

CHAPTER 5

Conclusion

This thesis has been investigated the synchronization problems for coupled NNs with time-varying delays and hybrid couplings. Firstly, this work has been to study and introduce a general array model of coupled delayed NNs with hybrid couplings for handling synchronization problem. Global synchronization for coupled NNs has been studied and analysed. The coupled NNs, with simultaneous presence of both the discrete and unbounded distributed delays, is studied in the case of constant and interval time-varying delay couplings. Furthermore, the problem of synchronization for coupled delayed NNs with hybrid couplings designed by feedback and sampled-data feedback controls. Based on the design of outer-coupling, inner-coupling, and some free matrices representing the relationships between the system matrices, some sufficient conditions are systematically established to meet the synchronization by constructing the improved Lyapunov-Krasovskii functionals and Kronecker product properties. The criteria are derived in terms of LMIs solved efficiently by standard convex optimization algorithms.

Secondly, the exponential synchronization for coupled NNs has been studied and analysed. For exponential synchronization, the coupled NNs with presence leakage delay has been studied in the case of constant, interval time-varying delay and distributed delay couplings. The designed controller ensures the exponential synchronization of hybrid couple delayed NNs are proposed via feedback and intermittent feedback controls. Based on the design of outer-coupling, inner-coupling, and some free matrices representing the relationships between the system matrices, some sufficient conditions are systematically established to meet the synchronization by constructing the improved Lyapunov-Krasovskii functionals and Kronecker product properties. The criteria are derived in terms of LMIs solved efficiently by standard convex optimization algorithms.

Finally, numerical examples with their simulations and the obtained results are given to show the effectiveness of the proposed method.

The following results are all main theorems of this thesis:

5.1 Global Synchronization of Hybrid Coupled Delayed Neural Networks with Feedback and Sampled-Data Feedback Controls

1) Assume that assumptions 3.1-3.3 hold. The system (3.1) is globally asymptotically synchronized under the controller (3.2), if there exist positive definite matrices, $P, Q_1, Q_2, Q_3, Q_4, R, S_1, S_2, S_3, Y$, three diagonal matrices $\Lambda = \text{diag}(\iota_1, \dots, \iota_n) > 0, \Omega = \text{diag}(\kappa_1, \dots, \kappa_n) > 0, \Delta = \text{diag}(\eta_1, \dots, \eta_n) > 0$ and any real matrices T_1, T_2 , with appropriate dimensions, such that the following linear matrix inequalities (LMIs) hold for all $1 = i < j = N$:

$$\begin{aligned}
 \Sigma_{1ij} &= \mathcal{W}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 \end{bmatrix}^T \\
 &\quad \times S_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 \end{bmatrix} < 0, \\
 \Sigma_{2ij} &= \mathcal{W}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 \end{bmatrix}^T \\
 &\quad \times S_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 \end{bmatrix} < 0, \\
 \Sigma_3 &= \begin{bmatrix} -0.1(S_1 + S_2) & 3Y^T & PE & T_1E \\ * & -3P & 0 & 0 \\ * & * & -P & 0 \\ * & * & * & -P \end{bmatrix} < 0, \\
 \Sigma_4 &= \begin{bmatrix} -T_2 & T_2E \\ * & -P \end{bmatrix} < 0, \\
 \mathcal{W}_{ij} &= \begin{bmatrix} W_{1,1} & W_{1,2} & W_{1,3} & W_{1,4} & \Omega J_2 & \Delta L_2 & W_{1,7} & S_1 & S_2 & W_{1,10} \\ * & W_{2,2} & T_2B & T_2A & 0 & 0 & W_{2,7} & 0 & 0 & T_2D \\ * & * & W_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\Lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & W_{5,5} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \bar{k}R - \Delta & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & W_{7,7} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & W_{8,8} & 0 & 0 \\ * & * & * & * & * & * & * & * & W_{9,9} & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{k}R \end{bmatrix},
 \end{aligned}$$

where $\bar{k} = \int_0^{+\infty} k(s) ds$ and

$$W_{1,1} = -PC - C^T P - Ng_{ij}^1 P \Gamma_1 - Ng_{ij}^1 \Gamma_1^T P - T_1 C - C^T T_1^T$$

$$\begin{aligned}
& -Ng_{ij}^1 T_1 \Gamma_1 - Ng_{ij}^1 \Gamma_1^T T_1^T + \sum_{i=1}^3 Q_i - 0.9S_1 - 0.9S_2 \\
& -\Lambda F_1 - \Omega J_1 - \Delta L_1, \\
W_{1,2} &= -T_1 - C^T T_2^T - Ng_{ij}^1 \Gamma_1^T T_2^T, \\
W_{1,3} &= (P + T_1)B, \\
W_{1,4} &= (P + T_1)A + \Lambda F_2, \\
W_{1,7} &= -Ng_{ij}^2 P \Gamma_2 - Ng_{ij}^2 T_1 \Gamma_2, \\
W_{1,10} &= (P + T_1)D, \\
W_{2,2} &= \tau_1^2 S_1 + \tau_2^2 S_2 + \eta^2 S_3 - T_2, \\
W_{2,7} &= -Ng_{ij}^2 T_2 \Gamma_2, \\
W_{3,3} &= -(1 - \mu)Q_4, \\
W_{5,5} &= Q_4 - \Omega, \\
W_{7,7} &= -(1 - \mu)Q_1 - 2S_3, \\
W_{8,8} &= -Q_2 - S_1 - S_3, \\
W_{9,9} &= -Q_3 - S_2 - S_3.
\end{aligned}$$

Moreover the memoryless feedback control is $K = -P^{-1}Y$.

2) Assume that assumptions 3.1-3.3 hold. The system (3.1) is globally asymptotically synchronized under the controller (3.27), if there exist positive definite matrices, $P, Q_1, Q_2, Q_3, Q_4, R, S_1, S_2, S_3, Z_1, Z_2, Y$, three diagonal matrices $\Lambda = \text{diag}(\iota_1, \dots, \iota_n) > 0, \Omega = \text{diag}(\kappa_1, \dots, \kappa_n) > 0, \Delta = \text{diag}(\eta_1, \dots, \eta_n) > 0$ and any real matrices T_1, T_2 , with appropriate dimensions, such that the following linear matrix inequalities (LMIs) hold for all $1 = i < j = N$:

$$\begin{aligned}
\Phi_{1ij} &= \mathcal{M}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 & 0 \end{bmatrix}^T \\
&\quad \times S_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 & 0 \end{bmatrix} < 0, \\
\Phi_{2ij} &= \mathcal{M}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
&\quad \times S_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 & 0 & 0 \end{bmatrix} < 0, \\
\Phi_3 &= \begin{bmatrix} -0.1(S_1 + S_2) & PE & T_1 E \\ * & -P & 0 \\ * & * & -P \end{bmatrix} < 0,
\end{aligned}$$

$$\begin{aligned}
\Phi_4 &= \begin{bmatrix} -T_2 & T_2 E \\ * & -P \end{bmatrix} < 0, \\
\Phi_5 &= \begin{bmatrix} -\frac{1}{h}Z_2 & 3Y^T \\ * & -3P \end{bmatrix} < 0, \\
\mathcal{M}_{ij} &= \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} & \cdots & M_{1,12} \\ * & M_{2,2} & M_{2,3} & \cdots & M_{2,12} \\ * & * & M_{3,3} & \cdots & M_{3,12} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & M_{12,12} \end{bmatrix},
\end{aligned}$$

where $\bar{k} = \int_0^{+\infty} k(s) ds$ and

$$\begin{aligned}
M_{1,1} &= -PC - C^T P - Ng_{ij}^1 P \Gamma_1 - Ng_{ij}^1 \Gamma_1^T P - T_1 C - C^T T_1^T \\
&\quad - Ng_{ij}^1 T_1 \Gamma_1 - Ng_{ij}^1 \Gamma_1^T T_1^T + \sum_{i=1}^3 Q_i - 0.9S_1 - 0.9S_2 \\
&\quad - \Lambda F_1 - \Omega J_1 - \Delta L_1 - \frac{1}{h}Z_2 + Z_1, \\
M_{1,2} &= -T_1 - C^T T_2^T - Ng_{ij}^1 \Gamma_1^T T_2^T, \\
M_{1,3} &= (P + T_1)B, \\
M_{1,4} &= (P + T_1)A + \Lambda F_2, \\
M_{1,5} &= \Omega J_2, \\
M_{1,6} &= \Delta L_2, \\
M_{1,7} &= -Ng_{ij}^2 P \Gamma_2 - Ng_{ij}^2 T_1 \Gamma_2, \\
M_{1,8} &= S_1, \\
M_{1,9} &= S_2, \\
M_{1,10} &= (P + T_1)D, \\
M_{1,11} &= \frac{1}{h}Z_2, \\
M_{2,2} &= \tau_1^2 S_1 + \tau_2^2 S_2 + \eta^2 S_3 - T_2 + hZ_2, \\
M_{2,3} &= T_2 B, \\
M_{2,4} &= T_2 A, \\
M_{2,7} &= -Ng_{ij}^2 T_2 \Gamma_2, \\
M_{2,10} &= T_2 D, \\
M_{3,3} &= -(1 - \mu)Q_4, \\
M_{4,4} &= -\Lambda,
\end{aligned}$$

$$\begin{aligned}
M_{5,5} &= Q_4 - \Omega, \\
M_{6,6} &= \bar{k}R - \Delta, \\
M_{7,7} &= -(1 - \mu)Q_1 - 2S_3, \\
M_{8,8} &= -Q_2 - S_1 - S_3, \\
M_{9,9} &= -Q_3 - S_2 - S_3, \\
M_{10,10} &= -\frac{1}{k}R, \\
M_{11,11} &= -\frac{1}{h}Z_2, \\
M_{11,12} &= \frac{1}{h}Z_2, \\
M_{12,12} &= -Z_1 - \frac{1}{h}Z_2, \\
0 &= M_{1,12} = M_{2,5} = M_{2,6} = M_{2,8} = M_{2,9} = M_{2,11} = M_{2,12} = M_{3,4} \\
&= M_{3,5} = M_{3,6} = M_{3,7} = M_{3,8} = M_{3,9} = M_{3,10} = M_{3,11} = M_{3,12} \\
&= M_{4,5} = M_{4,6} = M_{4,7} = M_{4,8} = M_{4,9} = M_{4,10} = M_{4,11} = M_{4,12} \\
&= M_{5,6} = M_{5,7} = M_{5,8} = M_{5,9} = M_{5,10} = M_{5,11} = M_{5,12} = M_{6,7} \\
&= M_{6,8} = M_{6,9} = M_{6,10} = M_{6,11} = M_{6,12} = M_{7,8} = M_{7,9} = M_{7,10} \\
&= M_{7,11} = M_{7,12} = M_{8,9} = M_{8,10} = M_{8,11} = M_{8,12} = M_{9,10} = M_{9,11} \\
&= M_{9,12} = M_{10,11} = M_{10,12}.
\end{aligned}$$

Moreover the memoryless feedback control is $K = -P^{-1}Y$.

5.2 Exponential Synchronization of Hybrid Coupled Delayed Neural Networks with Feedback and Intermittent Feedback Controls

1) Given $\alpha > 0$ and suppose that assumption 4.1-4.2 hold, the system (4.1) is exponentially synchronized under the controller (4.2) if there exist symmetric positive definite matrices, $P, Y, Q_1, Q_2, Q_3, Q_4, Q_5, R_l, S_l, (l = 1, 2, 3)$, three diagonal matrices $\Lambda = \text{diag}(\iota_1, \dots, \iota_n) > 0, \Omega = \text{diag}(\kappa_1, \dots, \kappa_n) > 0, \Delta = \text{diag}(\eta_1, \dots, \eta_n) > 0$ and real matrices T with appropriate dimensions, such that the following linear matrix inequalities (LMIs) hold for all $1 = i < j = N$:

$$\begin{aligned}
\Sigma_{1ij} &= \mathcal{M}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 \end{bmatrix}^T \\
&\quad \times e^{-2\alpha\tau_2} R_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 \end{bmatrix} < 0, \\
\Sigma_{2ij} &= \mathcal{M}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 & 0 \end{bmatrix}^T \\
&\quad \times e^{-2\alpha\tau_2} R_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 & 0 \end{bmatrix} < 0,
\end{aligned}$$

$$\Sigma_3 = \begin{bmatrix} -0.1e^{-2\alpha\tau_1}R_1 - 0.1e^{-2\alpha\tau_2}R_2 & 2Y^T & PE \\ * & -2P & 0 \\ * & * & -P \end{bmatrix} < 0,$$

$$\Sigma_4 = \begin{bmatrix} -T & TE \\ * & -P \end{bmatrix} < 0,$$

where

$$\mathcal{M}_{ij} = \begin{bmatrix} M_{1,1} & M_{1,2} & M_{1,3} & \cdots & M_{1,12} \\ * & M_{2,2} & M_{2,3} & \cdots & M_{2,12} \\ * & * & M_{3,3} & \cdots & M_{3,12} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & M_{12,12} \end{bmatrix},$$

$$\begin{aligned} M_{1,1} &= -Ng_{ij}^1 P\Gamma_1 - Ng_{ij}^1 \Gamma_1^T P + 2\alpha P + \sum_{r=1}^4 Q_r - 0.9e^{-2\alpha\tau_1}R_1 \\ &\quad - 0.9e^{-2\alpha\tau_2}R_2 - e^{-2\alpha\sigma}S_1 + kS_2 - \Omega F_1 - \Theta J_1 - \Delta L_1, \\ M_{1,2} &= -e^{-2\alpha\sigma}S_1 + C^T P, \\ M_{1,3} &= -Ng_{ij}^1 T\Gamma_1, \\ M_{1,4} &= B^T P, \\ M_{1,5} &= \Omega F_2, \\ M_{1,6} &= \Theta J_2 + A^T P, \\ M_{1,7} &= \Delta L_2, \\ M_{1,8} &= -Ng_{ij}^2 \Gamma_2^T P, \\ M_{1,9} &= e^{-2\alpha\tau_1}R_1, \\ M_{1,10} &= e^{-2\alpha\tau_2}R_2, \\ M_{1,12} &= -Ng_{ij}^3 \Gamma_3^T P + D^T P, \\ M_{2,2} &= -e^{-2\alpha\sigma}(Q_1 + S_1), \\ M_{2,3} &= -TC, \\ M_{3,3} &= \tau_1^2 R_1 + \tau_2^2 R_2 + \eta^2 R_3 + \sigma^2 S_1 - T, \\ M_{3,4} &= B^T T, \\ M_{3,6} &= A^T T^T, \\ M_{3,8} &= -Ng_{ij}^2 \Gamma_2^T T^T, \\ M_{3,12} &= -Ng_{ij}^3 \Gamma_3^T T^T + D^T T^T, \end{aligned}$$

$$\begin{aligned}
M_{4,4} &= -(1 - \mu_1)e^{-2\alpha\tau_2}Q_5, \\
M_{5,5} &= -\Omega - Q_5, \\
M_{6,6} &= -\Theta, \\
M_{7,7} &= kS_3 - \Delta, \\
M_{8,8} &= -(1 - \mu_1)e^{-2\alpha\tau_2}Q_4 - e^{-2\alpha\tau_2}R_3 - e^{-2\alpha\tau_2}R_3, \\
M_{8,9} &= -e^{-2\alpha\tau_2}R_3, \\
M_{8,10} &= -e^{-2\alpha\tau_2}R_3, \\
M_{9,9} &= -e^{-2\alpha\tau_1}(Q_2 + R_1) - e^{-2\alpha\tau_2}R_3, \\
M_{10,10} &= -e^{-2\alpha\tau_2}(R_2 + R_3 + Q_3), \\
M_{11,11} &= -\frac{(1 - \mu_2)}{k}e^{-2\alpha k}S_2, \\
M_{12,12} &= -\frac{(1 - \mu_2)}{k}e^{-2\alpha k}S_3, \\
0 &= M_{1,11} = M_{2,4} = M_{2,5} = M_{2,6} = M_{2,7} = M_{2,8} = M_{2,9} \\
&= M_{2,10} = M_{2,11} = M_{2,12} = M_{3,5} = M_{3,7} = M_{3,9} = M_{3,10} \\
&= M_{3,11} = M_{4,5} = M_{4,6} = M_{4,7} = M_{4,8} = M_{4,9} = M_{4,10} \\
&= M_{4,11} = M_{4,12} = M_{5,6} = M_{5,7} = M_{5,8} = M_{5,9} = M_{5,10} \\
&= M_{5,11} = M_{5,12} = M_{6,7} = M_{6,8} = M_{6,9} = M_{6,10} = M_{6,11} \\
&= M_{6,12} = M_{7,8} = M_{7,9} = M_{7,10} = M_{7,11} = M_{7,12} = M_{8,11} \\
&= M_{8,12} = M_{9,10} = M_{9,11} = M_{9,12} = M_{10,11} = M_{10,12} = M_{11,12}.
\end{aligned}$$

Moreover, the memoryless feedback control is $K = -P^{-1}Y$.

2) Given $0 < \alpha < \varepsilon$, and suppose that assumption 4.1-4.2 hold, the system (4.1) is exponentially synchronized under the controller (4.32) if there exist symmetric positive definite matrices, $P, Y, Q_1, Q_2, Q_3, Q_4, Q_5, R_l, S_l, (l = 1, 2, 3)$, three diagonal matrices $\Lambda = \text{diag}(\iota_1, \dots, \iota_n) > 0, \Omega = \text{diag}(\kappa_1, \dots, \kappa_n) > 0, \Delta = \text{diag}(\eta_1, \dots, \eta_n) > 0$ and real matrices T with appropriate dimensions, such that the following linear matrix inequalities (LMIs) hold for all $1 = i < j = N$:

$$\begin{aligned}
\Phi_{1ij} &= \mathcal{Z}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 \end{bmatrix}^T \\
&\quad \times e^{-2\alpha\tau_2}R_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 \end{bmatrix} < 0, \\
\Phi_{2ij} &= \mathcal{Z}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 & 0 \end{bmatrix}^T \\
&\quad \times e^{-2\alpha\tau_2}R_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 & 0 \end{bmatrix} < 0,
\end{aligned}$$

$$\begin{aligned}
\Phi_3 &= \begin{bmatrix} -0.1e^{-2\alpha\tau_1}R_1 - 0.1e^{-2\alpha\tau_2}R_2 & 2Y^T & PE \\ * & -2P & 0 \\ * & * & -P \end{bmatrix} < 0, \\
\Phi_4 &= \begin{bmatrix} -T & TE \\ * & -P \end{bmatrix} < 0, \\
\Phi_{5ij} &= \hat{Z}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 \end{bmatrix}^T \\
&\quad \times e^{-2\alpha\tau_2}R_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 \end{bmatrix} < 0, \\
\Phi_{6ij} &= \hat{Z}_{ij} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 & 0 \end{bmatrix}^T \\
&\quad \times e^{-2\alpha\tau_2}R_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & I & 0 & 0 & 0 \end{bmatrix} < 0,
\end{aligned}$$

and

$$-\alpha\delta + (\varepsilon - \alpha)(\omega - \delta) < 0,$$

where

$$\begin{aligned}
Z_{ij} &= \begin{bmatrix} Z_{1,1} & Z_{1,2} & Z_{1,3} & \cdots & Z_{1,12} \\ * & Z_{2,2} & Z_{2,3} & \cdots & Z_{2,12} \\ * & * & Z_{3,3} & \cdots & Z_{3,12} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & Z_{12,12} \end{bmatrix}, \\
\hat{Z}_{ij} &= \begin{bmatrix} \hat{Z}_{1,1} & Z_{1,2} & Z_{1,3} & \cdots & Z_{1,12} \\ * & Z_{2,2} & Z_{2,3} & \cdots & Z_{2,12} \\ * & * & \hat{Z}_{3,3} & \cdots & Z_{3,12} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & Z_{12,12} \end{bmatrix},
\end{aligned}$$

$$Z_{1,1} = -Ng_{ij}^1 P \Gamma_1 - Ng_{ij}^1 \Gamma_1^T P + 2\alpha P + \sum_{r=1}^4 Q_r - 0.9e^{-2\alpha\tau_1} R_1$$

$$-0.9e^{-2\alpha\tau_2} R_2 - e^{-2\alpha\sigma} S_1 + kS_2 - \Omega F_1 - \Theta J_1 - \Delta L_1,$$

$$Z_{1,2} = -e^{-2\alpha\sigma} S_1 + C^T P,$$

$$Z_{1,3} = -Ng_{ij}^1 T \Gamma_1,$$

$$Z_{1,4} = B^T P,$$

$$Z_{1,5} = \Omega F_2,$$

$$Z_{1,6} = \Theta J_2 + A^T P,$$

$$\begin{aligned}
Z_{1,7} &= \Delta L_2, \\
Z_{1,8} &= -Ng_{ij}^2 \Gamma_2^T P, \\
Z_{1,9} &= e^{-2\alpha\tau_1} R_1, \\
Z_{1,10} &= e^{-2\alpha\tau_2} R_2, \\
Z_{1,12} &= -Ng_{ij}^3 \Gamma_3^T P + D^T P, \\
Z_{2,2} &= -e^{-2\alpha\sigma} (Q_1 + S_1), \\
Z_{2,3} &= -TC, \\
Z_{3,3} &= \tau_1^2 R_1 + \tau_2^2 R_2 + \eta^2 R_3 + \sigma^2 S_1 - T, \\
Z_{3,4} &= B^T T, \\
Z_{3,6} &= A^T T^T, \\
Z_{3,8} &= -Ng_{ij}^2 \Gamma_2^T T^T, \\
Z_{3,12} &= -Ng_{ij}^3 \Gamma_3^T T^T + D^T T^T, \\
Z_{4,4} &= -(1 - \mu_1) e^{-2\alpha\tau_2} Q_5, \\
Z_{5,5} &= -\Omega - Q_5, \\
Z_{6,6} &= -\Theta, \\
Z_{7,7} &= kS_3 - \Delta, \\
Z_{8,8} &= -(1 - \mu_1) e^{-2\alpha\tau_2} Q_4 - e^{-2\alpha\tau_2} R_3 - e^{-2\alpha\tau_2} R_3, \\
Z_{8,9} &= -e^{-2\alpha\tau_2} R_3, \\
Z_{8,10} &= -e^{-2\alpha\tau_2} R_3, \\
Z_{9,9} &= -e^{-2\alpha\tau_1} (Q_2 + R_1) - e^{-2\alpha\tau_2} R_3, \\
Z_{10,10} &= -e^{-2\alpha\tau_2} (R_2 + R_3 + Q_3), \\
Z_{11,11} &= -\frac{(1 - \mu_2)}{k} e^{-2\alpha k} S_2, \\
Z_{12,12} &= -\frac{(1 - \mu_2)}{k} e^{-2\alpha k} S_3, \\
0 &= Z_{1,11} = Z_{2,4} = Z_{2,5} = Z_{2,6} = Z_{2,7} = Z_{2,8} = Z_{2,9} \\
&= Z_{2,10} = Z_{2,11} = Z_{2,12} = Z_{3,5} = Z_{3,7} = Z_{3,9} = Z_{3,10} \\
&= Z_{3,11} = Z_{4,5} = Z_{4,6} = Z_{4,7} = Z_{4,8} = Z_{4,9} = Z_{4,10} \\
&= Z_{4,11} = Z_{4,12} = Z_{5,6} = Z_{5,7} = Z_{5,8} = Z_{5,9} = Z_{5,10} \\
&= Z_{5,11} = Z_{5,12} = Z_{6,7} = Z_{6,8} = Z_{6,9} = Z_{6,10} = Z_{6,11} \\
&= Z_{6,12} = Z_{7,8} = Z_{7,9} = Z_{7,10} = Z_{7,11} = Z_{7,12} = Z_{8,11} \\
&= Z_{8,12} = Z_{9,10} = Z_{9,11} = Z_{9,12} = Z_{10,11} = Z_{10,12} = Z_{11,12},
\end{aligned}$$

$$\begin{aligned}
\hat{Z}_{1,1} &= -Ng_{ij}^1 P \Gamma_1 - Ng_{ij}^1 \Gamma_1^T P + 2\alpha P + \sum_{r=1}^4 Q_r - e^{-2\alpha\tau_1} R_1 - e^{-2\alpha\tau_2} R_2 \\
&\quad - e^{-2\alpha\sigma} S_1 + kS_2 - \Omega F_1 - \Theta J_1 - \Delta L_1 - 2\varepsilon P, \\
\hat{Z}_{3,3} &= \tau_1^2 R_1 + \tau_2^2 R_2 + \eta^2 R_3 + \sigma^2 S_1 - T - T^T.
\end{aligned}$$

Moreover, the memoryless feedback control is $K = -P^{-1}Y$.



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