APPENDICES

APPENDIX A

Risk, Return and International Portfolio Analysis: Entropy and Linear Belief Functions

Apiwat Ayusuk and Songsak Sriboonchitta

The original article was published in "Econometrics of Risk", Studies in
Computational
Intelligence Volume 583, pp 319 – 328. (2015).

Risk, Return and International Portfolio Analysis: Entropy and Linear Belief Functions

Apiwat Ayusuk and Songsak Sriboonchitta

Abstract In this study, we analyze the international portfolio with respect to risk and return aspects. We applied entropy methods to find the optimal portfolio weights. In this method, we used entropy as the objective function and we also compared our results with the conventional method. Moreover, we use the linear belief function to build a portfolio, which can represent market information and financial knowledge and then we use matrix sweepings to integrate the knowledge for evaluating portfolio performance. Overall, our empirical analysis indicates that all entropy methods performed better than Markowitz method, and the finding also suggests that the investor should take the benefit from ASEAN market.

1 Introduction

Risk and return are important factors when investing in the capital market. According to the risk and return trade-off, the capital invested in the market cannot make higher returns without the possibility of investment loss. In classical work, Markowitz [14] is a well-known for the foundation of modern portfolio theory; which is mean and variance based method to find the optimal portfolio weights. Several researches were extensively studied in both theoretical and empirical works. (See, Tobin [19], Markowitz [15], Hakansson [6], Zenios and Kang [20], Konno and Kobayashi [10], etc.)

The modern world of economic globalization has a quick changing impact on the capital markets that contributed to an increase in international capital flow across countries. Many researches on topics related to international diversification, including Cavaglia et al. [1], Li [18], Fletcher and Marshall [4], Chiou [2] and Herrero and Vzquez [7] recommend that international diversification improves

A. Ayusuk (✉) · S. Sriboonchitta
Faculty of Economics, Chiang Mai University, Chiang Mai 50200, Thailand
e-mail: pai.mr.flute@gmail.com

A. Ayusuk
Department of Business Economics, Faculty of Liberal Arts and Management Sciences, Prince of Songkla University, Suratthani 84000, Thailand

 319
V.-N. Huynh et al. (eds.), *Econometrics of Risk*, Studies in Computational Intelligence 583, DOI 10.1007/978-3-319-13449-9_22

portfolio performance. During the last decade, The Chinese economy has been rapidly developed and played an important role in Asia and the world. Jayasuriya [8] found evidence that the stock market behavior of China had an impact on the stock market behavior of the East Asia and Pacific. Zhou et al. [21] found that the impact of the Chinese stock market on Asian markets had become increasingly powerful after 2005. Glick and Hutchison [5] also found that the strength of the correlation of stock markets between China and other Asia countries has increased markedly during 2008–2010 and has remained high in 2010–2012. In 2015, the ASEAN Economic Community (AEC) will induce regional economic integration, which provides a competitive advantage and economic benefits. Hence, to take advantage of investment diversification an international level, this study focuses on the stock markets in ASEAN, China and The U.S., which is the world major stock market.

In our review of the literature, we found two specific research questions. First, what is the most efficient tool for portfolio allocation under international risk and return strategy? Second, which portfolio should we invest in? Therefore, the primary objective of this research is to suggest new portfolio selection methods under risk and return using an information theory to select the optimal portfolio and the linear belief function to combine evidence. The secondary objective is to evaluate international portfolio performance.

The remainder of this paper is organized as follows. We give more details about the portfolio optimization methods and portfolio analysis in the system of the linear belief function in Sect. 2. We examine the data selection, descriptive statistics and the results of portfolio analysis in Sect. 3. Finally provides a brief conclusion.

2 Methodology

2.1 Portfolio Selection Methods

In this section we present four different methods to determine the optimal portfolio based on risk-return framework. The basic notations are defined by: $r_{i,t}$ is the return of market i at time t, μ_i is the expected return of market i, $\sigma_{i,j}$ is the covariance between the market of i and j, p_i and p_j are the weights assigned to markets i and j, μ_p is the expected return of portfolio, σ_p^2 is the portfolio risk. While m denote number of markets in portfolio, then expected return and variance of return of portfolio can be described by $\mu_p = \sum_i^m p_i \mu_i$ and $\sigma_p^2 = \sum_{i=1}^m \sum_{j=1}^m p_i p_j \sigma_{i,j}$ respectively.

2.1.1 Mean-Variance Markowitz Method

The conventional work of MV method is well known for the portfolio optimization approach. The goal of an investor is to find the optimal weight determinations in a portfolio by minimizing risk subjecting to the expected return of the portfolio being greater than or equal to risk free rate. The problem can be stated as:

$$\begin{aligned} &Minimize\ \sigma_p^2 \\ &st.\ \ \sum_i^m p_i\mu_i \geq \mu_0, \quad \sum_i^m p_i = 1 \end{aligned} \tag{1}$$

2.1.2 Mean Entropy Method

Entropy is a one of the methods to measure uncertainty in random variables. This study uses the Shannon entropy, $S(p) = -\sum_i^m p_i ln(p_i)$ under the principle of maximum entropy introduced by Jaynes [9]. The optimization problem is to choose the probability (or weight) in a portfolio by maximizing entropy function subject to the expected return (mean) of the portfolio being greater than or equal to risk free rate.

$$\begin{aligned} &Maximize - \sum_i^m p_i ln(p_i) \\ &st.\ \ \sum_i^m p_i\mu_i \geq \mu_0, \quad \sum_i^m p_i = 1 \end{aligned} \tag{2}$$

2.1.3 Mean-Variance Entropy Method

As the constraint of the principle of maximum entropy can be flexible, then we can provide more information. This optimization problem becomes maximizing the Shannon entropy subject to the expected return condition and the risk limitation strategy.

$$\begin{aligned} &Maximize - \sum_i^m p_i ln(p_i) \\ &st.\ \ \sum_i^m p_i\mu_i \geq \mu_0, \quad \sum_{i=1}^m \sum_{j=1}^m p_i p_j \sigma_{i,j} \leq \sigma_p^2, \quad \sum_i^m p_i = 1 \end{aligned} \tag{3}$$

2.1.4 Sharpe Ratio Entropy Method

The Sharpe ratio is introduced by Sharpe [16] to measure the portfolio performance that is described in unit of return per unit of risk. Therefore, we propose this methodology based on the Sharpe ratio into the principle of maximum entropy. The optimization problem is maximizing the Shannon entropy with an additional criterion of excess return per unit of risk.

$$Maximize - \sum_{i}^{m} p_i ln(p_i)$$

$$st. \ \frac{\mu_p}{\sigma_p} \geq \frac{\mu_0}{\sigma_0}, \quad \sum_{i}^{m} p_i = 1 \tag{4}$$

After we complete the solutions from each method, we use the Sharpe ratio to compare the performance of the portfolio selection methods.

2.2 Linear Belief Function

According to Dempster [3] and Liu [11], Linear belief functions is a special type of belief functions in expert system such as linear equations, linear regressions and Kalman filters, and also including Gaussian distributions that explain probabilistic knowledge on a set of variables in the continuous case. Liu et al. [13] used matrix sweepings to combine information from the linear belief function. In this study, we extended by using the linear time series belief function. Consequently, this study considers the linear time series to model portfolio investment by using the reduced form vector autoregressive (VAR) model with market returns as follows:

$$r_{i,t} = \Phi_0 + \Phi_1 r_{i,t-1} + e_t \tag{5}$$

where $r_{i,t} = [r_{1,t}, r_{2,t}, r_{3,t}]'$ is a 3×1 market return vector at time t that consists of ASEAN (AS), Chinese (CN) and U.S. markets, respectively. Φ_0 is a 3×1 vector of intercepts, Φ_1 is a time-invariant 3×3, e_t is a 3×1 vector of error terms and assume Gaussian distribution $e_t \sim N(0, \Sigma)$ with satisfying $E(e_t) = 0$, $E\left(e_t, e'_{t-1}\right) = 0$ is no serial correlation in error term and $E\left(e_t, e'_t\right) = \Sigma_{ij}$ is the variance-covariance matrix of error term that allowing non-zero correlation between error terms. The parameters of the VAR model can be estimated consistently by the OLS method when sample size is large. We construct a graphical structure for international portfolio analysis as follows:

In Fig. 1 we used the linear relationship from a VAR model combining with optimal portfolio weight to construct a graphical structure of international portfolio. There are ten variable nodes: $AS_t, CN_t, US_t, AS_{t-1}, CN_{t-1}, US_{t-1}, E_{AS_t}, E_{CN_t}, E_{US_t}, P$ and ten belief function nodes. Four linear belief functions represent the relationship between the variables, e.g. $Bel(AS_t, AS_{t-1}, CN_{t-1}, US_{t-1}, E_{AS_t})$ is a linear belief function of the ASEAN return that depended on the first lag of past returns of itself, China, and the U.S. and a residual variable. $Bel(P, AS_t, CN_t, US_t)$ is a linear belief function of portfolio return that is integrated with three market returns and optimal portfolio weights from best methods. And six linear belief functions represent an individual variable, e.g. $Bel(E_{AS_t})$ is the true value of a residual variable from

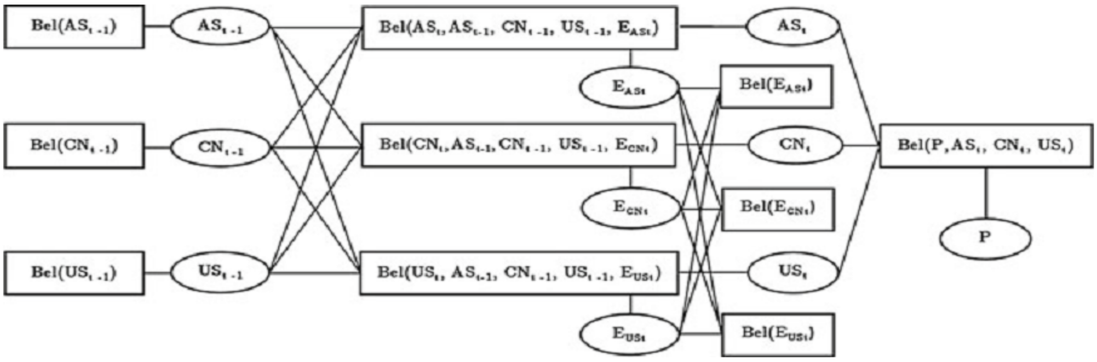

Fig. 1 A graphical structure of international portfolio

first function. Thus, we can analyze the linear belief function into the moment matrix approach.

The concept of Dempster's rule used to combine multiple focal elements that are independent evidence from several sources. Liu [11] proved the combination rule in Gaussian linear belief function of variable space was equivalent to that of Dempster [3]'s for continuous case. Liu [12] also proved that combination and marginalization of Gaussian linear belief function satisfies the axioms of Shenoy and Shafer [17] and showed Dempster's rule for the combination could be interpreted by matrix sweepings.

According to the matrix sweeping technique for Gaussian linear belief function is a matrix operation or a matrix transformation that including forward sweep and reverse sweep to consider. We can sweep a matrix from variance and covariance matrix move to conditional representation. Let r_i be a random variable representing the market returns that are assuming Gaussian distribution with expected mean: $E(r_i) = \mu_i$, variance: $Var(r_i) = \Sigma_{ii}$ and covariance $Cov(r_i, r_j) = \Sigma_{ij}, i, j = 1, 2, \ldots, n$, then we can write the moment matrix as $m = \begin{pmatrix} \mu_j \\ \Sigma_{ij} \end{pmatrix}$, This matrix represents the distribution of the random variables. We can define the operation on moment matrices by definitions below.

Definition 1 (*Marginalization*) Liu [12], the marginalization of a linear belief function is simply a projection in variable space. Let r_1 and r_2 are two random variables in the moment matrix: $m(r_1, r_2)$, its marginal to r_1 as

$$m^{\downarrow r_1}(r_1, r_2) = \begin{pmatrix} \mu_j \\ \Sigma_{ij} \end{pmatrix} \quad (6)$$

where $m^{\downarrow r_1}$ is the marginalization of the moment matrix that represent to the conditional moment matrix of linear regression coefficient.

Definition 2 (*Forward sweep*) Liu [12], Forward sweeping is the transformation of the moment matrix to be the conditional moment matrix. Let n market returns in portfolio and then we can operate a forward sweep of $m(r_1, , r_n)$ from r_s as follows:

$$m(r_1, \ldots, r_{s-1}, \overrightarrow{r_s}, r_{s+1}, \ldots, r_n) = \begin{pmatrix} \mu_{j,s} \\ \Sigma_{ij,s} \end{pmatrix} \tag{7}$$

where

$$\mu_{j,s} = \begin{cases} \mu_j - \mu_s \Sigma_{ss}^{-1} \Sigma_{sj}, & for\ j \neq s \\ \mu_s \Sigma_{ss}^{-1}, & for\ j = s \end{cases}$$

$$\Sigma_{ij,s} = \begin{cases} -\Sigma_{ss}^{-1}, & for\ i = s = j \\ \Sigma_{is} \Sigma_{ss}^{-1}, & for\ j = s \neq i \\ \Sigma_{ss}^{-1} \Sigma_{sj}, & for\ i = s \neq j \\ \Sigma_{ij} - \Sigma_{is} \Sigma_{ss}^{-1} \Sigma_{sj}, & for\ \ otherwise \end{cases}$$

Definition 3 (*Reverse sweep*) Liu [12], Let n market returns in portfolio and then we can operate a reverse sweep of $m(\overrightarrow{r_1}, , \overrightarrow{r_n})$ from r_s as follows:

$$m(\overrightarrow{r}_1, \ldots, \overrightarrow{r}_{s-1}, r_s, \overrightarrow{r}_{s+1}, \ldots, \overrightarrow{r}_n) = \begin{pmatrix} \mu_{j,s} \\ \Sigma_{ij,s} \end{pmatrix} \tag{8}$$

where

$$\mu_{j,s} = \begin{cases} \mu_j - \mu_s \Sigma_{ss}^{-1} \Sigma_{sj}, & for\ j \neq s \\ -\mu_s \Sigma_{ss}^{-1}, & for\ j = s \end{cases}$$

$$\Sigma_{ij,s} = \begin{cases} -\Sigma_{ss}^{-1}, & for\ i = s = j \\ -\Sigma_{is} \Sigma_{ss}^{-1}, & for\ j = s \neq i \\ -\Sigma_{ss}^{-1} \Sigma_{sj}, & for\ i = s \neq j \\ \Sigma_{ij} - \Sigma_{is} \Sigma_{ss}^{-1} \Sigma_{sj}, & for\ \ otherwise \end{cases}$$

Definition 4 (*The combined linear belief function*) Liu [12] The combination of two linear belief functions is the sum of fully swept matrices: $\overrightarrow{m} = \overrightarrow{m_1} \oplus \overrightarrow{m_2}$ and then we can write this combination as follows:

$$\overrightarrow{m} = \overrightarrow{m_1} \oplus \overrightarrow{m_2} = \begin{pmatrix} \overrightarrow{\mu_1} + \overrightarrow{\mu_2} \\ \overrightarrow{\Sigma_1} + \overrightarrow{\Sigma_2} \end{pmatrix} \tag{9}$$

3 An Application to International Portfolio Evaluation

The research is performed as follows: Firstly, we calculate the optimal weights of international portfolio. There are different methods to optimize the portfolio selection problem; Mean-Variance Markowitz (MV) method, Mean Entropy (ME) method, Mean-Variance Entropy (MVE) method. Secondly, we use the Sharpe ratio to measure the portfolio performance and select the best performance method. Thirdly, we

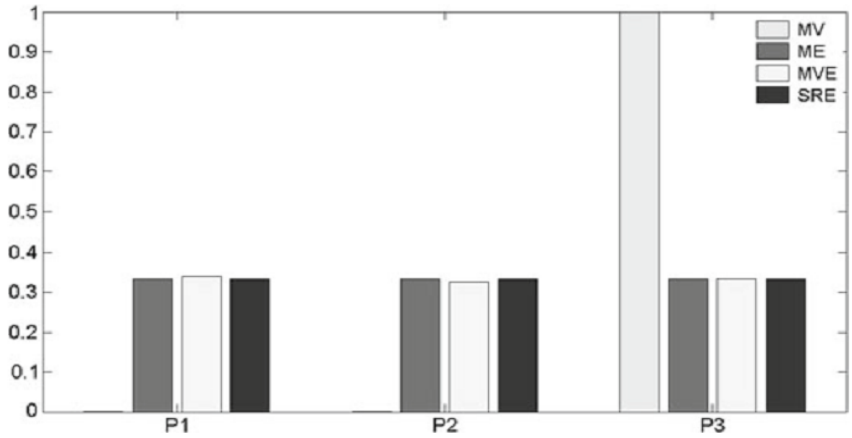

Fig. 2 The Optimal portfolio weight for four methods

construct the network of portfolio structure by using linear time series belief function. Finally, applying matrix sweepings to integrate the knowledge and information from second and third to evaluate the portfolio performance.

We collected daily data from January of 2009 to December of 2013 from Data Stream. As mentioned before, we considered a portfolio selection problem from three attractive markets, which are ASEAN (FTSE/ASEAN index), China (The Shanghai Composite index) and the U.S. (The S&P 500 index) markets.

Figure 2 presents the optimal weights of portfolios that are computed from four different methods. Table 1 presents the performances of the portfolio selection methods. From the results of Sharpe ratios, ME, MVE and SRE perform better than MV. MVE is better than other considered methods. Its optimal weights are 34.0 %, 32.6 % and 33.4 % in ASEAN, China and The U.S. markets respectively.

Table 2 shows the results of the parameter estimates in a VAR model. It represents the relationship between international markets, which should have an influence on each other. The results show that the first lag of the U.S. has high influence in ASEAN and China.

According to the financial information and market knowledge from the above results, we use MVE to optimize the portfolio weights because this method performs better than others. The optimal weight can represent by using the partially swept

Table 1 Comparison results for the portfolio selection methods

Methods	Portfolio returns	Portfolio variance	Sharpe ratios
MV	0.000549	0.000145	0.0373
ME	0.000421	0.000066	0.0396
MVE	0.000424	0.000065	0.0402
SRE	0.000421	0.000066	0.0396

MV Mean-Variance Markowitz method, *ME* Mean Entropy method,
MVE Mean-Variance Entropy method, *SRE* Sharpe Ratio Entropy method

Table 2 Estimates of a VAR model

Methods	ASEAN	China	U.S
ASEAN(−1)	−0.010810	−0.067352	0.004051
	[0.02801]	[0.04710]	[0.04308]
China(−1)	−0.040520	0.002412	0.015236
	[0.01770]	[0.02977]	[0.02722]
U.S.(−1)	0.330312	0.221640	−0.092744
	[0.01870]	[0.03145]	[0.02876]
Constant	0.000431	0.000031	0.000596
	[0.00022]	[0.00037]	[0.00033]
R-squared	0.202821	0.036987	0.008427
Schwarz SC	−6.846264	−5.806809	−5.985372

In parentheses are standard errors of the coefficient estimates

matrix as

$$m(P, \overrightarrow{AS}_t, \overrightarrow{CN}_t, \overrightarrow{US}_t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.3403 & 0.3255 & 0.3255 \\ 0.3403 & 0 & 0 & 0 \\ 0.3255 & 0 & 0 & 0 \\ 0.3255 & 0 & 0 & 0 \end{pmatrix} \quad (10)$$

Figure 1 $Bel(AS_t, AS_{t-1}, CN_{t-1}, US_{t-1}, E_{AS_t})$, we can define by the partially swept matrix from the first equation in a VAR model, $m(AS_t, \overrightarrow{AS}_{t-1}, \overrightarrow{CN}_{t-1}, \overrightarrow{US}_{t-1}, \overrightarrow{E}_{AS_t})$ with $Var(e_{AS_t}) = 0.00006$ is a variance of residual and $Cov(e_{AS_t}, e_{CNt_t}) = 0.00004$, $Cov(e_{AS_t}, e_{CN_t}) = 0.00004$ are covariance of residual as follows:

$$m(AS_t, \overrightarrow{AS}_{t-1}, \overrightarrow{CN}_{t-1}, \overrightarrow{US}_{t-1}, \overrightarrow{E}_{AS_t})$$
$$= \begin{pmatrix} 0.000431 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.01081 & -0.04052 & 0.33031 & 1 & 1 & 1 \\ -0.01081 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.04052 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.33031 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0.00006 & 0.00004 & 0.00004 \\ 1 & 0 & 0 & 0 & 0.00004 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0.00004 & 0 & 0 \end{pmatrix} \quad (11)$$

Therefore, to analyze the portfolio performance when the markets are related by using the linear belief function. We use six step method of Liu, Shenoy and Shenoy [13] to integrate knowledge using the combination of matrix sweeping.

Table 3 The results for moment matrix in portfolio

	Portfolio	ASEAN	China	U.S
Return	0.000584	0.0005942	0.000110	0.000551
Var-Cov	0.000051			
	0.000058	0.000077		
	0.000007	0.000011	0.000180	
	0.000031	−0.000005	−0.000003	0.000146
Sharpe ratio	0.067433			

Table 3 presents portfolio performance using the linear belief function. The result shows risks and returns in a portfolio: the ASEAN return 0.0594 % is highest, the standard deviation of ASEAN 0.8775 % is smallest, and the portfolio return is 0.0584 % with the standard deviation 0.7141 %.

4 Conclusions

This study provided empirical example for ASEAN, China and the U.S. markets between January 2009 and December 2013. We use three entropy methods base on Sharnon measure, which are ME, MVE and SRE, to select the optimal weights and compare its performances with conventional method, which is MV. Moreover, we use the linear belief function to extend portfolio evaluation because the belief function method can allow us to add information on the conditions of the relationship between international markets. There are two main findings from this study. First, all entropy methods perform better than MV because the entropy method well handles uncertainty information from simulations. Moreover, we found that MVE has higher performed than ME and SRE since MVE has added more information in constrains. Second, after integrating the information between the optimal portfolio strategy and international market relationship using linear belief function, we found that the portfolio risk is decreased and the portfolio return is increased. This implies that the relationship of international markets affects portfolio performance and this finding suggests that an investor should increase the investment proportion in the ASEAN market.

Acknowledgments The authors are very grateful to Prof. Thierry Denoeux for his comments and Prof. Amos Golan for the concept of Entropy Econometrics. This study was supported from Prince of Songkla University-PhD Scholarship.

References

1. Cavaglia, S., Hodrick, R., Vadim, M., Zhang, X.: Pricing the Global Industry Portfolios. Working Paper, National Bureau of Economic Research (2002)
2. Chiou, W.J.P.: Benefits of international diversification with investment constraints: an over-time perspective. J. Multinatl. Financ. Manag. **19**(2), 93–110 (2009)

3. Dempster, A.P.: Normal Belief Functions and the Kalman Filter Research Report Department of Statistics. Harvard University, Cambridge (1990)
4. Fletcher, J., Marshall, A.: An empirical examination of the benefits of international diversification. J. Int. Financ. Mark. **15**(5), 455–468 (2005)
5. Glick, R., Hutchison, M.: Chinasfinancial linkages with Asia and the globalfinancial crisis. J. Int. Money Financ. **39**, 186–206 (2013)
6. Hakansson, N.: Capital growth and the mean-variance approach to portfolio selection. J. Financ. Quant. Anal. **6**(1), 517–557 (1971)
7. Herrero, A.G., Vázquez, F.: International diversification gains and home bias in banking. J. Bank. Financ. **37**, 2560–2571 (2013)
8. Jayasuriya, S.A.: Stock market correlations between China and its emerging market neighbors. Emerg. Mark. Rev **12**(4), 418–431 (2011)
9. Jaynes, E.T.: Information theory and statistical mechanics. In: Statistical Physics, New York pp. 181–218 (1963)
10. Konno, H., Kobayashi, K.: An integrated stock-bond portfolio optimization model. J. Econ. Dyn. Control **21**, 1427–1444 (1997)
11. Liu, L.: A theory of Gaussian belief functions. Int. J. Approx. Reason. **14**(2–3), 95–126 (1996)
12. Liu, L.: Local computation of Gaussian belief functions. Int. J. Approx. Reason. **22**(3), 217–248 (1999)
13. Liu, L., Shenoy, C., Shenoy, P.P.: Knowledge representation and integration for portfolio evaluation using linear belief functions. IEEE Trans. Syst. Man, Cybern. Ser. A **36**(4), 774–785 (2006)
14. Markowitz, H.: Portfolio selection. J. Financ. **7**(1), 77–91 (1952)
15. Markowitz, H.: Portfolio Selection: Efficient Diversification of Investments. Wiley, New York (1959)
16. Sharpe, W.F.: Mutual fund performance. J. Bus. **39**(S1), 119–138 (1966)
17. Shenoy, P.P., Shafer, G.: Axioms for probability and belief-function propagation. In: Shachter, R.D., Levitt, T.S., Kanal, L.N., Lemmer, J.F. (eds.) Uncertainty in Artificial Intelligence, vol. 4, pp. 169–198. North-Holland, Amsterdam (1990)
18. Li, L.: An economic measure of diversification benefits. Working Paper, Yale International Center for Finance (2003)
19. Tobin, J.: Liquidity preference as behavior towards risk. Rev. Econ. Stud. **25**(2), 65–86 (1958)
20. Zenios, S.A., Kang, P.: Mean-absolute deviation portfolio optimization for mortgage backed securities. Ann. Oper. Res. **45**, 433–450 (1993)
21. Zhou, X., Zhang, W., Zhang, J.: Volatility spillovers between the Chinese and world equity markets. Pacific-Basin Financ. J. **20**(2), 247–270 (2012)

APPENDIX B

Risk Analysis in Asian Emerging Markets using Canonical Vine Copula and Extreme Value Theory

Apiwat Ayusuk and Songsak Sriboonchitta

The original article was published in “Special Issue on: Copula Mathematics and Econometrics” Thai Journal of Mahematics pp 59-72. (2014).

Thai Journal of Mathematics (2014) 59–72
Special Issue on : Copula Mathematics and Econometrics
http://thaijmath.in.cmu.ac.th
Online ISSN 1686-0209

Risk Analysis in Asian Emerging Markets using Canonical Vine Copula and Extreme Value Theory

Apiwat Ayusuk [†] [♭] [1] **and Songsak Sriboonchitta** [†]

[†]Faculty of Economics, Chiang Mai University, Thailand
e-mail : songsak@econ.cmu.ac.th
[♭] Department of Business Economics
Faculty of Liberal Arts and Management Sciences
Prince of Songkla University, Thailand
e-mail : pai.mr.flute@gmail.com

Abstract : Normal distributions are appropriate to describe the behavior of stock market returns only when returns do not exhibit extreme behavior. This study examined extreme value theory (EVT) to capture more precisely the tail distribution of market risk with vine copula and to identify the dependence structures between Asian emerging markets. We used value at risk (VaR) and conditional value at risk (CVaR), based on simulation method, to measure the market risk and portfolio optimization. Our empirical findings are that the conditional dependence between asymmetric volatility among five markets are positive and have the dependence between Indian and Thai stronger than other markets. The results of VaR and CVaR show that the Chinese market has the highest risk.

Keywords : Copula; GARCH; EVT; Value at Risk; Stock

2010 Mathematics Subject Classification : 62P20; 91B84 (2010 MSC)

[1]Corresponding author.

1 Introduction

Volatility implies uncertainty that has implications for investment decisions. Hence, the investors can find opportunities in gaining benefit with the situation when they implement efficient information and tools. In the context of risk modeling, Engle[14] and Bollerslev[8] proposed the econometric modeling of volatility that assumed the conditional on variance, namely, GARCH, which is taking into account the conditional heteroskedasticity inherent in time series. The GARCH models are able to yield VaR and CVaR estimates. The recent financial situation has experienced extreme risk or crises in the last two decades such as the Asian financial crisis in 1997, the U.S. Subprime crisis in 2007, and the EU debt crisis in 2009. Such studies conducted by McNeil and Frey [29], Bali[4] and Marimoutou et al.[27] have applied EVT for an alternative of effective framework to estimate the tail of a distribution. In the EVT based method, the GARCH models can estimate the volatility of the return series, then EVT is used to capture the tail of the standardized residual distribution of the GARCH models before estimating VaR. Bali and Neftci[5], Bystrom [10], Fernandez [18] and Chan and Gray [11] also found that the GARCH-EVT model had a more accurate estimation of VaR than that obtained from the parametric families.

Bollerslev[9] and Engle[17] improved the GARCH models to estimated the conditional linear dependence of volatility in pattern of multivariate random series and assumed multivariate normality. Subsequently, Lee et al.[26], Chiang et al[12], Syllignakis et al.[37], Ayusuk[3] and Hwang et al.[22] applied this methods in topics that were related to the international diversification from the perspective of market dependence. Gupta and Guidi[21] suggested that the conditional correlations between India and Asian markets have increased especially during the periods of international crisis. If dependence is not limited to linear correlation, then the usual correlation of returns may not provide sufficient information. Skarlar[33] proposed the copulas function to describe the joint dependence of random uniform marginal distribution. Copulas are flexible to analyze the dependence structure more than Gaussian or t distribution. According to the studies of copula in financial, Embrechts et al.[15] introduced copula in finance to relax the assumption of dependence structures between random returns. Patton[31] explained an overview of copula based models in financial applications. Bedford and Cook [6][7] and Kurowicka and Cooke [25] developed graphical model to determine the copula networks which can be called pair copulas, then Aas et al.[2] induced D and C-vine copula for inferential statistics. Subsequently, Nikoloulopoulos et al.[30], Zhang[41] and Sriboonchitta et al.[36] applied vine copula in the empirical studies for the international diversification of stock markets.

According to Asian markets, Wang[39] suggested that East Asian markets are less responsive to the shocks in the USA after the global financial crisis. The Chinese economy has been rapidly becoming one of the important role in the Asian market. Jayasuriya[22], Zhou et al. [43] and Glick and Hutchison[20] found evidence that the Chinese market had an impact on the Asian market. To take advantage of the portfolio allocation for international diversification and making it an alternative choice for investment, we focused on portfolio diversification based on risk analysis in the application of the Asian emerging markets.

In this paper, we focus on two aims. For the primary aim, we used conditional EVT or GARCH-EVT with canonical vine copula to study the dependence across Asian emerging markets. As for the secondary aim, we will compute the market risk and the international portfolio performance using VaR and CVaR technique. The remainder of this paper is organized as follows. We give more details about copula, EVT and portfolio optimization technique in Section 2. We exhibit the data selection, descriptive statistics and the results in Section 3. In the last section a conclusion has been provided.

2 Methodology

Using a three stage approaches, we estimated AR-GARCH model for the conditional volatility in stage one. To create the GARCH residuals, this study used generalized Pareto distribution (GPD) to capture the standardized residuals in the extreme tails and Gaussian distribution in the interior. The vine copula is used for the analysis of the dependence structures between markets in stage two. Finally, We used simulation procedure to generate the dependent return series for calculating VaR and CVaR.

2.1 The GARCH-EVT model

We use the simplified AR-GARCH model with mean equation as a first autoregressive process and the conditional variance equation as a GJR-GARCH (1,1) for modeling asymmetric volatility collecting.

$$r_{it} = \beta_{i0} + \beta_{i1} r_{it-1} + \varepsilon_{it} \tag{2.1a}$$

$$\sigma_{it}^2 = \mu_i + \alpha_i \varepsilon_{it-1}^2 + \gamma_i \varepsilon_{it-1}^2 I_{it-1} \text{ and } z_{it} \sim iid. \tag{2.1b}$$

where $I_{it-1} = 0$ if $\varepsilon_{it-1} \geq 0$, $I_{it-1} = 1$ if $\varepsilon_{it-1} \leq 0$, $r_{it} = [r_{1t}, r_{2t}, r_{3t}, r_{4t}, r_{5t}]'$ is a 5×1 market returns vector at time t, $\beta_{i0}, \beta_{i1}, \mu_i, \alpha_i, \theta_i, \gamma_i$ are parameters, $\varepsilon_{it} = \sigma_{it} z_{it}$ is return residuals and z_{it} is standardized residuals and it must satisfies independently and identically distributed, then the marginal distribution of standardized residuals can define as Gaussian distribution distribution is $g(z) = \varphi(z)$ for general situations and GPD to select the extreme situations that are peaks over threshold (POT).

The GPD was introduced by Pickands [32] ,which is defined as

$$g(z) = 1 - \frac{1}{n}\left(1 + \eta(\frac{z-u}{\vartheta})\right)^{-\eta^{-1}} \tag{2.2}$$

for $z > u$ that given a threshold u, where n is the number of observation, and k is the number of observations that excess over the threshold u, ϑ is the scale parameter, η is the shape parameter that can estimated by maximum likelihood. For $\eta = 0$, this distribution close to the Gumbel distribution, for $\eta < 0$, the distribution close to the Weibull distribution, for $\eta > 0$, the distribution belongs to the heavy-tailed distribution.

2.2 The Vine Copula

Sklar[35] proposed the copula theory, which is a function that links univariate marginal to their multivariate distribution and it can also be the models for the dependence between random variables by copulas. Let $x = [x_1, \cdots, x_n]'$ be random variables for $i = 1, \cdots, n$, the continuous marginal distributions are $F_1(x_1, \cdots, F_n(x_n)$ and $F(x_1, \cdots, x_n)$ be a multivariate distribution, then n-dimensio copula $C(\cdot) : [0,1]^n \to [0,1]$ can defined by

$$C(u_1, \cdots, u_n) = F(F_1^{-1}(u_1), \cdots, F_n^{-1}(u_n)) \tag{2.3}$$

then we can write

$$F(x_1, \cdots, x_n) = C(F_1(x_1), \cdots, F_n(x_n)) \tag{2.4}$$

Let F is continuous and strictly increasing. The probability density function of x can be defined as

$$f_{1,\cdots,n}(x) = \prod_{i=1}^{n} f_i(x_i) \cdot c_{1,\cdots,n}(F_i(x_1)), \cdots, F_n(x_n)) \tag{2.5}$$

We can write left hand side (6) as

$$f_{1,\cdots,n}(x) = f_1(x_1) f_{2|1}(x_2|x_1) \cdots f_{i|1\cdots n-1}(x_n|x_1, \cdots, x_{n-1}) \tag{2.6}$$

where $c_(1, \cdots, n)$ is the copula density and $f_i, i = 1, \cdots, n$ are the corresponding marginal pdf. Bedford and Cooke[6][7], who introduced canonical (C) and drawable (D) vines. Chollete et al.[13], Sriboonchitta et al.[37] suggested that C-vine copula dominate alternative dependence structures.

For this study we used a five-dimensional variable which has 240 options to design the possible pair copula constructions. Let us consider the five dimensional using C-vines, the density function which can be expressed as the following:

$$\begin{aligned} f(x_1, x_2, x_3, x_4, x_5) = &\prod_{i=1}^{5} f_i(x_i) \cdot c_{12}(F_1, F_2) \cdot c_{13}(F_1, F_3) \cdot c_{14}(F_1, F_4) \\ &\cdot c_{15}(F_1, F_5) \cdot c_{23|1}(F_{2|1}, F_{3|1}) \cdot c_{24|1}(F_{2|1}, F_{4|1}) \\ &\cdot c_{25|1}(F_{2|1} \cdot F_{5|1}) \cdot c_{34|12}(F_{3|12}, F_{4|12}) \cdot c_{35|12}(F_{3|12}, F_{5|12}) \\ &\cdot c_{45|123}(F_{4|123}, F_{5|123}) \end{aligned} \tag{2.7}$$

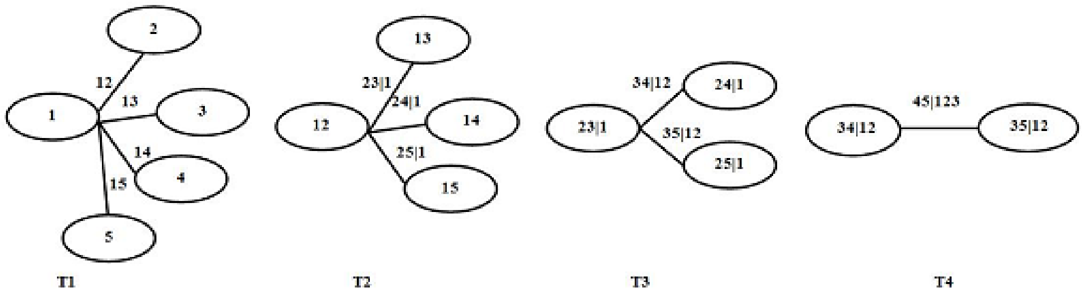

Figure 1: Five dimensional canonical vine construction

Aas and Berg[1] showed that the conditional distribution functions are computed by using partial derivatives of the bivariate copulas at the previous level as the following:

$$F(x_2|x_1) = \partial C_{12}(F_1, F_2)/\partial F_1 \tag{2.8a}$$
$$F(x_3|x_1) = \partial C_{13}(F_1, F_3)/\partial F_1 \tag{2.8b}$$
$$F(x_4|x_1) = \partial C_{14}(F_1, F_4)/\partial F_1 \tag{2.8c}$$
$$F(x_5|x_1) = \partial C_{15}(F_1, F_5)/\partial F_1 \tag{2.8d}$$
$$F(x_3|x_1, x_2) = \partial C_{23|1}(F_{2|1}, F_{3|1})/\partial F_{1|2} \tag{2.8e}$$
$$F(x_4|x_1, x_2) = \partial C_{24|1}(F_{2|1}, F_{4|1})/\partial F_{1|2} \tag{2.8f}$$
$$F(x_5|x_1, x_2) = \partial C_{25|1}(F_{2|1}, F_{5|1})/\partial F_{1|2} \tag{2.8g}$$
$$F(x_4|x_1, x_2, x_3) = \partial C_{34|12}(F_{3|12}, F_{4|12})/\partial F_{12|3} \tag{2.8h}$$
$$F(x_5|x_1, x_2, x_3) = \partial C_{35|12}(F_{3|12}, F_{5|12})/\partial F_{12|3} \tag{2.8i}$$
$$F(x_5|x_1, x_2, x_3, x_4) = \partial C_{45|123}(F_{4|123}, F_{5|123})/\partial F_{123|4} \tag{2.8j}$$

2.3 Value at Risk, Conditional Value at Risk and Portfolio Optimization

In this section, we present the portfolio analysis determined by the risk measure. In classical work, Markowitz[28] provided a quantitative procedure for measuring risk and return that used mean returns and variances to derive an efficient frontier where an investor could either maximize the expected return for a given variance as well as minimize the variance for a given expected return. Over the past decade, the VaR is a very popular model [see Duffie and Pan[14], RiskMetrics[33] Gourieroux et al.[19] and etc.] to measure risk; it means the maximum amount of loss that are not exceed on a given confidence level (q) over a time horizon. We can perform the following equation (2.9)

$$VaR_q(w) = min\{\gamma \in \mathbf{R} : P[f(w,r) \leq \gamma] \geq q\} \tag{2.9}$$

Let $q \in (0,1)$ is the confidence level, the probability of $f(w,r) = -w^T r$ not exceeding a given threshold γ, An alternative method which is defined risk by the expected loss of VaR, it called the conditional value at risk (or expected short fall) that calculated by

$$CVaR_q(w) = \gamma + \frac{1}{1-q} \int_{f(w,r) \geq VaR_q(w)} [f(w,r) - \gamma]^+ p(r) dr \tag{2.10}$$

Rockafellar and Uryasev[34] introduced the optimization portfolio problem using CVaR minimizing, which is able to be simplified as the following formulas:

$$\text{minimize} \begin{cases} CVaR_q(w) \\ \text{st. } w^T r = r_p \text{ and } e^T w = 1 \end{cases} \tag{2.11}$$

where r is a vector of the expected market return , r_p is the expected return of portfolio, w is a vector of the portfolio weight.

3 Empirical Applications

We collected daily data from January of 2008 to December of 2013 from the DataStream. With regards to the literature review, we considered a portfolio problem from five attractive markets in Asian emerging countries, which are 1. China (the Shanghai composite index: SC) 2. India (the Bombay stock exchange: BE) 3. Korea (Korea exchange: KE) 4. Taiwan (the Taiwan stock exchange: TE) 5. Thailand (the stock exchange of Thailand: SET) The stock return series are generated by $r_{it} = \log(p_{it}) - \log(p_{it-1})$

Table 1: Data descriptive and statistics

Statistics	SC	BE	KE	TE	SET
Min	-0.080437	-0.116044	-0.148764	-0.067351	-0.110902
Max	0.090343	0.159900	0.202302	0.065246	0.075487
Mean	-0.000588	0.000022	0.000021	0.000022	0.000278
S.D.	0.016770	0.016805	0.025203	0.013533	0.014206
Skewness	-0.179702	0.271395	0.544274	-0.293334	-0.662498
Kurtosis	6.960423	11.98158	15.06218	6.173628	9.822351
JB	1023.306	5239.008	9491.497	674.0081	3125.418
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ADF Test	-39.94339	-37.57615	-42.45914	-37.55748	-38.26316
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Note: In parentheses are the p-value of the test statistics

Table 1 reports the descriptive statistics for daily returns. The mean daily return is mostly positive, mostly positive skewness and the non-normality of all distribution which is rejected the null hypothesis by the high Jarque-Bera statistics. The ADF test confirmed that all return series are stationary at level.

Table 2: Parameter Estimates for AR(1) GARCH-EVT models

	SC	BE	KE	TE	SET
Mean eq.					
β_0	-0.000134	4.62e-06	0.000371	0.00040783	0.001011
	[0.000301]	[0.000274]	[0.000382]	[0.000241]	[0.000255]
β_1	-0.018851	0.03723	-0.039035	0.041258	0.0095189
	[0.023660]	[0.02703]	[0.026614]	[0.026476]	[0.027025]
Variance eq.					
μ	3.80e-07	2.29e-06	3.65e-06	5.01e-07	4.27e-06
	[3.48e-07]	[6.82e-07]	[1.24e-06]	[2.94e-07]	[1.15e-06]
α	0.9795	0.9139	0.9324	0.95462	0.86876
	[0.00479]	[0.012078]	[0.011657]	[0.008196]	[0.018241]
θ	0.0061551	0.006155	0.000000	0.000000	0.047426
	[0.007331]	[0.007331]	[0.012735]	[0.009873]	[0.018295]
γ	0.025594	0.025594	0.11359	0.078569	0.11959
	[0.009471]	[0.009471]	[0.021114]	[0.014977]	[0.028061]
EVT					
u_r	0.0177	0.0182	0.0230	0.0143	0.0145
η_r	-0.0006	0.1155	-0.1087	0.0373	0.0397
ϑ_r	0.5559	0.5173	0.5886	0.4166	0.4737
u_l	-0.0194	-0.0181	-0.0248	-0.0163	-0.0166
η_l	0.0147	-0.059	-0.0198	-0.0177	-0.0438
ϑ_l	0.7031	0.5953	0.6851	0.6471	0.6845

Note: In parentheses are standard errors of the coefficient estimates

Table 2 shows the estimated parameters for mean and variance equations of AR(1)GJR-GARCH with Gaussian kernel and generalized Pareto distribution, that is called the "semiparametric" distribution. Figure 1 presents the in-sample conditional volatility that is calculated by equation (2) and it is noticed that the five markets were highly volatile during the global financial crisis.

Table 3: Diagnostic statistics

Statistics	SC	BE	KE	TE	SET
JB	1288.06 [0.000]	926.236 [0.000]	1912.282 [0.000]	1215.095 [0.000]	414.347 [0.000]
Q(3)	4.1238 [0.248]	1.8630 [0.601]	2.3566 [0.652]	0.7779 [0.855]	4.1495 [0.246]
Q(6)	6.0856 [0.414]	4.3268 [0.633]	2.5839 [0.859]	4.6500 [0.589]	7.1647 [0.306]
KS test	0.0315 [0.0922]	0.0222 [0.4277]	0.01467 [0.8935]	0.037 [0.1215]	0.0305 [0.112]

Note: In parentheses are the p-value of the test statistics

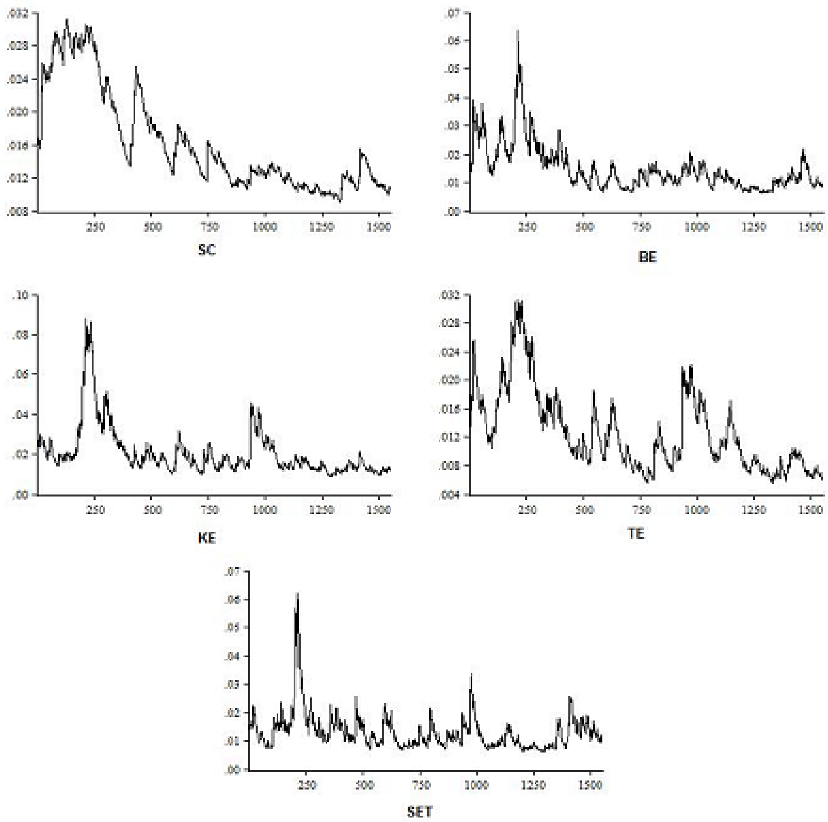

Figure 2: Estimated conditional volatility of SC, BE, KE, TE, SET in sample period

For the diagnostics test in table 3, the standardized residuals are non-normality distribution from the high Jarque-Bera statistics. Meanwhile, the standardized residuals satisfy the i.i.d. assumption because the Q-statistics accept the null hypothesis which implied that each series are not serially correlated. These findings confirm that the GARCH model should apply EVT on the standardized residuals. We transform the standardized residuals into uniform [0,1] based on empirical processes, then we also use Kolmogorov-Smirnov (KS) statistics to confirm that the data are uniformly distributed.

Table 4: Estimated parameters for five dimensional C-vine copula decomposition

Copula Family	Margin	$\hat{\theta}_0$	$\hat{\theta}_1$	Kendall′s tau	AIC
Gaussian	C_{12}	0.265650 [0.023600]	-	0.171173	-103.17465
Gaussian	C_{13}	0.239088 [0.024060]	-	0.153697	-82.96829
BB1	C_{14}	0.3550667 [0.050364]	1.100088 [0.027343]	0.228032	-222.21381
BB7	C_{15}	1.119008 [0.030929]	0.259820 [0.039628]	0.164719	-116.32319
t	$C_{23\|1}$	0.313743 [0.023594]	19.780978 [10.003315]	0.203166	-146.38576
BB1	$C_{24\|1}$	0.111484 [0.041479]	1.142413 [0.026810]	0.170877	-120.53664
t	$C_{25\|1}$	0.431295 [0.020672]	15.560217 [6.296301]	0.283887	-295.95646
Frank	$C_{34\|12}$	1.624721 [0.158124]	-	0.175962	-103.67280
Frank	$C_{35\|12}$	0.938040 [0.158465]	-	0.1033231	-33.06613
Gaussian	$C_{45\|123}$	0.207901 [0.023890]	-	0.1333264	-65.88185
Total					-1290.18

Note: 1 = SC , 2 = BE, 3 = KE, 4 = TE , 5 = SET and in parentheses are standard errors of the coefficient estimates

To model the dependence structures of Asian emerging markets, we considered using canonical vine model to analysis. Table 3 shows the dependence structures between the markets. We selected the best fitting copula family by the Akaike information criterion (AIC). The result shows that the copula families which are Gaussian and t copula in the linear sense as $C_{12}, C_{13}, C_{(45|123)}, C_{(23|1)}$ and $C_{(25|1)}$ for other copula, are far away from normality and difficult to compare with the results. From this information, we used the copula parameter to approximate the rank correlation as the Kendalls tau coefficient. This coefficient has a range between $-1 \leq \tau \leq 1$. If markets are fully independent, then the coefficient takes value close to zero. The result shows that there are the highest conditional dependency between Indian and Thai, Chinese and Taiwan, respectively

Table 5: VaR and CVaR estimation in each market

	SC	BE	KE	TE	SET
N = 10000					
$VaR_{0.99}$	0.028919	0.026666	0.026291	0.026943	0.028389
$VaR_{0.95}$	0.016419	0.015650	0.015684	0.015883	0.016463
$VaR_{0.99}$	0.035704	0.034472	0.033570	0.033629	0.034859
$VaR_{0.95}$	0.023938	0.022782	0.022298	0.022747	0.023549
N=20000					
$VaR_{0.99}$	0.027537	0.026911	0.027432	0.026676	0.026897
$VaR_{0.95}$	0.016219	0.015914	0.016185	0.015950	0.016202
$VaR_{0.99}$	0.034268	0.033979	0.034437	0.034330	0.033916
$VaR_{0.95}$	0.023273	0.022688	0.023099	0.022632	0.022907

Note: N = number of simulated data

Table 6: Optimal portfolio with CVaR minimization

	SC	BE	KE	TE	SET	CVaR
N = 10000						
99%	0.1857	0.2171	0.2034	0.1841	0.2097	0.0136
95%	0.1850	0.2099	0.2082	0.1982	0.1986	0.0097
N=20000						
99%	0.1829	0.2104	0.2036	0.2107	0.1924	0.0132
95%	0.1982	0.2032	0.1947	0.2044	0.1995	0.0096

Note: N = number of simulated data at 95% and 99% confidence level

Given the copula parameters in table 4, we used algorithms belong to Aas et al.(2009)[2] in CDVine package to generate 10,000 and 20,000 dependent uniform random variables over a holding period of one day. After generating the data, we converse the uniform series into the returns of each market and then we computed the risk measure with the daily data ,Table 4 reports the value at risk and conditional value at risk for different confidence interval. The VaR are used to measure the maximum possibility loss of the market value over a holding period of one day. The VaR is computed as 0.028919 at 99% confident interval, which implies the daily loss will not exceed 2.8919% in the Chinese market. Simultaneously, the CVaR is estimated as 0.035704 at 99% confident interval, which implies that the expected loss at 3.5704% would be exceed the VaR at 99% confident interval. The Chinese market has the highest VaR and CVaR based on comparison with the overall market. Table 5 shows the results of portfolio optimization in Asian emerging markets. The optimal weights suggest that investors should focus on Indian and Taiwan markets more than the other markets that are involved in the big picture.

4 Conclusions

We examined an empirical study of China ,India ,Korea ,Taiwan and Thai in Asian emerging stock markets from the period of 2008 to 2013 that covered the global financial crisis. Methodologically, we applied conditional EVT to capture the tails of the standardized residuals in each market return which are over the threshold when the tails have high risk. Then, we used C-vine copula to analyze the dependence of diversification measures. Empirically, the results show that the five stock markets have a positive dependence. The two highest dependences are the Indian and Thai markets as well as the Chinese and Taiwanese markets, respectively. Moreover, we have extended C-vine copula by applying the Monte Carlo simulation to generate series for measuring the VaR and CVaR in each markets. The results suggested that the Chinese market has the highest risk. For performing portfolio optimization, the results suggested that investors should pay attention to the Indian and Taiwanese markets.

Acknowledgements : The authors are very grateful to Prof.Hung Nguyen for his valuable teaching in the copula concept. This study was supported from Prince of Songkla University-PhD Scholarship.

References

[1] K. Aas, D. Berg, Modeling dependence between financial returns using PCC. In: D.Kurowicka, H.Joe, (Eds.), (Org.), Dependence modeling: Vine copula handbook, World Scientific (2011) 305-328.

[2] K. Aas, C. Czado, A. Frigessi, H. Bakken, Pair-copula constructions of multiple dependence, Insurance: Mathematics and Economics 44(2) (2009) 182-198.

[3] A. Ayusuk, The linkage of foreign stock markets on the stock exchange of Thailand: Empirical evidence from the Subprime crisis period, Nida Development Journal 52(1)(2012) 25-39.

[4] T.G. Bali, An extreme value approach to estimating volatility and value at risk, Journal of Business 76 (2003) 83-107.

[5] T.G. Bali, S. N. Neftci, Disturbing extremal behavior of spot price dynamics, Journal of Empirical Finance 10 (2003) 455-477.

[6] T. Bedford, ,R. Cooke, Probabilistic density decomposition for conditionally dependent random variables modeled by vines, Annals of mathematics and Artificial Intelligence 32 (2001) 245-268.

[7] T. Bedford, R. Cooke, Vines - a new graphical model for dependent random variables, Annals of Statistics 30(4) (2002) 1031-1068.

[8] T. Bollerslev, Generalized Autoregressive Conditional Heteroskedasticity, Journal of Econometrics 31 (1986) 307-327.

[9] T. Bollerslev, Modeling the Coherence in short-run nominal exchange rates: A multivariate generalized ARCH model, The Review of Economics and Statistics 72(3) (1990) 498-505.

[10] H. Bystrom, Extreme value theory and extremely large electricity price changes, International Review of Economics and Finance 14 (2005) 41-55.

[11] K.F. Chan, P. Gray, Using extreme value theory to measure value-at-risk for daily elec-tricity spot prices, International Journal of Forecasting 22(2) (2006) 283-300.

[12] T.C. Chiang, B.N. Jeon, H. Li, Dynamic correlation analysis of financial conta-gion: Evidence from Asian markets, Journal of International Money and Finance 26 (2007) 1206-1228.

[13] L. Chollete, A. Heinen, A. Valdesogo, Modeling international financial returns with a multivariate regime-switching copula, Journal of Financial Econometrics 7(4) (2009) 437-480.

[14] D. Duffie, J. Pan, An overview of value at risk, Journal of Derivatives 4 (1997) 7-49.

[15] P. Embrechts, A. McNeil, A. Straumann, Correlation and dependence properties in risk management: properties and pitfalls, in M. Dempster, ed., risk management: Value at risk and beyond, Cambridge University Press (2002)

[16] R.F. Engle, Autoregressive Conditional Heteroskedasticty with estimates of the variance of U. K. Inflation, Econometrica 50 (1982) 987-1008.

[17] R.F. Engle, Dynamic conditional correlation - A simple class of multivariate GARCH models, Journal of Business and Economic Statistics 20(3) (2002) 339-350.

[18] V. Fernandez, Risk management under extreme events, International Review of Financial Analysis 14 (2005) 113-148.

[19] C. Gourieroux, J.P. Laurent , O. Scaillet, Sensitivity analysis of values at risk, Journal of Empirical Finance 7 (2000) 225-245.

[20] R. Glick, M. Hutchison, Chinas financial linkages with Asia and the global financial cri-sis, Journal of International Money and Finance 39 (2013) 186-206.

[21] R. Gupta, F. Guidi, Cointegration relationship and time varying co-movements among Indian and Asian developed stock markets, International Review of Financial Analysis 21 (2012) 10-22.

[22] E. Hwang, H.G. Min, B.H. Kim, H. Kim, Determinants of stock market co-movements among US and emerging economics during the US financial crisis, Economic Modelling 35 (2013) 338-348.

[23] S.A. Jayasuriya, Stock market correlations between China and its emerging market neighbors, Emerging Markets Review 12(4) (2011) 418-431.

[24] G. Kim, M.J. Silvapulle, P. Silvapulle, Comparisons of semiparametric and paramet-ric methods for estimating copulas, Computational Statistics & Data Analysis 51 (2007) 2836-2850.

[25] D. Kurowicka, R. Cooke, Uncertainty analysis with high dimensional dependence modelling, Wiley Series in Probability and Statistics, 1st edition (2006)

[26] M.C. Lee, J.S. Chiou, C.M. Lin, A study of value-at-risk on portfolio in stock return using DCC multivariate GARCH, Applied Financial Economics Letters 2 (2006) 183-188.

[27] V. Marimoutou, B. Raggad, A. Trabesi, Extreme value theory and value at risk: Application to oil market, Energy Economics 31(4) (2009) 519-530.

[28] H. Markowitz, Portfolio selection, Journal of Finance 7(1) (1952) 77-91.

[29] A. McNeil, R. Frey, Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach, Journal of empirical Finance 7(3-4) (2000) 271-300.

[30] A.K. Nikoloulopoulos, H. Joe, H. Li, Vine copulas with asymmetric tail dependence and applications to financial return data, Computational Statistics & Data Analysis 56(11) (2012) 3659-3673.

[31] A.J. Patton, Copula-based models for Financial time series, in T.G. Andersen, R.A. Davis, J.-P. Kreiss and T. Mikosch (eds.) Handbook of Financial Time Series, Springer Verlag (2009)

[32] J. Pickands, Statistical inference using extreme order statistics, Annals of Statistics 3 (1975) 119-131.

[33] Risk Metrics TM, Technical Document , 4-th edition, J.P. Mogan (1996)

[34] R.T. Rockafellar, S. Uryasev, Optimization of conditional value-at-risk, The Journal of Risk 2(3) (2000) 21-41.

[35] A. Sklar, Fnctions de rpartition n dimensions et leurs marges. Publ. Inst. Statist. Univ. Paris 8 (1959) 229-231.

[36] S. Sriboonchitta, J. Liu, V. Kreinovich, H.T. Nguyen, A vine copula approach for analyzing financial risk and comovement of the Indonesian, Philippine and Thailand stock markets, Modeling Dependence in Econometrics, Springer Verlag, Berlin, Heidelberg (2014a) 241-254.

[37] S. Sriboonchitta, J. Liu, A. Wiboonpongse, Vine copula-cross entropy evaluation of dependence structure and financial risk in agricultural commodity index returns, Modeling Dependence in Econometrics, Springer Verlag, Berlin, Heidelberg (2014b) 275-287.

[38] M.N. Syllignakis, G.P. Kouretas, Dynamic correlation analysis of financial contagion: Evidence from the Central and Eastern European markets, International Review of Economics & Finance 20(4) (2011) 717-732.

[39] L. Wang, Who moves East Asian stock markets? The role of the 2007-2009 global financial crisis, Journal of International Financial Markets, Institution and Money 28 (2014) 182-203.

[40] C.C. Wu, H. Chung, Y.H. Chang, The economic value of comovement between oil price and exchange rate using copula based GARCH models, Energy Economics 34 (2012) 270-282.

[41] D. Zhang, Vine copulas and applications to the European Union sovereign debt analysis, International Review of Financial Analysis (2014)(in press)

[42] S.A. Zenios, P. Kang, Mean-absolute deviation portfolio optimization for mortgage backed securities, Annals of Operations Research 45 (1993) 433-450.

[43] X. Zhou, W. Zhang, J. Zhang, Volatility spillovers between the Chinese and world equity markets, Pacific-Basin Finance Journal 20(2) (2012) 247-270.

[44] B.J. Ziobrowskyi, Exchange rate risk and internationally diversified portfolios, Journal of International Money and Finance 14(1) (1995) 65-81.

(Received 30 May 2014)
(Accepted 10 September 2014)

APPENDIX C

Copula based Volatility Models and Extreme Value Theory for Portfolio Simulation with an Application to Asean stock markets

Apiwat Ayusuk and Songsak Sriboonchitta

The original article was accepted in "Causal Inference in Econometrics" Studies in Computational Intelligence, pp. 223-235. (2016)

Copula based volatility models and extreme value theory for portfolio simulation with an application to Asean stock markets

A. Ayusuk and S. Sriboonchitta

Abstract Many empirical works used risk modeling under the assumption of Gaussian distribution to investigate the market risk. The Gaussian assumption may not be appropriate for risk estimation techniques in some situations. In this study, we used the extreme value theory (EVT) to examine more precisely the tail distribution of market risk and incorporate high dimensional copulas to explore the dependence between stock markets. We gathered data of stock markets from Asean countries (Thailand, Singapore, Malaysia, Indonesia and the Philippines) to simulate the portfolio analysis during and post subprime crisis. The results found that D-vine copula GARCH-EVT model can simulate the efficient frontier of portfolios greater than other models. Furthermore, we also found the positive dependence for the overall markets.

1 Introduction

For asset allocation models, the risk-return characteristics are the most important issue for investors to consider. The conventional portfolio theory uses standard deviation and linear correlation coefficient to measure portfolio risk under multivariate normal distribution. To construct the optimal portfolio, this theory uses the risk-return framework to allocate assets by minimizing the risk of the portfolio subject to the portfolio return being greater or equal to the risk free rate.

Apiwat Ayusuk
Faculty of Economics, Chiang Mai University, Chiang Mai 50200 Thailand
Department of Business Economics, Faculty of Liberal Arts and Management Sciences,
Prince of Songkla University, Suratthani 84000 Thailand, e-mail: pai.mr.flute@gmail.com

Songsak Sriboonchitta
Faculty of Economics, Chiang Mai University, Chiang Mai 50200 Thailand,e-mail: songsakecon@gmail.com

The Value at Risk (VaR) is one of the most important and popular tool to measure the financial risk. It measures the maximum amount of loss that is not exceeded on a given confidence interval. An alternatively risk measure is the Condtional VaR (CVaR), which is used to estimate the expected loss from VaR. Rockafellar and Uryasev[30] showed a representation of CVaR based approachs to optimize portfolios. Moreover, Artzner et al.[3] and Rockafellar and Uryasev[31] explained that VaR is not coherence whereas CVaR satisfies the properties of the risk of a diversified portfolio, which are the sub-additive and convex properties. For these reasons, CVaR has the advantages over VaR.

The most widely used econometric approach to volatility modeling is the family of autoregressive conditional heteroscedasticity (ARCH), which is introduced by Engle[10]. It assumes that the conditional variance takes into account the conditional heteroskedasticity inherent in time with the assumption of normally distributed innovations. Bollerslev[6] then improved the ARCH to generalized ARCH (GARCH) model, which can yields VaR and CVaR as well.

In recent years, the EVT has been utilized to analyze financial data. It is a statistical tool to examine the extreme deviations from the median of probability distribution. It is very popular and useful for modeling in rare events. Hence, the EVT can be an alternative for an effective framework to estimate the tail of financial series when there are extreme financial events, such as the Asian financial crisis, Subprime crisis and European debt crisis. Embrechts et al.[9] provided examples for applications of EVT in finance and insurance. Bali[5], Wang et al.[34], Ren and Giles[29] and Jess et al.[16] applied EVT to calculate VaR for risk management.

The EVT based method combines ideas from the GARCH models with the tail of the innovations distribution using EVT to estimate VaR and CVaR. Exemplary works by McNeil and Frey[24] introduced EVT based method (or conditional EVT models) to forecast VaR. Karmakar[18] applied this method to estimate VaR in different percentiles for negative and positive BSE India returns. Furio and Climent[11] found that GARCH-EVT model is more accurate than the GARCH models assuming Gaussian or Student′s t distribution innovations for VaR simulation analysis. Meanwhile, Allen et al.[2] used both unconditional and conditional EVT models to forecast VaR. Marimoutou et al.[22] found that this model performs better than other methods without EVT, such as conventional GARCH, historical simulation and filtered historical simulation.

To study the dependence among stock markets using traditional methods, Pearsons correlation has been the most commonly used in empirical works. However, Pearsons correlation used to measure the degree of linear dependence between multivariate normally distributed data. More precisely, Copulas can relax the dependence structure beyond normal distribution. Moreover, the copula is flexible as it can be used to analyze linear, nonlinear or tail dependence. In the context of the copula in financial studies, Embrechts et al.[8] introduced copula in finance to relax the assumption of dependence structures between random returns. Patton[28] explained an overview of copula based models for financial applications. In the study of multivariate copulas, Kole et al.[19] and Wang et al.[35] found that the multivariate t copula is the best measure of the dependence structure between multiple assets because it can capture the dependence both in the center and the tails. Aas et al.[1] introduced the flexible way to set the pair copula construction, namely, D and C-vine copula. The recent study such as Low et al.[21], Hernandez[14], Ayusuk and Sriboonchitta[4], Mensi et al.[25] have applied vine copula with applications to portfolio management.

There are researchers on the effects of the subprime crisis. Hemche et al.[13] found the dynamic linkages between the US and developed stock markets (as France, Mexico, Italy and the UK) with strong comovements in times of financial crisis. The correlation between the US and other markets (as China, Japan, Tunisia, Egypt and Morocco) were weak and thus they suggested that the investors should also invest in some emerging countries. Moralesa and Callaghan[26] and Wang[33] suggested that the US stock markets are less generating effects into the Asian stock markets. In 2015, Asean Economic Community (AEC) is set to be implemented. There will be free trade of goods, services, skilled labor and investment capital following the liberalization and most countries in AEC are still emerging economies. Hence, to take advantage of the portfolio allocation for international stock market, this study focused on VaR and CVaR based on the econometric approaches with the application on the Asean stock markets during and post subprime crisis.

In this paper, the primary objective is to compare the econometric approaches to portfolio simulation. These econometric approaches include the multivariate t copula GARCH-EVT, C-vine copula GARCH-EVT and D-vine copula GARCH-EVT. The secondary objectives to measure the dependence among Asean stock markets.

The remainder of this paper is organized as follows. In section 2, we provide details about the GARCH model, EVT, copulas and the portfolio simulation procedure. In section 3, discuss the data selection, descriptive statistics and the results of the empirical work. In the final section, we present concluding remarks.

2 Methodology

2.1 Marginal Models

Generally, data on market returns present conditional heteroscedasticity. Hence, this study focuses on the marginal returns through the autoregressive conditional heteroskedasticity model. To capture the asymmetry property under the sense that shocks not have the exact same impact on volatility in between negative and positive shocks, we used the GJR GARCH model that was proposed by Glosten et al.[12].

$$r_t = \beta_0 + \beta_1 r_{t-1} + \varepsilon_t = \beta_0 + \beta_1 r_{t-1} + \sigma_t z_t \tag{1}$$

$$\sigma_t^2 = \mu + \alpha \varepsilon_{t-1}^2 + \theta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \tag{2}$$

where r_t is a market return at time t, $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$,$I_{t-1} = 0$ if otherwise, $\beta_0, \beta_1, \mu, \alpha, \theta, \gamma$ are parameters. For stationarity and positivity, the GJR GARCH model has the following properties: $\alpha > 0, \theta > 0, \gamma > 0, \alpha + \gamma > 0$ and $\alpha + \theta + \gamma/2 < 1$, $\varepsilon_t = \sigma_t z_t$ is residual return ,σ_t is the volatility of the return and z_t is standardized residual that must satisfy independently and identically distributed.Traditionally, the standardized residuals follow a normal distribution.

2.2 The distributions of standardized residuals

In this study, we focus on EVT, which is an appropriate approach to define the behavior of extreme tail observations. We apply the semi parametric approach to generate the standardized residuals of the GJR GARCH model. To capture the extreme tails, we use the generalized Pareto distribution (GPD) to select the extreme tails that are peaks over the threshold. To capture the interior distribution, we define by using the Gaussian kernel distribution $(\varphi(z))$. The distribution is given by

$$F(z) = \begin{cases} \frac{k_{u^L}}{n}\left(1 + \eta^L\left(\frac{u^L - z}{\vartheta^L}\right)\right)^{-(\eta^L)^{-1}} & ,z < u^L \\ \varphi(z) & ,u^L < z < u^R \\ 1 - \frac{k_{u^R}}{n}\left(1 + \eta^R\left(\frac{u^R - z}{\vartheta^R}\right)\right)^{-(\eta^R)^{-1}} & ,z > u^R \end{cases} \quad (3)$$

where u^L and u^R are lower (L) and upper (R) thresholds, z is the standardized residuals that excess over the thresholds, k_{u^L} and k_{u^R} are the number of observations that excess over thresholds, n is the number of observation, ϑ^L and ϑ^R are the scale parameters, η^L and η^R are the shape parameters.

2.3 Copula Approach

A copula is a function that connects univariate marginals to construct the multivariate distribution with uniformly distributed marginals $U(0,1) \longrightarrow [0,1]$. It also can be used to portray the dependency of random variables in each event. This study used the copula approach for describing the dependence between international markets. Originally, Sklar[32]introduced the important theorem for copula function as follows

Theorem 1. *Let $x_1,...,x_n$ are random variables for $i = 1,...,n, F_1(x_1),...,F_n(x_n)$ are the continuous marginal distributions and $F(x_1,...,x_n)$ be a multivariate distribution. Then, $n-$dimensional copulas $C(\cdot) : [0,1]^n \longrightarrow [0,1]$ can be defined by*

$$F(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)) \quad (4)$$

Inversely, equation(4) can be written as

$$C(u_1,...,u_n) = F(F_1^{-1}(u_1),...,F_n^{-1}(u_n)) \tag{5}$$

where $F_i^{-1}(u_i)$ are the inverse distribution function of the marginals and $u_i \in [1,0]$.

We can determine the copula density $c(u_1,...,u_n)$ by using n order partial derivative as follows

$$c(u_1,...,u_n) = \frac{\partial^n C(u_1,...,u_n)}{\partial u_1,...,\partial u_n} \tag{6}$$

According to the joint density function $f(x_1,...,x_n)$, it can be defined by n order partial derivative of equation (6) as follows

$$f(x_1,...,x_n) = \frac{\partial^n F(x_1,...,x_n)}{\partial x_1,...,\partial x_n} \tag{7}$$

$$f(x_1,...,x_n) = \prod_{i=1}^{n} f_i(x_i) \frac{\partial^n C(u_1,...,u_n)}{\partial u_1,...,\partial u_n} \tag{8}$$

$$f(x_1,...,x_n) = \prod_{i=1}^{n} f_i(x_i) \cdot c(u_1,...,u_n) \tag{9}$$

Equation (9) shows that the joint density function is the combination between the copula density and the product of marginal densities. In the study of copulas, Mashal and Zeevi[23], Breymann et al.[7], Kole et al.[19] and Wang et al.[35] suggested that t copula is the better measure of the dependency structure for multiple assets. Hence, this study considered t copula for measuring the market dependence. We can define a multivariate t copula for n dimensional as follows

$$C^t(u_1,...,u_n) = t_{\nu,\Sigma}(t_\nu^{-1}(u_1),...,t_\nu^{-1}(u_n)) \tag{10}$$

where $t_{\nu,\Sigma}$ is the standardized residual of the multivariate t copula, Σ is a correlation matrix and ν is the degree of freedom. Moreover, This study also applied C and D-vine structures with t copula to determine the market dependence. The two vine copulas were introduced by Aas et al.[1] . In n dimensions, $n(n-1)$ is the number of pair copula, $n-1$ is the number of trees in vine copulas and $n!$ is the number of possible tree structures. To select the tree structures, this study determines the appropriate ordering of the tree structures by choosing the maximum of absolute empirical Kendall$'$s tau values for all bivariate copula. C and D-vine density functions can be defined by

$$f(x_1,...,x_n) = \prod_{i=1}^{n} f_i(x_i) \prod_{j=1}^{n-1} \prod_{k=1}^{n-j} c_{j,j+k|1,...,j-1}\left(F(x_j|\mathbf{v_1}), F(x_{j+k}|\mathbf{v_1})\right) \tag{11}$$

$$f(x_1,...,x_n) = \prod_{i=1}^{n} f_i(x_i) \prod_{j=1}^{n-1} \prod_{k=1}^{n-j} c_{k,k+j|k+1,...,k+j-1}\left(F(x_k|\mathbf{v_2}), F(x_{k+j}|\mathbf{v_2})\right) \tag{12}$$

where $\mathbf{v_1} = x_1,...,x_{j-1}$, $\mathbf{v_2} = x_{k+1},...,x_{k+j-1}$, j is the tree in vine copulas, k is the edge in each tree, $c_{j,j+k|1,...,j-1}$ in equation (11) and $c_{k,k+j|k+1,...,k+j-1}$ in equation (12) are bivariate copula densities. In order to compute the conditional distribution functions $F(x|\mathbf{v})$ in equation (11) and (12) by following Joe [17], as in equation (13)

$$F(x|\mathbf{v}) = \frac{\partial C_{xv_j|\mathbf{v_{-j}}}\left(F(x|\mathbf{v_{-j}}), F(v_j|\mathbf{v_{-j}})\right)}{\partial F(v_j|\mathbf{v_{-j}})} \tag{13}$$

where the vector v_{-j} is the vector v_j that excludes the component v_j. $C_{xv_j|\mathbf{v_{-j}}}$ is the bivariate copula distribution between x and v_j that is taken conditional on v_{-j}. The estimated dependence parameters of various copulas are obtained by maximum likelihood (see Aas et al.[1]).

2.4 Portfolio Simulation

We forecast one-day-ahead for VaR, CVaR based on t copula GARCH-EVT at 95% and 99% confidence level with the procedures as follows:
(1) We estimate the parameters of the GARCH model for each market return series. We obtain the standardized residuals over the threshold follow the generalized Pareto distribution (GPD), because GPD can capture the upper and lower tails. Additionally, we also use the Gaussian kernel estimation for the interior part.
(2) We transform each standardized residuals (z_t) of each univariate distribution to approximate i.i.d. uniform data (u_t) on $[0,1]$ by using empirical distribution functions and then fit t copula for estimating its parameter.
(3) Given the parameters of copula function, we simulate the uniform series 100,000 dimensional time series and obtain the standardized residuals by using the inverse functions of the estimated marginals.
(4) We converse the standardized residuals from step (3) into the returns at $t+1$, calculate the empirical one-day-ahead VaR, CVaR at 95% and 99% confidence level, and optimize the portfolio based on CVaR minimization problem at 99% confidence level (or $Min_{w \in W} CVaR$) by following the procedure of Rockafellar and Uryasev[30][31].

3 Empirical Results

We used the daily data of five main stock market indices in Asean countries from DataStream: The indices composed of SET index (Thailand:TH), Straits Times index (Singapore:SP), KLSE Composite index (Malaysia:MS), JSX Composite index (Indonesia:ID) and PSE Composite(the Philippines:PP). We defined the market returns by $r_t = log(p_t) - log(p_{t-1})$. Following Horta et al.[15]and Lee et al.[20], this study focuses on subprime crises period and then we divide it into sub periods: the subprime crisis period (1 August 2007 to 29 December 2009) and the post subprime crisis period (4 January 2010 to 29 December 2014).

Table 1: Descriptive measures for Asean markets

Index	TH	SP	MS	ID	PP
			A: crisis period		
Mean	-0.000221	-0.000341	-9.38E-05	0.000210	-0.000220
Max	0.086167	0.102705	0.057165	0.190719	0.083854
Min	-0.085892	-0.129279	-0.102374	-0.257802	-0.136399
S.D.	0.018798	0.020622	0.012513	0.026211	0.019945
Skewness	-0.104720	-0.034376	-0.873542	-1.404863	-0.687729
Kurtosis	6.331662	8.140576	13.30447	27.81636	9.093367
Jarque-Bera	243.3070	577.0605	2384.952	13618.47	851.9586
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ADF statistics	-20.55669	-21.93688	-21.54766	-22.41699	-21.35143
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
			B: after crisis period		
Mean	0.000667	0.000141	0.000304	0.000651	0.000818
Max	0.057515	0.029001	0.047228	0.070136	0.055419
Min	-0.058119	-0.037693	-0.026757	-0.092997	-0.069885
S.D.	0.011965	0.008567	0.006207	0.012800	0.011732
Skewness	-0.367571	-0.423877	0.108370	-0.822461	-0.500209
Kurtosis	6.294566	4.813557	8.424547	10.33282	6.756750
Jarque-Bera	509.4336	179.1765	1317.675	2524.951	675.7222
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ADF statistics	-30.01431	-30.38082	-29.13288	-22.97933	-22.97933
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Note:In parentheses are standard errors of the coefficient estimates

Table 1 shows summary statistics. We found that almost all markets of the average yield (mean of market return) are negative during the subprime crisis. SP has the most negative returns. After the subprime crisis, the average yield has a positive sign in every market and the standard deviation (SD) is less than a period of the subprime. The Jarque-Bera rejects the null hypothesis which indicated that returns of the markets are not following the normality assumption. The ADF test approved the stationary property of all markets.

Table 2: Parameter Estimates for AR(1)-GJR GARCH-EVT models

Index	TH	SP	MS	ID	PP
			A: crisis period		
Mean equation					
β_0	0.000549	0.000135	0.000199	0.000541	0.000142
	[0.000665]	[0.000651]	[0.000392]	[0.000688]	[0.000678]
β_1	0.009793	0.001870	0.065287	0.11054	0.086547
	[0.045931]	[0.046813]	[0.044449]	[0.046162]	[0.046992]
Variance equation					
μ	1.69e-005	3.22e-006	1.17e-005	0.000137	7.79e-005
	[8.32e-006]	[3.16e-006]	[5.24e-006]	[3.16e-005]	[2.59e-005]
α	0.83871	0.91095	0.80145	0.38876	0.57183
	[0.051076]	[0.023375]	[0.059008]	[0.083337]	[0.097427]
θ	0.045528	0.035349	0.018407	0.000000	0.063385
	[0.036726]	[0.020904]	[0.028609]	[0.040953]	[0.048436]
γ	0.13214	0.1004	0.23656	0.88391	0.30257
	[0.064542]	[0.040892]	[0.095621]	[0.25269]	[0.12236]
Q(2)	9.0621	0.8644	0.8474	4.3689	0.3434
	[0.0108]	[0.6491]	[0.6546]	[0.1125]	[0.8423]
Q(6)	16.4338	13.7795	3.5713	6.9328	5.8217
	[0.0116]	[0.0322]	[0.7345]	[0.3271]	[0.4435]
KS-statistics	0.0027	0.003	0.003	0.0036	0.0032
	[0.4771]	[0.3156]	[0.3438]	[0.1599]	[0.2709]
Jarque-Bera	82.0629	85.3962	789.3312	1316.8	231.8684
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
			B: after crisis period		
Mean equation					
β_0	0.001186	0.000277	0.000306	0.000988	0.000791
	[0.000282]	[0.000210]	[0.000146]	[0.000289]	[0.000293]
β_1	0.027503	0.010368	0.083798	0.004194	0.085756
	[0.032322]	[0.03082]	[0.028665]	[0.032237]	[0.033591]
Variance equation					
μ	5.95e-006	8.02e-007	1.97e-006	6.29e-006	8.96e-006
	[1.68e-006]	[3.40e-007]	[6.75e-007]	[1.99e-006]	[2.63e-006]
α	0.83591	0.92912	0.85629	0.87223	0.81495
	[0.029077]	[0.015984]	[0.03229]	[0.027457]	[0.036924]
θ	0.035536	0.014864	0.024771	0.019357	0.020671
	[0.024681]	[0.018464]	[0.021801]	[0.025768]	[0.025987]
γ	0.16857	0.085624	0.14827	0.125	0.18828
	[0.041524]	[0.0243]	[0.043269]	[0.04037]	[0.048925]
Q(2)	3.0135	0.2932	2.4969	4.1248	0.7148
	[0.2216]	[0.8636]	[0.2870]	[0.1271]	[0.6995]
Q(6)	6.5855	7.2224	3.3907	24.2223	8.8284
	[0.3609]	[0.3008]	[0.7585]	[0.0005]	[0.1835]
KS-statistics	0.0161	0.0164	0.0227	0.0208	0.0154
	[0.9339]	[0.945]	[0.6376]	[0.7443]	[0.9651]
Jarque-Bera	114.0268	40.9454	1319.4	968.1900	160.1151
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Note: In parentheses are standard errors of the coefficient estimates
In parentheses of Q, KS and JB-statistics are p-value for testing the null hypothesis

Table 2 shows GJR GARCH parameter estimation. The mean equation is in the simplest form of first autoregressive ($AR(1)$). The Q-statistics confirm that the marginals mostly accept the null hypotheses which suggested that there are no serial correlations and satisfy an i.i.d. assumption for almost all the markets. Then, we transform standardized residuals into the uniform ($U[0,1]$) by using the empirical distribution functions. The Kolmogorov-Smirnov test (KS-test) is used to test the null hypothesis that the transformed data are uniformly distributed, because all data series support the null hypothesis and use this results to carry out the copula procedure. Jarque-Bera statistics suggested that the standardized residuals of are non-normality distribution. These findings from statistical testing confirm that the GJR GARCH model can apply EVT to handle on the standardized residuals.

Table 3 shows parameter estimation of extreme value theory, we use the GPD in our study where ϑ, η are the scale parameter and the shape parameter and we fixed the threshold value u at 10% level of confidence. Figure 1 is a sample of the CDF by using semi-parametric form of Singapore market, in the subprime period, obviously, the valued of upper tail was higher than the after-crisis period.

Table 3: GPD estimation of each markets residuals

Index	TH	SP	MS	ID	PP
			A: crisis period		
u_r	1.1663	1.2225	1.1225	1.1281	1.1864
ϑ_r	0.4929	0.7693	0.5947	0.5580	0.4916
η_r	0.1237	-0.0712	0.1764	0.1827	0.1447
u_l	-1.3113	-1.3290	-1.1794	-1.2537	-1.2979
ϑ_l	0.4835	0.4938	0.3622	0.6470	0.6831
η_l	0.0875	0.0625	0.4554	0.2680	-0.0168
			B: after crisis period		
u_r	1.2222	1.2172	1.0998	1.0367	1.1718
ϑ_r	0.5245	0.4665	0.4004	0.4673	0.4097
η_r	0.0178	-0.0380	0.3282	0.0805	0.1942
u_l	-1.3012	-1.2407	-1.1444	-1.1542	-1.2279
ϑ_l	0.6973	0.7277	0.7322	0.6031	0.5931
η_l	-0.0715	-0.1496	0.0092	0.1908	0.0282

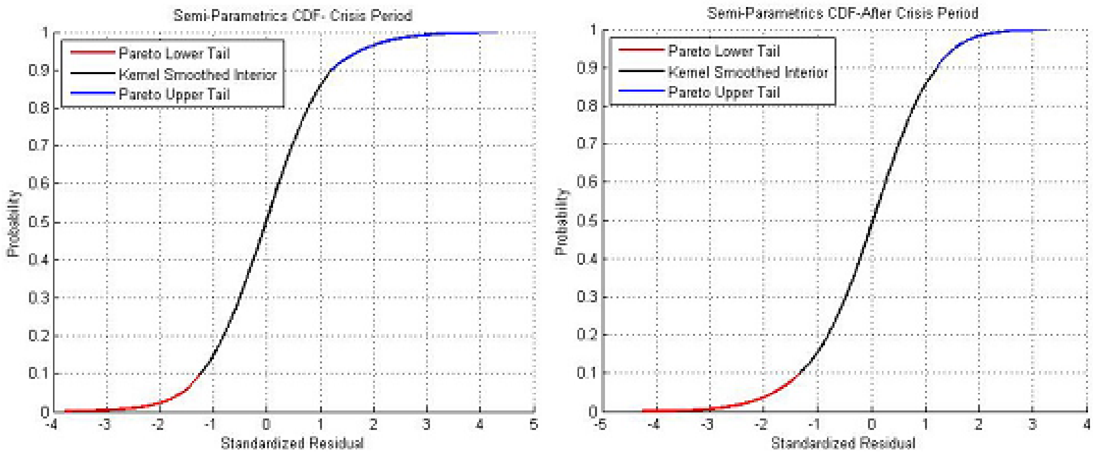

Fig. 1: Semi-parametic CDFs of Singapone residuals

Table 4: The matrixes of the Kendall′s tau from the multivariate t copula

Index	TH	SP	MS	ID	PP
			A: crisis period		
TH	1				
SP	0.4410	1			
MS	0.3524	0.4289	1		
ID	0.3970	0.4866	0.3741	1	
PP	0.2306	0.2808	0.3498	0.2721	1
			B: after crisis period		
TH	1				
SP	0.3435	1			
MS	0.2520	0.3291	1		
ID	0.3206	0.3831	0.3550	1	
PP	0.2274	0.2574	0.2662	0.2890	1

Table 4 shows the values of Kendall′s rank correlation,which were computed by using the parameter of the multivariate t copula function from equation (10). The results show that five markets have a monotonic relationship because of the Kendall′s tau is more than zero. During the crisis, the highest relationship is SP & ID, SP & TH and SP & MS, respectively. While, PP & ID has the weakest relationship. After the crisis, the strongest relationship is still SP & ID, ID & MS and SP & MS, respectively. While, TH & PP has the weakest relationship.

Table 5: The matrixes of the Kendall′ s rank correlation from C and D vine copula

A:crisis period				B:after crisis period			
C-vine copola		D-vine copula		C-vine copula		D-vine copula	
τ_{51}	0.2501	τ_{41}	0.4024	τ_{51}	0.2386	τ_{31}	0.2654
τ_{52}	0.3101	τ_{43}	0.3668	τ_{52}	0.2681	τ_{41}	0.3262
τ_{53}	0.3685	τ_{51}	0.2500	τ_{53}	0.2760	τ_{52}	0.2681
τ_{54}	0.2749	τ_{52}	0.3101	τ_{54}	0.2965	τ_{53}	0.2760
$\tau_{31\|5}$	0.2851	$\tau_{21\|5}$	0.3876	$\tau_{31\|5}$	0.1931	$\tau_{32\|5}$	0.2676
$\tau_{32\|5}$	0.3307	$\tau_{31\|4}$	0.2094	$\tau_{32\|5}$	0.2676	$\tau_{43\|1}$	0.2836
$\tau_{43\|5}$	0.2826	$\tau_{54\|1}$	0.1817	$\tau_{43\|5}$	0.2865	$\tau_{51\|3}$	0.1522
$\tau_{21\|53}$	0.3100	$\tau_{42\|51}$	0.3007	$\tau_{21\|53}$	0.2447	$\tau_{21\|53}$	0.2448
$\tau_{41\|53}$	0.2655	$\tau_{53\|41}$	0.2548	$\tau_{41\|53}$	0.2057	$\tau_{54\|31}$	0.1488
$\tau_{42\|531}$	0.2627	$\tau_{32\|541}$	0.1576	$\tau_{42\|531}$	0.1892	$\tau_{42\|531}$	0.1886

Note: 1 = TH, 2 = SP, 3 = MS, 4 = ID, 5 = PP

Table 5 shows the values of Kendall′s tau that compute by using the parameter of vine copula function follow equation (11) and (12). In table 5, we found that five markets have a positive dependence. The highest relationship is ID & TH by D-vine copula in during and after the crisis period.

Table 6: Portfolio risk of the equally weighted market strategy

	The multivariate t copula GARCH-EVT	C-vine copula GARCH-EVT	D-vine copula GARCH-EVT
	A: crisis period		
$VaR_{0.95}$	0.0148	0.0144	0.0147
$VaR_{0.99}$	0.0246	0.0217	0.0224
$CVaR_{0.95}$	0.0212	0.0215	0.0218
$CVaR_{0.99}$	0.0330	0.0325	0.0328
	B: after crisis period		
$VaR_{0.95}$	0.0137	0.0137	0.0141
$VaR_{0.99}$	0.0207	0.0208	0.0215
$CVaR_{0.95}$	0.0183	0.0203	0.0205
$CVaR_{0.99}$	0.0263	0.0306	0.0307

Table 6 shows the simulation results of one step ahead forecasting in portfolio risk using the multivariate t copula GARCH-EVT, C-vine copula GARCH-EVT and D-vine copula GARCH-EVT under the same strategies for all markets. The simulation results found that VaR and CVaR values during the crisis are higher than after-crisis at 0.95 and 0.99 significant level. In the crisis period, the multivariate t copula GARCH-EVT gives the values of VaR higher than both vine copula GARCH-EVT and D-vine copula GARCH-EVT gives the values of VaR and CVaR higher then C-vine copula GARCH-EVT . Then we also found that the computational of VaR and

CVaR using D-vine copula GARCH-EVT gives higher value than the multivariate t copula GARCH-EVT and the C-vine copula GARCH-EVT after the crisis period.

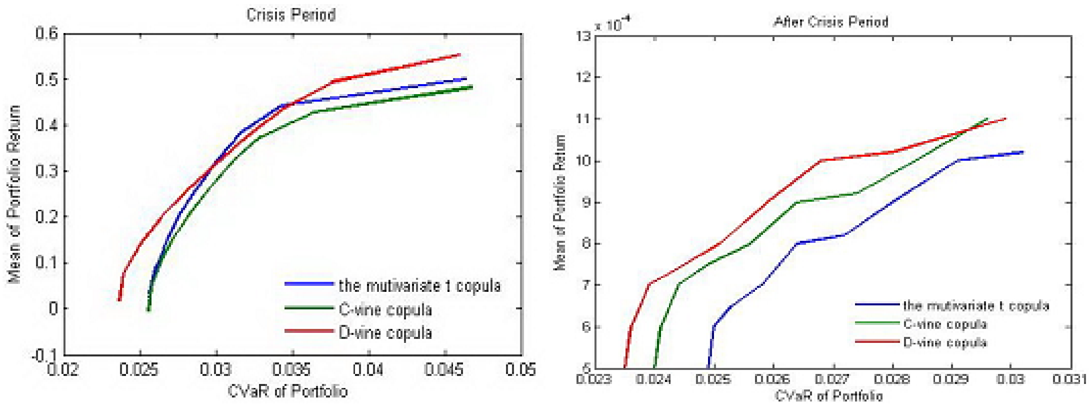

Fig. 2: Efficient frontier from minimizing CVaR at 99% confidence level

Figure 2 shows the efficient frontier of Asean portfolio by minimizing portfolio risk. A subfigures (a) is the efficient frontier during the crisis period and a subfigure (b) is the efficient frontier after the crisis period. From the figure 2, we can conclude that at the same level of CVaR, D-vine copula GARCH-EVT generates portfolio return higher than the multivariate t copula GARCH and C-vine copula GARCH-EVT during the crisis period. After the crisis period, D-vine copula GARCH-EVT gives portfolio return higher than C-vine copula GARCH-EVT and the multivariate t copula GARCH-EVT at the same level of CVaR. Finally, We calculate the optimal weights of the portfolio at the efficient frontier as Table 7. All three approaches generate the return and CVaR of the portfolio in the crisis period higher than the post crisis period.

Table 7: the optimal portfolio weights in efficient frontier for Asean markets

Portfolios	TH	SP	MS	ID	PP	Return	$CVaR_{0.99}$
A:crisis period by the multivariate t copula GARCH-EVT							
1	0.0483	0.0483	0.6923	0.1032	0.1080	0.0343	0.0257
2	0.1129	0.1129	0.4703	0.1306	0.1734	0.1507	0.0268
3	0.1752	0.1752	0.2400	0.1801	0.2295	0.2671	0.0289
4	0.2646	0.2646	0.0000	0.0000	0.4709	0.4417	0.0343
B:crisis period by C-vine copula GARCH-EVT							
1	0.1506	0.0000	0.5981	0.0000	0.2513	0.0506	0.0258
2	0.1982	0.0589	0.3768	0.0000	0.3661	0.1584	0.0273
3	0.2372	0.1306	0.1612	0.0000	0.4711	0.2661	0.0297
4	0.0000	0.5310	0.0000	0.0000	0.4690	0.4277	0.0365
C: crisis period by D-vine copula GARCH-EVT							
1	0.2902	0.1128	0.0000	0.0000	0.5969	0.0781	0.0240
2	0.2700	0.2532	0.0784	0.0000	0.3984	0.1966	0.0264
3	0.2566	0.3569	0.1833	0.0000	0.2032	0.3152	0.0300
4	0.0083	0.5311	0.4606	0.0000	0.0000	0.5524	0.0376
D:after crisis period by the multivariate t copula GARCH-EVT							
1	0.0000	0.0000	0.5158	0.3826	0.1016	0.0006	0.0250
2	0.0000	0.0000	0.3575	0.5084	0.1341	0.0007	0.0258
3	0.0000	0.0000	0.1997	0.6349	0.1654	0.0008	0.0272
4	0.0000	0.0000	0.0000	0.8604	0.1396	0.0010	0.0302
E: after crisis period by C-vine copula GARCH-EVT							
1	0.3534	0.0642	0.5117	0.0707	0.0000	0.0006	0.0241
2	0.4165	0.1553	0.3626	0.0656	0.0000	0.0007	0.0249
3	0.4930	0.2324	0.2099	0.0648	0.0000	0.0009	0.0264
4	0.6017	0.3682	0.0000	0.0301	0.0000	0.0011	0.0296
F:after crisis period by D-vine copula GARCH-EVT							
1	0.4440	0.0423	0.3284	0.0000	0.1853	0.0006	0.0236
2	0.2887	0.1340	0.3893	0.0000	0.1880	0.0007	0.0244
3	0.1309	0.2273	0.4455	0.0000	0.1964	0.0009	0.0259
4	0.0000	0.4152	0.5826	0.0000	0.0022	0.0011	0.0299

4 Conclusions

In this study, we adopt copula based volatility models to measure the dependence between Asean stock markets and then we used a semi-parametric approach from extreme value theory to capture the tail distribution of standardized residuals from the data in the context of the subprime crisis. We examine the portfolio simulation produced by each model and emphasize comparing three models. The models consist of the multivariate t copula GARCH-EVT, C-vine copula GARCH-EVT and D-vine copula GARCH-EVT. Regarding dependence, all copulas provide evidence of positive dependence in every pair. The dependences are mostly strong between Singapore and other markets by the multivariate t copula, which may imply that Singapore market plays an important role in Asean markets. Meanwhile, the risk measure was simulated with equally weighted strategy. This result indicates that D-copula GARCH model-EVT can be estimates VaR and CVaR greater than C-copula GARCH model-EVT and the multivariate t copula GARCH-EVT in the post subprime crisis period. The values of VaR and CVaR during the subprime crisis are higher than those after the subprime crisis. Moreover, the results of the portfolio optimization problem using CVaR objective show that D-vine copula GARCH-EVT is a more efficient tool to simulate the portfolio optimization. Finally, the optimal portfolio weights suggest that the international investors should concentrate on the Malaysian market at the high risk and return, and should invest in Thailand market at the low portfolio risk and return after the subprime crisis.

Acknowledgements The authors are thankful to Dr. Supanika Leurcharusmee and Dr. Kittawit Autchariyapanitkul for reviewing the manuscript. First author was supported from Prince of Songkla University-PhD Scholarship.

References

1. Aas, K.,Czado, C., Frigessi,A., Bakken,H.:Pair-copula constructions of multiple dependence. Insurance: Mathematics and Economics 44 (2), 182-198 (2009)
2. Allen D.E., Singh A.K., Powell R.J.: EVT and tail-risk modelling: Evidence from market indices and volatility series, North American Journal of Economics and Finance 26, 355-369 (2013)
3. Artzner, P., Delbaen, F., Eber, J.M., Heath, D.: Coherent measures of risk. Mathematical Finance 9, 203-228 (1999)
4. Ayusuk, A., Sriboonchitta, S.:Risk Analysis in Asian Emerging Markets using Canonical Vine Copula and Extreme Value Theory. Thai Journal of Mathematics, 59-72 (2014)
5. Bali, T.G.:An extreme value approach to estimating volatility and value at risk. Journal of Business 76, 83-108 (2003)
6. Bollerslev, T.:Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics 31, 307-327 (1986)
7. Breymann, W., Dias, A., Embrechts, P.:Dependence structures for multivariate high-frequency data in finance. Quantitative Finance 3, 1-14.(2003)
8. Embrechts, P., McNeil, A.: Straumann, Correlation and dependence properties in risk management: properties and pitfalls, in M. Dempster, ed., risk management: Value at risk and beyond, Cambridge University Press (2002)
9. Embrechts, P., Resnick, S., Samorodnitsky, G.: Extreme value theory as a risk man-agement tool. North American Actuarial Journal 3, 30-41(1999)
10. Engle, R.F.: Autoregressive Conditional Heteroskedasticty with Estimates of the Variance of U. K. Inflation. Econometrica 50, 9871008 (1982)
11. Furi, D.,Climent, F.J.: Extreme value theory versus traditional GARCH approaches ap-plied to financial data: a comparative evaluation. Quantitative Finance 13, 45-63 (2013)
12. Glosten, L.R., Jagannathan, R., Runkle, D.E.:On the relation between the expected value and the volatility of the nominal excess return on stock. Journal of Finance 48, 1779-1801 (1993)
13. Hemche,O.,Jawadi, F., Maliki, S.B., Cheffou, A.I.:On the study of contagion in the context of the subprime crisis: A dynamic conditional correlationmultivariate GARCH approach. Economic Modelling, (2014) in press
14. Hernandez, J.A.: Are oil and gas stocks from the Australian market riskier than coal and uranium stocks? Dependence risk analysis and portfolio optimization. Energy Economics 45, 528-536 (2014)

15. Horta, P. , Lagoa,S. and Martins,L.:The impact of the 2008 and 2010 financial crises on the Hurst exponents of international stock markets: Implications for efficiency and contagion. International Review of Financial Analysis 35, 140153 (2014)
16. Jess R.,Ortiz, E., Cabello A.: Long run peso/dollar exchange rates and extreme value behavior: Value at Risk modeling. The North American Journal of Economics and Finance 24, 139-152 (2013)
17. Joe, H.: Families of m-variate distributions with given margins and $m(m-1)/2$ bivariate dependence parameters. Distributions with fixed marginals and related topics. Institute of Mathematical Statistics, California (1996)
18. Karmakar, M.:Estimation of tail-related risk measures in the Indian stock market: An Extreme value approach. Review of Financial Economics 22, 7985 (2013)
19. Kole, E., Koedijk, E.,Verbeek, M.: Selecting copulas for risk management. Journal of Banking & Finance 31(8), 2405-2423 (2007)
20. Lee, Y.H., Tucker ,A.L., Wang, D.K., Pao, H.T.: Global contagion of market sentiment during the US subprime crisis. Global Finance Journal 25, 1726(2014)
21. Low, R.K.Y., Alcock, J., Faff, R., Brailsford, T.: Canonical vine copulas in the context of modern portfolio management :Are they worth it ?. Journal of Banking & Finance 37, 3085-3099 (2013)
22. Marimoutou, V., Raggad, B., Trabesi, A.:Extreme value theory and value at risk: Application to oil market.Energy Economics 31(4), 519-530 (2009)
23. Mashal, R., Zeevi, A.:Beyond correlation: extreme co-movements between financial assets. Unpublished, Columbia University. (2002)
24. McNeil, A., Frey, R. Estimation of tail-related risk measures for heteroscedastic financial time series: An extreme value approach. Journal of empirical Finance 7(3-4), 271-300 (2000)
25. Mensi, W., Hammoudeh, S., Reboredo, J.C., Nguyen, D.K.: AreSharia Stocks, Gold and U.S. Treasuries Hedges and Safe Havens for the Oil-Based GCC Markets?.Emerging Markets Review (2015) in press
26. Moralesa,L., Andreosso-OCallaghan, B.: The current global financial crisis: Do Asian stock markets show contagion or interdependence effects?. Journal of Asian Economics 23,616-626 (2012)
27. Naoui, K., Liouane, N., Brahim, S.:A dynamic conditional correlation analysis of financial contagion: the case of the subprime credit crisis. International Journal of Economics and Finance 2 (3), 85-96 (2010)
28. Patton, A.J.:Copula-based models for Financial time series, in T.G. Andersen, R.A. Davis, J.-P. Kreiss and T. Mikosch (eds.) Handbook of Financial Time Series, Springer Verlag (2009)
29. Ren, F., Giles, D.E.:Extreme value analysis of daily Canadian crude oil prices. Applied Financial Economics 20(12), 941-954 (2010)
30. Rockafellar, R.T., Uryasev, S.: Optimization of conditional value-at-risk. Journal of Risk 2, 21-41 (2000)
31. Rockafellar, R.T., Uryasev, S.: Conditional value-at-risk for general loss distributions. Journal of Banking & Finance 26, 1443-1471(2002)
32. Sklar, A.: Fonctions de rpartition n dimensions et leurs marges. Publ. Inst. Statist. Univ. Paris 8,229-231 (1959)
33. Wang, L.:Who moves East Asian stock markets? The role of the 2007-2009 global fi-nancial crisis. Journal of International Financial Markets, Institution and Money 28, 182-203 (2014)
34. Wang Z., Wu W., Chen C., Zhou,Y., The exchange rate risk of Chinese yuan: Using VaR and ES based on extreme value theory, Journal of Applied Statistics 37, 265-282 (2010)
35. Wang, Z.R., Chen, Z.R. , Jin, Z.R., Zhou, Z.R.: Estimating risk of foreign exchange portfolio: using VaR and CVaR based on GARCHEVT-Copula model. Physica A 389,4918-4928 (2010)

CURRICULUM VITAE

Author's Name	Mr. Apiwat Ayusuk
Date of Birth	20th Febuary 1983
Education	2007 Master of Economics (Development Economics) National Institute of Development Administration, Bangkok, Thailand
	2005 Bachelor of Arts (Economics) 2nd Class Honors The University of The Thai Chamber of Commerce, Bangkok, Thailand
Experience	Lecturer, Department of Business Economics, Faculty of Liberal Arts and Management Sciences, Prince of Songkla University, Surathani, Thailand
	Part Time Lecturer, Faculty of Management Sciences Chiang Mai Rajabhat University, Chiang Mai, Thailand
	Part Time Lecturer, Faculty of Management Sciences Surathani Rajabhat University, Surathani, Thailand
	Relationship Officer (ROC), Bangkok Bank, Bangkok, Thailand
Scholarships	Funded by Princeof Songkla University-PhD Scholarship