, \quad \mathbf{f} = \beta(\mathbf{z}) \\ \mathbf{y} &= \mathbf{C}_y\mathbf{W} \end{aligned} \tag{6.31}$$

The output $\mathbf{y}$ can again include weighted components of vibration states and control forces with $\mathbf{C}_y$ as defined in (6.9).

For controller design, a norm-bound condition (6.7) is considered and a Lyapunov function candidate is adopted as in (6.21). Following the some derivation given in Section 6.2, the stability condition $\dot{\mathbf{V}}(\omega) \leq 0$ is equivalent to

$$\mathbb{V}_0 = \begin{bmatrix} \mathbf{V}_{11} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22} \end{bmatrix} < 0 \tag{6.32}$$

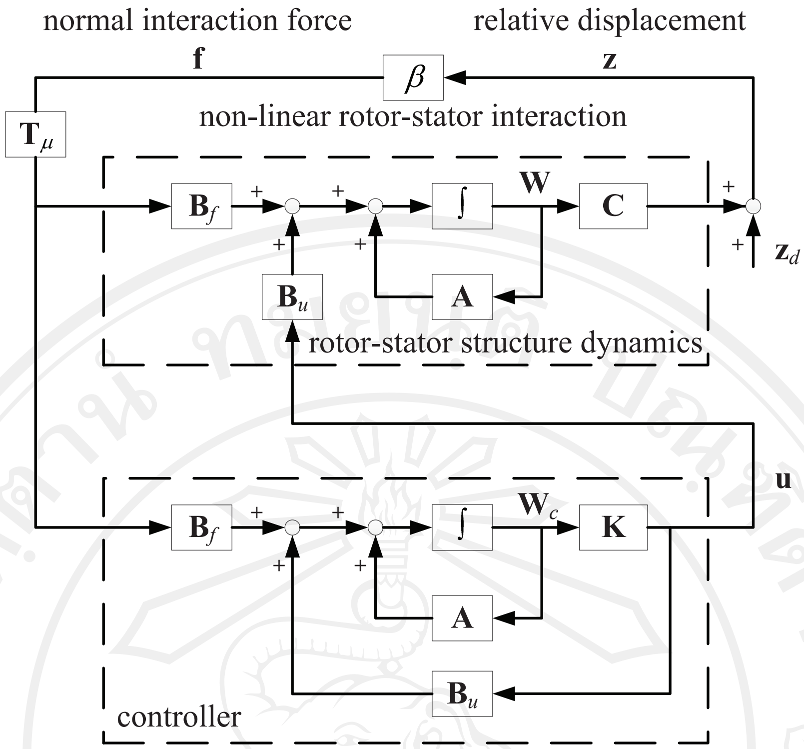

Figure 6.3: Control scheme based on feedback of rotor-stator interaction forces with friction at interaction plane

where

$$
\begin{aligned}
\mathbf{V}_{11} &= \mathbf{P}(\mathbf{A}+\mathbf{B}_u\mathbf{K}) + (\mathbf{A}+\mathbf{B}_u\mathbf{K})^T\mathbf{P} + \mathbf{K}^T\mathbf{K} + \alpha^2\mathbf{C}^T\mathbf{C}, \\
\mathbf{V}_{21} = \mathbf{V}_{12}^T &= \mathbf{T}_\mu^T\mathbf{B}_f^T\mathbf{P}^T - \nu(\mathbf{CA} + \omega\Pi\mathbf{C}) - \sigma\mathbf{C}, \\
\mathbf{V}_{22} &= -2\sigma k^{-1}\mathbf{I}
\end{aligned}
$$

To obtain a controller solution using standard LMI solvers, (6.32) must be transform using lemma 1. Similar to previous cases, the stability condition can be obtained as

$$
\tilde{\mathbb{V}}_0(\omega) = \begin{bmatrix} \tilde{\mathbf{V}}_{11} & \tilde{\mathbf{V}}_{12} & \tilde{\mathbf{V}}_{13} \\ \tilde{\mathbf{V}}_{21} & \tilde{\mathbf{V}}_{22} & \tilde{\mathbf{V}}_{23} \\ \tilde{\mathbf{V}}_{31} & \tilde{\mathbf{V}}_{32} & \tilde{\mathbf{V}}_{33} \end{bmatrix} < 0 \qquad (6.33)
$$

where

$$\begin{aligned}
\tilde{\mathbf{V}}_{11} &= \mathbf{AS} + \mathbf{SA}^T + \mathbf{B}_u\mathbf{B}_u^T, \\
\tilde{\mathbf{V}}_{21} = \tilde{\mathbf{V}}_{12}^T &= \zeta\mathbf{T}_\mu^T\mathbf{B}_f^T - \eta(\mathbf{CA} + \omega\Pi\mathbf{C})\mathbf{S} - \mathbf{CS}, \\
\tilde{\mathbf{V}}_{22} &= -2\zeta k^{-1}\mathbf{I}, \\
\tilde{\mathbf{V}}_{31} = \tilde{\mathbf{V}}_{13}^T &= \mathbf{CS}, \\
\tilde{\mathbf{V}}_{33} &= -\alpha^{-2}\mathbf{I}.
\end{aligned}$$

For a nominal angular speed ω, the optimal gain matrix $\mathbf{K}^*$ can be found by solving the following LMI optimization problem:

Controller synthesis problem. Minimize γ over $\mathbf{S}$, ζ and η subject to $\mathbf{S} = \mathbf{S}^T > 0$, $\zeta > 0$, $\eta > 0$, $\tilde{\mathbb{V}}_0(\omega) < 0$ and

$$\begin{bmatrix} -\gamma^2\mathbf{I} & \mathbf{W}_0^T \\ \mathbf{W}_0 & -\mathbf{S} \end{bmatrix}$$

From the solution to this problem we obtain $\mathbf{K}^* = -\mathbf{B}_u^T\mathbf{S}^{-1}$ and the state space controller $(\mathbf{A}_c, \mathbf{B}_c, \mathbf{C}_c)$ follows as

$$\begin{aligned}
\mathbf{A}_c &= \mathbf{A} - \mathbf{B}_u\mathbf{B}_u^T\mathbf{S}^{-1}, \\
\mathbf{B}_c &= \mathbf{T}_\mu\mathbf{B}_f, \\
\mathbf{C}_c &= -\mathbf{B}_u^T\mathbf{S}^{-1}.
\end{aligned}$$

6.4 Summary

In this chapter, a dynamic force feedback controller has been proposed based on a state-estimation concept. A model-based analysis and synthesis approach has been derived covering three different situations and accounting fully for the possibility of contact interaction between the rotor and stator:

1) Stabilization of the rotor equilibrium point
2) Stabilization of a circular whirl response (friction-free case)
3) Stabilization of a circular whirl response with significant friction at the contact interface

Implementation of the controller requires actuators to apply control forces to the rotor (i.e. an active magnetic bearing) and a means to measure rotor-stator contact forces in the expected plane of contact. For the control structure proposed, a feedback gain matrix could be obtained from the solution to a set of LMI constraints. The next chapter will describe results for validation of the control approach obtained by model-based simulation and experimental testing.