CHAPTER 1

Introduction

The phenomena of time-delays are often encountered in many practical systems like process control systems, manufacturing, networked control systems, economic systems and so on. The existence of these delays may be the source of instability and serious deterioration in the performance of the closed-loop systems. The problems of stabilization and H_∞ control of linear time-delay systems has received increasing attention, see for examples [6, 8, 30]. For the H_∞ control problem, appropriate method for linear systems usually make use of the Lyapunov functional approach, whereby the H_∞ condition are obtained via solving either linear matrix inequalities (LMIs) or algebraic Riccati-type equation (RDE). Time-delay systems are frequently encountered in various areas such as chemical engineering systems, biological modelling, economic systems and networked control systems. H_∞ control for time-delay system have received considerable attention for the past few decades.

Recently, delay-dependent stability for interval time-varying delay was investigated in [14, 15, 48, 51, 52, 59, 61, 64]. Interval time-varying delay is a time delay that varies in an interval in which the lower bound is not restricted to be 0. In order to reduce further the conservatism introduced by descriptor model transformation and bounding techniques, a free-weighting matrices method is proposed in [11, 13, 14, 37, 51]. The advantage of this transformation is to transform the original system to an equivalent descriptor form representation and will not introduce additional dynamics in the sense defined in [11]. In [13], based on the descriptor model transformation, decomposition technique of coefficient matrix and the novel robust stability criteria are derived. In [48], Shao presented a new delay-dependent stability criterion for linear systems with interval time-varying delay, stability criteria are derived in terms of LMIs without introducing any free-weighting matrices. Tian and Zhou [51] considered the delay-dependent asymptotic stability criteria for neural networks (NNs) with time-varying interval delay; by introducing a novel Lyapunov functional stability criteria of asymptotic stability is derived in terms of LMIs. Delay-dependent robust exponential stabilization criteria for interval time-varying delay systems with norm-bounded uncertainties are proposed in [52], by using Lyapunov-Krasovskii functionals combined with the free-weighting matrices.

In the past few decades, the H_∞ control problem for linear uncertain systems has attracted increasing attention(see, for instance,[7, 26, 47, 49]and the references therein), and still many questions remain unsolved. The standard H_∞ control problem is to find conditions that guarantee the existence of feedback controller stabilizing given system and satisfies a prescribed γ-suboptimal level on perturbations/uncertainties. There are some solution techniques which have been proposed for the problem. In the H_∞ control problem for linear autonomous systems,the appropriate methods make use of Lypunov-Krasovskii function approach and the sufficient conditions are obtained via solving either LMIs [18, 36], or RDEs [47, 49]. However, this approach may not be readily applied to time-varying systems due to the fact that the solution of LMIs which depend on infinite time cannot be solved in general. For linear time-varying systems, the investigation of the stability and control problem becomes more complicated. For instance, in contrast to linear autonomous systems, the stability (and instability) of linear time-varying system may not be determined from the spectral property of their system matrix $A(t)$. It was shown that the real parts of eigenvalues of system matrix $A(t)$ for every t are negative does not imply the asymptotic stability, and there is a linear time-varying system which is stable with positive eigenvalues. In spite of this, fore linear time-varying systems, in the literature to date, there are several papers dealing with stability and control problem. Stabilization problem of linear time-varying systems has been investigated in [39, 40, 41] via RDEs. In order to find H_∞ controller for linear time-varying systems, the state-space approach is used in [44]. The paper [17] proposed H_∞ control condition in terms of the solution of two couple RDEs. However, the problem of existence of solution of RDEs is still under active investigations. One of the efficient approaches to this problem is the controllability approach introduced by Kalman et al. [20]. Some sufficient conditions for global stabilisation of linear time-varying systems using controllability assumption are given in [39, 40, 45]. The controllability approach was also used in [9, 28] as a systematic method for finding the solution of partial differential equation.

Existing results on H_∞ control for time-delay systems can be classified into two types : that is, delay-dependent and delay-independent results. The delay-dependent H_∞ control results were improved by using the new technique,a new Lyapunov-Krasovskii functional and new condition for the solvability of the delay-dependent H_∞ control problem, see in [6, 8, 30, 56]. However, conservatism still remains in these results, which motivates the present study. They are improved delay-dependent bounded real lemma (BRL) for time-delay systems is established in term of a LMI. The problem of delay-range-dependent robust H_∞ for uncertain systems with time-varying delays and norm-bounded parameter uncertainties has been studied by [57]. Some new delay-range dependent stability and stabilization criteria are proposed by exploiting a new LyapunovKrasovskii functional and by making use of novel techniques for time-delay systems.

In practice the system often contains some uncertainties because it is too difficult to obtain an exact mathematical model due to environmental condition, uncertain or slowly varying parameters, etc. Recently, the stability analysis for time-varying delay has been provided to the systems with nonlinear perturbation. Moreover, the proposed result also has been expanded to H_∞ controller/filter design. Recently, Jiang and Han [19] proposed the problem of robust H_∞ control for system with interval time-varying delay in a range by employing free weighting matrix method. However some useful terms in estimating the derivative of Lyapunov functional are neglected. In [56], some less conservative delay-dependent robust H_∞ control conditions for uncertain linear system with state delay and parameter uncertainties have been obtained by constructing new Lyapunov-Krasovskii functional. However, the range of time-varying delay considered in these paper is from 0 to an upper bound. In [57], some new delay-range dependent stability and stabilization criteria are proposed by introducing some free weighting matrices and new estimate on the upper bound of the derivative of Lyapunov functional without ignoring some useful term that take into account information of the lower and upper bounds for the time delay. In [43] the authors studied the problem of stability analysis of the linear systems with nonlinear perturbation by partitioning the delay interval into two segments of equal length. In these, the time-varying delay within the lower and upper bounds and the delay-derivative have only the upper bound. In [50], the authors proposed new delay-dependent sufficient condition of H_∞ control problem for nonlinear systems with interval time-varying delay by using Jensen's inequality and reciprocally convex combination technique.

Goal

In this thesis, we shall investigate the H_∞ control problem for certain dynamical systems with time-varying delay.

Firstly, we study H_∞ control problem for a class of linear uncertain time-varying systems with time-varying delay via the solution of certain RDE. We show that the solution to this problem can be verified by the existence of solution of this RDE. The obtained result holds for system without delay and in this case, the solution to H_∞ control problem is verified by global null controllability of linear control system. The feedback stabilizing controller for the problem is constructed via the solution of a RED.

Secondly, we consider robust H_∞ control for a class of linear systems with time-varying delay. The time delay is a continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, but the delay function is not necessarily differentiable. Based on Lyapunov-Krasovskii theory combined with Leibniz-Newton's formula, new delay-dependent sufficient conditions for the exponential stabilization and a prescribed H_∞ performance level of the closed-loop system for all admissible uncertainties, are established in terms of LMIs.

Finally, we consider the problem of asymptotic stabilization and H_∞ control of nonlinear system with time-varying delay. By constructing a set of improved Lyapunov-Krasovskii functional which includes some integral terms of the form $\int_{t-h}^{t}(h-t-s)^j \dot{x}^T(s)R_j \dot{x}(s)ds$ $(j = 1, 2)$, a matrix-based on quadratic convex, combined with Wirtinger inequalities and some useful integral inequality. H_∞ controller is designed via memoryless state feedback control and new sufficient conditions for the existence of the H_∞ state-feedback for the system are given in terms of linear matrix inequalities (LMIs).

In the following, we will provide a guideline of what to expect in each chapter of this thesis. The remainder of this thesis is organized in six chapters as follows.

Chapter 1

In chapter 1, we investigates about general introduction and reason for H_∞ control problem for certain dynamical systems with time-varying delay. Finally, we present the goal and structure of the thesis.

Chapter 2

In chapter 2, we will introduce some basic concepts, notations and mathematical definitions of dynamical systems with time-varying delay. We give some general concepts of stability and H_∞ control problem , important definitions, lemma, propositions and results which will be used in later chapters.

Chapter 3

In this chapter, we investigate the H_∞ control problem for certain dynamical systems with time-varying delay. We consider the H_∞ control problem for a class of linear uncertain time-varying systems with time-varying delay via the solution of certain RED. We propose to give a new condition for the existence of delay-dependent H_∞ state feedback control for uncertain time-varying system with time-varying delay in terms of solutions of certain RDEs. An advantage of our approach is that the obtained condition can still be used for the system without delay and in this case the obtained condition follows from global null controllability of linear control system which is easy to verify.

Chapter 4

In This chapter, we investigate the robust H_∞ control for a class of linear systems with time-varying delay. First, we are consider the exponential stabilization,the time delay is a continuous function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available, but the delay function is not necessarily differentiable. Based on the constructing of improved Lyapunov-Krasovskii functionals combined with Leibniz-Newton's formula and the technique of dealing with some integral terms, new delay-dependent sufficient conditions for the exponential stabilization of the systems are first established in terms of LMIs without introducing any free-weighting matrices. Second, we are consider the H_∞ performance for the system. Numerical examples are given to illustrate the effectiveness of the theoretical results.

Chapter 5

In this chapter, we investigate the problem of asymptotic stabilization and H_∞ control of nonlinear system with time-varying delay. By constructing a set of improved Lyapunov-Krasovskii functional which includes some integral terms in the form $\int_{t-h}^{t}(h-t-s)^j\dot{x}^T(s)R_j\dot{x}(s)ds$ $(j=1,2)$ a matrix-based on quadratic convex, combined with Wirtinger inequalities and some useful integral inequality. H_∞ controller is designed via memoryless state feedback control and new sufficient conditions for the existence of the H_∞ state-feedback for the system are given in terms of LMIs. Numerical examples are given to illustrate the effectiveness of the obtained result.

Chapter 6

The last chapter provides the conclusion of the thesis.