CHAPTER 1

Introduction

The time-delay phenomenon are frequently encountered in various areas, such as biological modeling, chemical engineering systems, neural network, physical networks, economics, nuclear reactor, and many others, see [7], [8] and [10].

It is well known that the existence of time delay in a system may cause instability and oscillations systems, for examples, in [7], [8] and [10].

Besides, many practical systems always involve time-varying delays. So the stability analysis of time-delay systems have received considerable attention for the last few years. But most of the works of the systems are required the restriction on the derivative of the delay, namely $\dot{h}(t) < 1$. Besides, time-delays are time-varying continuous functions which vary from 0 to given upper bound. This condition can lead to conservativeness.

In practice, systems with neutral-type delay (the delay is in derivatives of states of systems) can be found in many fields, such as heat exchanges, population ecology and distributed networks containing lossless transmission lines. Hence, many researchers have studied neutral systems and sufficient conditions of such systems have been provided to guarantee the stability of neutral systems, for examples, in [5], [11], [13], [15], [16] and [19].

A neural network is a computational or mathematical model inspired by the structural and functional aspects of the network of neurons in the human brain , see [12].

In the past few decades, there has been a great interest in neutral-type neural networks because of their wide range of applications in the real world processes, such as pattern recognition, signal processing, associative memory, and combinatorial optimization. Therefore, the stability of neutral-type neural networks has been investigated by several researchers, see [13, 19].

Another important type of time delay is distributed delayed where stability analysis of neural network with distributed delayed has been paid attention extensively recently , see [12].

In 2011, T. Botmart, P. Niamsup, and V.N. Phat [2] studied the delay-dependent exponential stabilization for uncertain linear systems with interval non-differentiable time-varying delays described by

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [D + \Delta D(t)]x(t - h(t)) + [B + \Delta B(t)]u(t),$$
$$x(t) = \phi(t), t \in [-h_2, 0], \tag{1.1}$$

where $x(t) \in R^n$ is the state vector; $A, D \in R^{n\times n}$ and $B \in R^{n\times m}$ are constant matrices; $\Delta A(t), \Delta D(t)$ and $\Delta B(t)$ are unknown real matrices of appropriate dimensions and satisfy

$$\Delta A(t) = E_1F_1H_1,\ \Delta D(t) = E_2F_2H_2,\ \Delta B(t) = E_3F_3H_3, \tag{1.2}$$

where $E_i,\ H_i,\ i = 1, 2, 3$ are known real constant matrices of appropriate dimensions and $F_i(t),\ i = 1, 2, 3$ are unknown matrices function with Lebesgue measurable elements satisfying

$$F_i^T(t)F_i(t) \leq I, \quad i = 1, 2, 3 \ \ \forall t \geq 0. \tag{1.3}$$

$\phi(t)$ is the initial condition of system (1.1). $h(t)$ is a continuous time-varying delay function satisfying

$$0 \leq h_1 \leq h(t) \leq h_2, \tag{1.4}$$

where h_1 and h_2 are two constants. The following theorem is the main result in [2].

Theorem 1.1 [2] *Given $\alpha > 0$. The system (1.1) is $\alpha-$exponentially stabilizable if there exist symmetric positive definite matrices P,Q,R,U and $\epsilon_i > 0$, $i = 1, 2, ..., 6$ such that the following LMI holds*

$$M_1 = M - \begin{bmatrix} 0 & 0 & 0 & -I & I \end{bmatrix}^T \times e^{-2\alpha h_2}U \begin{bmatrix} 0 & 0 & 0 & -I & I \end{bmatrix} < 0, \tag{1.5}$$

$$M_2 = M - \begin{bmatrix} 0 & 0 & I & 0 & -I \end{bmatrix}^T \times e^{-2\alpha h_2}U \begin{bmatrix} 0 & 0 & I & 0 & -I \end{bmatrix} < 0, \tag{1.6}$$

$$M_3 = \begin{bmatrix} M_{11} & PH_1^T & PH_1^T & \frac{BH_3^T}{2} & \frac{BH_3^T}{2} \\ * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & -\epsilon_2 I & 0 & 0 \\ * & * & * & \frac{-\epsilon_5 I}{2} & 0 \\ * & * & * & * & \frac{-\epsilon_6 I}{2} \end{bmatrix} < 0, \tag{1.7}$$

$$M_4 = \begin{bmatrix} M_{11} & PH_2^T & PH_2^T \\ * & -\epsilon_3 I & 0 \\ * & * & -\epsilon_4 I \end{bmatrix} < 0. \tag{1.8}$$

In 2012, G. Liu, S.X. Yang and W. Fu [13] studied the robust stability of uncertain neutral-type neural networks with discrete interval and distributed time -varying delays given by

$$\begin{aligned} \dot{x}(t) =& -[A+\Delta A(t)]x(t) + [W_1+\Delta W_1(t)]f(x(t)) + [W_2+\Delta W_2(t)]f(x(t-\tau(t))), \\ & + [W_3+\Delta W_3(t)]\dot{x}(t-h(t)) + [W_4+\Delta W_4(t)]\int_{t-r(t)}^{t} f(x(s))\,ds, \\ x(t) =& \phi(t),\ t\in[-\delta,0],\ \delta=\max\{\tau_2,h,r\}, \end{aligned} \tag{1.9}$$

where $x(t)\in R^n$ is the state vector; A, W_1, W_2, W_3 and $W_4 \in R^{n\times n}$ are constant matrices; $\Delta A(t), \Delta W_1(t), \Delta W_2(t), \Delta W_3(t)$ and $\Delta W_4(t)$ are unknown real matrices of appropriate dimensions and satisfy

$$\begin{aligned} \Delta A(t) &= HF(t)B_1,\ \Delta W_1(t) = HF(t)B_2, \\ \Delta W_2(t) &= HF(t)B_3,\ \Delta W_3(t) = HF(t)B_4, \\ \Delta W_4(t) &= HF(t)B_5, \end{aligned} \tag{1.10}$$

where $H,\ B_i,\ i=1,2,3,4,5$ are known real constant matrices of appropriate dimensions and $F(t)$ is unknown matrix function with Lebesgue measurable elements satisfying

$$F^T(t)F(t) \le I, \quad \forall t \ge 0, \tag{1.11}$$

$\phi(t)$ is the initial condition of system (1.9). The time-varying delay functions $\tau(t), r(t), h(t)$ satisfy the conditions

$$0 < \tau_1 \le \tau(t) \le \tau_2,\ \dot{\tau}(t) \le \tau_d, \tag{1.12}$$

$$0 \le r(t) \le r, \tag{1.13}$$

$$0 < h(t) \le h,\ \dot{h}(t) \le h_d < 1, \tag{1.14}$$

where $\tau_1,\ \tau_2,\ r,\ h,\ \tau_d,\ h_d$ are constants. The following theorem is the main result of [13].

Theorem 1.2 [13] *The system (1.9) is globally robustly stable if there exist symmetric positive definite matrices* $P,\ C,\ Q_i,\ i=1,2,...,6,\ R_i,\ i=1,2,3,$ *matrices* $U_i,\ M_i,\ N_i,\ S_i,\ i=1,2,...,10$ *of appropriate dimension, diagonal matrices* $K,\ T_i,\ i=1,2$ *and* $\epsilon>0$ *such that* *the following LMI holds*

$$\begin{bmatrix} \tilde{\psi} & U & M & N & \Gamma \\ * & \psi_2 & 0 & 0 & 0 \\ * & * & \psi_3 & 0 & 0 \\ * & * & * & \psi_4 & 0 \\ * & * & * & * & -\epsilon I \end{bmatrix} < 0. \tag{1.15}$$

In 2012, W. Weera and P. Niamsup [19] studied the novel delay-dependent exponential stability criteria for neutral-type neural networks with non-differentiable time-varying discrete and neutral delays, described by

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t-h(t)) + A_2\dot{x}(t-\eta(t)) + f_0(t,x(t)) + f_1(t,x(t-h(t))), \\ &\quad + f_2(t,\dot{x}(t-\eta(t))) \\ x(t) &= \phi(t),\ t \in [-d,0],\ d = \max\{h_2,\eta_2\}, \end{aligned} \tag{1.16}$$

where $x(t) \in R^n$ is the state vector, $\eta(t)$ is the neutral delay, h(t) is time-varying continuous function which satisfy

$$0 \le \eta_1 \le \eta(t) \le \eta_2, \tag{1.17}$$

$$0 \le h_1 \le h(t) \le h_2, \tag{1.18}$$

where $h_1,\ h_2,\ \eta_1,\ \eta_2$ are constants. $\phi(t),\ \varphi(t)$ are the initial functions that are continuously differentiable on $[-d,0]$. $f_0(x(t),t), f_1(t,x(t-h(t))), f_2(t,\dot{x}(t-\eta(t)))$ are unknown nonlinear perturbations satisfying $f_0(0,t)=0, f_1(0,t)=0, f_2(0,t)=0$, and

$$\begin{aligned} f_0^T(x(t),t)f_0(x(t),t) &\le \beta_0^2x^T(t)x(t), \\ f_1^T(x(t-h(t)),t)f_1(x(t-h(t)),t) &\le \beta_1^2x^T(t-h(t))x(t-h(t)), \\ f_2^T(x(t-\eta(t)),t)f_2(x(t-\eta(t)),t) &\le \beta_2^2x^T(t-\eta(t))x(t-\eta(t)), \end{aligned} \tag{1.19}$$

where $\beta_0 \ge 0,\ \beta_1 \ge 0$ and $\beta_2 \ge 0$ are given constants. The following theorem is the main result in [19].

Theorem 1.3 [19] *Given $\alpha > 0$. The system (1.16) is $\alpha-$exponentially stabilizable if there exist symmetric positive definite matrices $P_1,S,D,Q,R,U,T,M,N, matrices P_i$, $i = 2,3,...,27$ of aproppriate dimension and $\epsilon_0,\epsilon_1,\epsilon_2 > 0$ such that the following LMI holds*

$$
\begin{aligned}
\omega_1 =& \omega - \begin{bmatrix} 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
& \times e^{-2\alpha h_2} U \begin{bmatrix} 0 & 0 & 0 & 0 & I & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0,
\end{aligned} \tag{1.20}
$$

$$
\begin{aligned}
\omega_2 =& \omega - \begin{bmatrix} 0 & 0 & 0 & -I & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
& \times e^{-2\alpha h_2} U \begin{bmatrix} 0 & 0 & 0 & -I & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0,
\end{aligned} \tag{1.21}
$$

$$
\begin{aligned}
\omega_3 =& \omega - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & I & 0 & 0 & 0 \end{bmatrix}^T \\
& \times e^{-2\alpha \eta_2} N \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 & I & 0 & 0 & 0 \end{bmatrix} < 0,
\end{aligned} \tag{1.22}
$$

$$
\begin{aligned}
\omega_4 =& \omega - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & -I & 0 & 0 & 0 \end{bmatrix}^T \\
& \times e^{-2\alpha \eta_2} N \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & -I & 0 & 0 & 0 \end{bmatrix} < 0,
\end{aligned} \tag{1.23}
$$

$$
\omega_5 = \begin{bmatrix} -0.1D & D^T \\ * & -S \end{bmatrix} < 0. \tag{1.24}
$$

In summary, the sufficient conditions for robust stability of uncertain neutral-type neural networks with discrete interval and distributed time-varying delays were proposed in [13] but their activation functions and discrete and neutral delays are too restrictive. In this thesis, we propose exponential stability criteria of neutral-type neural networks with interval non-differentiable and distributed time-varying delays and generalized activation functions by applying technics used in [2,19]. In Chapter 3, we give sufficient conditions for exponential stability of neutral-type neural networks with interval non-differentiable and distributed time-varying delays. Numerical examples are illustrated to show the efficiency of our theoretical results. Conclusion is provided in Chapter 4.