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ข้อความแห่งการริเริ่ม

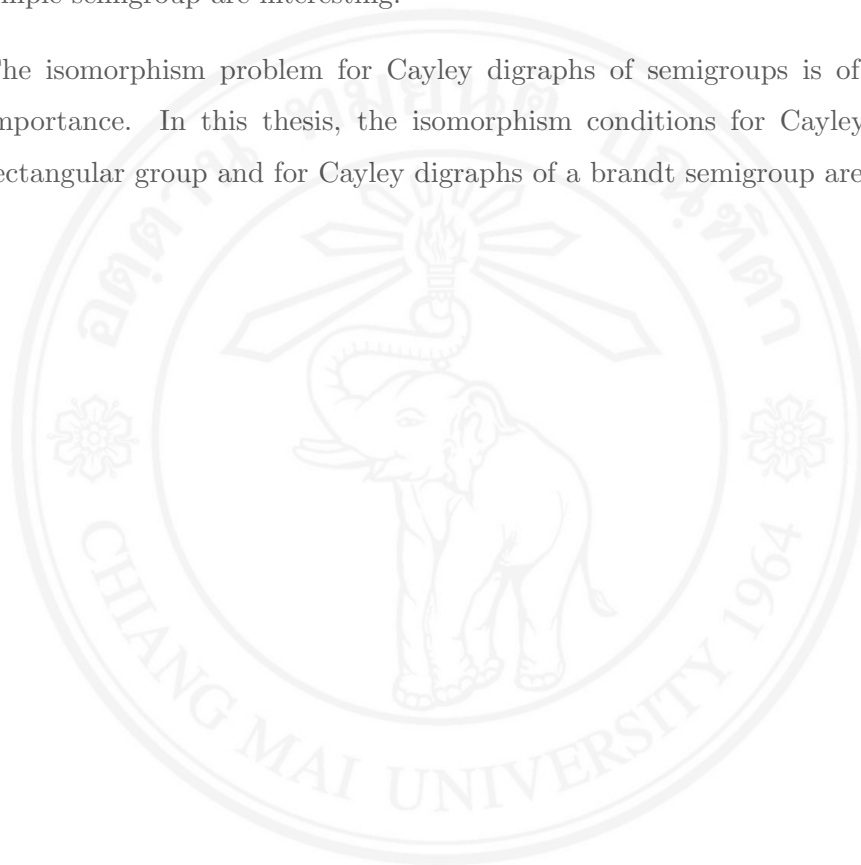
- 1) ไคกราฟเคย์เลย์ของกึ่งกรุปเชิงเดียวบริบูรณ์ได้รับการศึกษาอย่างแพร่หลาย ดังนั้นเงื่อนไขที่ใช้ระบุว่าไคกราฟใดจะเป็นไคกราฟเคย์เลย์ของกึ่งกรุปเชิงเดียวบริบูรณ์จึงเป็นเรื่องที่น่าสนใจศึกษา
- 2) ปัญหาการสมมติฐานสำหรับไคกราฟเคย์เลย์ของกึ่งกรุปเป็นปัญหาพื้นฐานที่มีความสำคัญ ในวิทยานิพนธ์นี้ได้นำเสนอเงื่อนไขการสมมติฐานสำหรับไคกราฟเคย์เลย์ของกรุปสี่เหลี่ยมมุมฉากและสำหรับไคกราฟเคย์เลย์ของกึ่งกรุปแบรนคัท



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STATEMENT OF ORIGINALITY

- 1) Cayley digraphs of completely simple semigroups have been studied extensively. Thus, the conditions for digraphs to be Cayley digraphs of a given completely simple semigroup are interesting.
- 2) The isomorphism problem for Cayley digraphs of semigroups is of fundamental importance. In this thesis, the isomorphism conditions for Cayley graphs of a rectangular group and for Cayley digraphs of a brandt semigroup are proposed.



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