CHAPTER 3

Risk, Return and International Portfolio Analysis: Entropy and Linear Belief Functions

This chapter is developed from the original article namely, “Risk, return and international portfolio analysis: entropy and linear belief functions”. The contents are extracted from the original article, which was published in “Econometrics of Risk”, Studies in Computational Intelligence Volume 583, pp 319 – 328 and this article can be found in the appendix A. The methodology used in this study was explained in Chapter 2.

3.1 Introduction

Risk and return are important factors when investing in the capital market. According to the risk and return trade-off, the capital invested in the market cannot make higher returns without the possibility of investment loss. In classical work, Markowitz (1952) is a well-known for the foundation of modern portfolio theory; which is mean and variance based method to find the optimal portfolio weights. Several researches were extensively studied in both theoretical and empirical works. (See, Tobin (1958), Markowitz (1959), Hakansson (1971), Zenios and Kang (1993), Konno and Kobayashi (1997), etc.)

The modern world of economic globalization has a quick changing impact on the capital markets that contributed to an increase in international capital flow across countries. Many researches on topics related to international diversification, including Cavaglia et al. (2002), Li (2003), Fletcher and Marshall (2005), Chiou (2009) and Herrero and Vzquez (2013) recommend that international diversification improves portfolio performance. During the last decade, The Chinese economy has been rapidly developed and played an important role in Asia and the world. Jayasuriya (2011)

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found evidence that the stock market behavior of China had an impact on the stock market behavior of the East Asia and Pacific. Zhou et al. (2012) found that the impact of the Chinese stock market on Asian markets had become increasingly powerful after 2005. Glick and Hutchison (2013) also found that the strength of the correlation of stock markets between China and other Asia countries has increased markedly during 2008–2010 and has remained high in 2010-2012. In 2015, the ASEAN Economic Community (AEC) will induce regional economic integration, which provides a competitive advantage and economic benefits. Hence, to take advantage of investment diversification an international level, this study focuses on the stock markets in ASEAN, China and The U.S., which is the world major stock market.

In our review of the literature, we found two specific research questions. First, what is the most efficient tool for portfolio allocation under international risk and return strategy? Second, which portfolio should we invest in? Therefore, the primary objective of this research is to suggest new portfolio selection methods under risk and return using an information theory to select the optimal portfolio and the linear belief function to combine evidence. The secondary objective is to evaluate international portfolio performance.

The remainder of this chapter is organized as follows. We give more details about the portfolio optimization methods and portfolio analysis in the system of the linear belief function in the methodology section. We examine the data selection, descriptive statistics and the results of portfolio analysis in data section. Finally provides a brief conclusion.

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3.2 Methodology

3.2.1 Portfolio Selection Methods

In this section we present four different methods to determine the optimal portfolio based on risk-return framework. The basic notations are defined by: r_{it} is the return of market i at time t, μ_i is the expected return of market i, σ_{ij} is the covariance between the market of i and j, p_i and p_j are the weights assigned to markets i and j, μ_p is the expected return of portfolio, σ_p^2 is the portfolio risk. While m denote number of markets in portfolio, then expected return and variance of return of portfolio can be described by $\mu_p = \sum_i^m p_i\mu_i$ and $\sigma_p^2 = \sum_{i=1}^m \sum_{j=1}^m p_i p_j \sigma_{ij}$ respectively

Mean-Variance Markowitz method: The conventional work of MV method is well known for the portfolio optimization approach. The goal of an investor is to find the optimal weight determinations in a portfolio by minimizing risk subjecting to the expected return of the portfolio being greater than or equal to risk free rate. The problem can be stated as:

$$\begin{cases} Min & \sigma_p^2 \\ s.t. & \sum_{i=1}^m p_i\mu_i \geq \mu_0 \quad , \quad \sum_{i=1}^m p_i = 1 \end{cases} \tag{3.1}$$

Mean Entropy method: Entropy is a one of the methods to measure uncertainty in random variables. This study uses the Shannon entropy, $S(p) = -\sum_i^m p_i \ln(p_i)$ under the principle of maximum entropy introduced by Jaynes (1963). The optimization problem is to choose the probability (or weight) in a portfolio by maximizing entropy function subject to the expected return (mean) of the portfolio being greater than or equal to risk free rate.

$$\begin{cases} Max & -\sum_{i=1}^m p_i \ln(p_i) \\ s.t. & \sum_{i=1}^m p_i\mu_i \geq \mu_0 \quad , \quad \sum_{i=1}^m p_i = 1 \end{cases} \tag{3.2}$$

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Mean-Variance Entropy method: As the constraint of the principle of maximum entropy can be flexible, then we can provide more information. This optimization problem becomes maximizing the Shannon entropy subject to the expected return condition and the risk limitation strategy.

$$\begin{cases} Max \quad -\sum_{i=1}^{m} p_i \ln(p_i) \\ s.t. \quad \sum_{i=1}^{m} p_i \mu_i \geq \mu_0 \,, \quad \sum_{i=1}^{m}\sum_{j=1}^{m} p_i p_j \sigma_{ij} \leq \sigma_0^2 \,, \quad \sum_{i=1}^{m} p_i = 1 \end{cases} \tag{3.3}$$

Sharpe Ratio Entropy method: The Sharpe ratio is introduced by Sharpe (1966) to measure the portfolio performance that is described in unit of return per unit of risk. Therefore, we propose this methodology based on the Sharpe ratio into the principle of maximum entropy. The optimization problem is maximizing the Shannon entropy with an additional criterion of excess return per unit of risk.

$$\begin{cases} Max \quad -\sum_{i=1}^{m} p_i \ln(p_i) \\ s.t. \quad \frac{\mu_p}{\sigma_p} \geq \frac{\mu_0}{\sigma_0} \,, \quad \sum_{i=1}^{m} p_i = 1 \end{cases} \tag{3.4}$$

After we complete the solutions from each method, we use the Sharpe ratio to compare the performance of the portfolio selection methods.

3.2.2 Linear Belief Function

According to Dempster (1990) and Liu (1996), Linear belief functions is a special type of belief functions in expert system such as linear equations, linear regressions and Kalman filters, and also including Gaussian distributions that explain probabilistic knowledge on a set of variables in the continuous case. Liu et al. (2006) used matrix sweepings to combine information from the linear

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belief function. In this study, we extended by using the linear time series belief function. Consequently, this study considers the linear time series to model portfolio investment by using the reduced form vector autoregressive (VAR) model with market returns as follows:

$$r_{it} = \Phi_0 + \Phi_1 r_{it-1} + e_t \tag{3.5}$$

where $r_{it} = [r_{1t}, r_{2t}, r_{3t}]'$ is a 3×1 market return vector at time t, Φ_0 is a 3×1 vector of intercepts, $\Phi_1$is a time-invariant 3×3 matrix , e_t is a 3×1 vector of error terms and assume Gaussian distribution $e_t \sim N(0, \Sigma)$ with satisfying $E(e_t) = 0, E(e_t, e'_{t-1}) = 0$ is no serial correlation in error term and $E(e_t, e'_t) = \Sigma_{ij}$ is the variance-covariance matrix of error term that allowing non-zero correlation between error terms. The parameters of the VAR model can be estimated consistently by the OLS method when sample size is large. We construct a graphical structure for international portfolio analysis as follows:

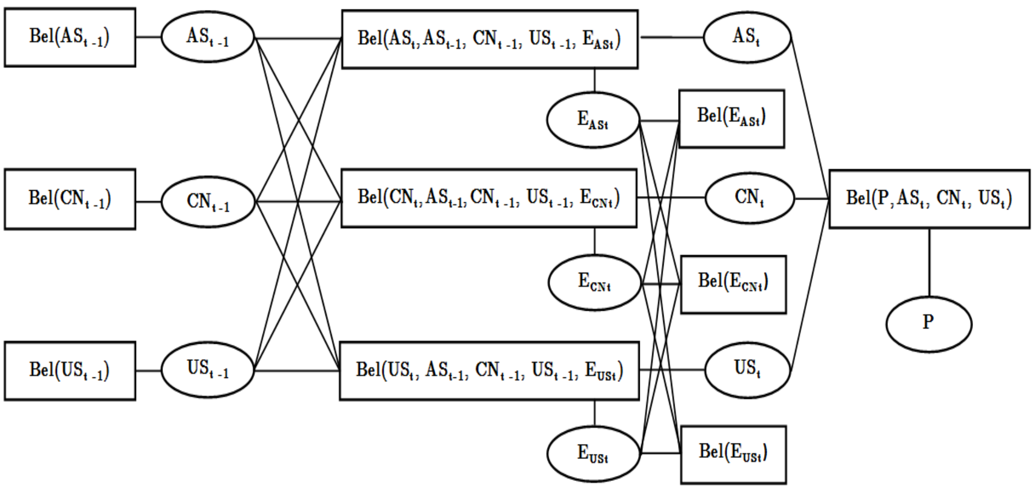

Figure 3.1 A graphical structure of international portfolio

In Figure3.1 we used the linear relationship from a VAR model combining with optimal portfolio weight to construct a graphical structure of international portfolio. There are ten variable nodes $AS_t, CN_t, US_t, AS_{t-1}, CN_{t-1}, US_{t-1}, E_{AS_t}, E_{CN_t}, E_{US_t}, P$ and ten belief function nodes. Four linear belief functions represent the relationship between the variables, eg. $Bel(AS_t, AS_{t-1}, CN_{t-1}, US_{t-1}, E_{AS_t})$ is a linear belief function of the ASEAN return that depended on the first lag of past returns of itself, China, and the U.S.and a residual variable. $Bel(P, AS_t, CN_t, US_t)$ is a linear belief function of portfolio return that is integrated with three market returns and optimal portfolio weights from best methods. And six linear belief functions represent an individual variable, e.g. $Bel(E_{AS_t})$ is the true value of a residual variable from first function. Thus, we can analyze the linear belief function into the moment matrix approach.

The concept of Dempster's rule used to combine multiple focal elements that are independent evidence from several sources. Liu (1996) proved the combination rule in Gaussian linear belief function of variable space was equivalent to that of Dempster (1990)'s for continuous case. Liu (1999) also proved that combination and marginalization of Gaussian linear belief function satisfies the axioms of Shenoy and Shafer (1990) and showed Dempster's rule for the combination could be interpreted by matrix sweepings.

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According to the matrix sweeping technique for Gaussian linear belief function is a matrix operation or a matrix transformation that including forward sweep and reverse sweep to consider. We can sweep a matrix from variance and covariance matrix move to conditional representation. Let r_i be a random variable representing the market returns that are assuming Gaussian distribution with expected mean: $E(r_i) = \mu_i$, variance: $Var(r_i) = \Sigma_{ii}$ and covariance $Cov(r_i, r_j) = \Sigma_{ij}$ $, i, j = 1,2, \ldots, n$, then we can write the moment matrix as $m = \begin{pmatrix} \mu_j \\ \Sigma_{ij} \end{pmatrix}$ This matrix represents the distribution of the random variables. We can define the operation on moment matrices by definitions below.

Definition 3.1: (Marginalization) Liu (1999), the marginalization of a linear belief function is simply a projection in variable space. Let r_1 and r_2 are two random variables in the moment matrix: $m(r_1, r_2)$, its marginal to r_1 as

$$m^{\downarrow r_1}(r_1, r_2) = \begin{pmatrix} \mu_j \\ \Sigma_{ij} \end{pmatrix} \tag{3.6}$$

where $m^{\downarrow r_1}$ is the marginalization of the moment matrix that represent to the conditional moment matrix of linear regression coefficient.

Definition 3.2: (Forward sweep) Liu (1999), Forward sweeping is the transformation of the moment matrix to be the conditional moment matrix. Let n market returns in portfolio and then we can operate a forward sweep of $m(r_1, \ldots, r_n)$ from r_s as follows:

$$m(r_1, \ldots, r_{s-1}, \vec{r_s}, r_{s+1}, \ldots, r_n) = \begin{pmatrix} \mu_{j,s} \\ \Sigma_{ij,s} \end{pmatrix}$$

where

$$\mu_{j,s} = \begin{cases} \mu_j - \mu_s \Sigma_{ss}^{-1} \Sigma_{sj} & \text{, for } j \neq s \\ \mu_s \Sigma_{ss}^{-1} & \text{, for } j = s \end{cases} \tag{3.7}$$

$$\Sigma_{ij,s} = \begin{cases} -\Sigma_{ss}^{-1} & \text{, for } i = s = j \\ \Sigma_{is}\Sigma_{ss}^{-1} & \text{, for } j = s \neq i \\ \Sigma_{ss}^{-1}\Sigma_{sj} & \text{, for } i = s \neq j \\ \Sigma_{ij} - \Sigma_{is}\Sigma_{ss}^{-1}\Sigma_{sj} & \text{, for otherwise} \end{cases}$$

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Definition 3.3 (Reverse sweep) Liu (1999), Let n market returns in portfolio and then we can operate a reverse sweep of $m(\vec{r_1}, \ldots, \vec{r_n})$ from r_s as follows:

$$m(\overrightarrow{r_1}, \dots, \vec{r}_{s-1}, r_s, \vec{r}_{s+1}, \dots, \overrightarrow{r_n}) = \begin{pmatrix} \mu_{j,s} \\ \Sigma_{ij,s} \end{pmatrix}$$

where

$$\mu_{j,s} = \begin{cases} \mu_j - \mu_s \Sigma_{ss}^{-1} \Sigma_{sj} & \text{,for j} \neq \text{s} \\ -\mu_s \Sigma_{ss}^{-1} & \text{,for j} = \text{s} \end{cases} \quad (3.8)$$

$$\Sigma_{ij,s} = \begin{cases} -\Sigma_{ss}^{-1} & \text{,for i} = \text{s} = \text{j} \\ -\Sigma_{is}\Sigma_{ss}^{-1} & \text{,for j} = \text{s} \neq \text{i} \\ -\Sigma_{ss}^{-1}\Sigma_{sj} & \text{,for i} = \text{s} \neq \text{j} \\ \Sigma_{ij} - \Sigma_{is}\Sigma_{ss}^{-1}\Sigma_{sj} & \text{,for otherwise} \end{cases}$$

Definition 3.4 (The combined linear belief function) Liu (1999) Let $\overrightarrow{m}_1 = m_1(\overrightarrow{r_1}, \overrightarrow{r_2})$ and $\overrightarrow{m}_2 = m_2(\overrightarrow{r_1}, \overrightarrow{r_2})$ are fully swept matrices. The combination of two linear belief functions is the sum of fully swept matrices: $\overrightarrow{m} = \overrightarrow{m}_1 \oplus \overrightarrow{m}_2$ then we can write this combination as follows:

$$\overrightarrow{m} = \overrightarrow{m}_1 \oplus \overrightarrow{m}_2 = \begin{pmatrix} \overrightarrow{\mu_1} + \overrightarrow{\mu_2} \\ \overrightarrow{\Sigma_1} + \overrightarrow{\Sigma_2} \end{pmatrix} \quad (3.9)$$

3.3 Data

We collected daily data from January of 2009 to December of 2013 from DataStream. As mentioned before, we considered a portfolio selection problem from three attractive markets, which are ASEAN (FTSE/ASEAN index), China (The Shanghai Composite index) and the U.S. (The S&P 500 index) markets. For market returns are calculated as $r_t = \ln(P_t) - \log(P_{t-1})$ where P_t is the closing price at time t. The data source is from DataStream.

Table 3.1 Summary statistics of international portfolio analysis

Classes	ASEAN	China	U.S.
Market indexes	The FTSE/ASEAN	The Shanghai Composite index	S&P 500 index
Max	0.048444	0.059344	0.068366
Min	-0.042920	-0.069856	-0.068958
Mean	0.000600	0.000113	0.000549
S.D.	0.008741	0.013373	0.012054
ADF-statistics	-32.93067	-35.79939	-39.47482
Jarque-Bera	609.7368	393.5131	1050.559

Note: ADF - Augmented Dickey and Fuller test, the 1% critical value is -3.4352

Table 3.1 shows summary statistics of the three market returns. The mean returns are the highest in the U.S. and China has the highest deviation of market returns. The results from using the ADF test indicate that we cannot reject the null hypothesis of unit roots for all market returns; it means the market returns are stationary.

3.4 Empirical Results

The research is performed as follows: Firstly, we calculate the optimal weights of international portfolio. There are different methods to optimize the portfolio selection problem; Mean-Variance Markowitz (MV) method, Mean Entropy (ME) method, Mean-Variance Entropy (MVE) method. Secondly, we use the Sharpe ratio to measure the portfolio performance and select the best performance method. Thirdly, we construct the network of portfolio structure by using linear time series belief function. Finally, applying matrix sweepings to integrate the knowledge and information from second and third to evaluate the portfolio performance

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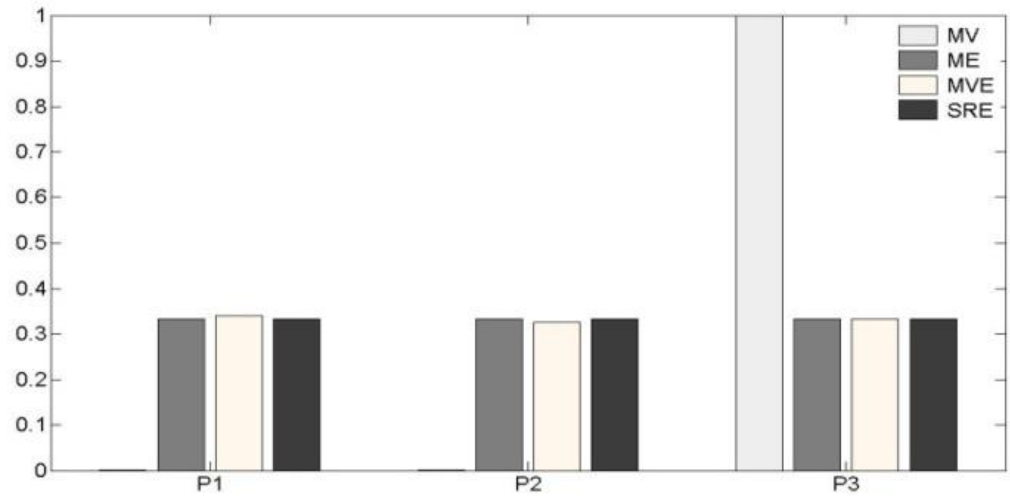

Figure 3.2 The optimal portfolio weight for four methods

Figure.3.2 presents the optimal weights of portfolios that are computed from four different methods. Table 3.2 presents the performances of the portfolio selection methods. From the results of Sharpe ratios, ME, MVE and SRE perform better than MV. MVE is better than other considered methods. Its optimal weights are 34.0%, 32.6% and 33.4% in ASEAN, China and The U.S. markets respectively.

Table 3.2 Comparison results for the portfolio selection methods

Methods	Portfolio returns	Portfolio variance	Sharpe ratios
MV	0.000549	0.000145	0.0373
ME	0.000421	0.000066	0.0396
MVE	0.000424	0.000065	0.0402
SRE	0.000421	0.000066	0.0396

Note: MVMean-Variance Markowitz method,MEMean Entropy method, MVEMean-Variance Entropy method,SRESharpe Ratio Entropy method

Table 3.3 shows the results of the parameter estimates in a VAR model. It represents the relationship between international markets, which should have an influence on each other. The results show that the first lag of the U.S. has high influence in ASEAN and China.

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Table 3.3 Estimates of a VAR model

Methods	ASEAN	China	U.S.
ASEAN(-1)	-0.010810	-0.067352	0.004051
	[0.02801]	[0.04710]	[0.04308]
China(-1)	-0.040520	0.002412	0.015236
	[0.01770]	[0.02977]	[0.027223]
U.S.(-1)	0.330312	0.221640	-0.092744
	[0.01870]	[0.03145]	[0.02876]
Constant	0.000431	0.000031	0.000596
	[0.00022]	[0.00037]	[0.00033]
R- squared	0.202821	0.036987	0.008427
Schwarz SC	-6.846264	-5.806809	-5.985372

Note: In parentheses are standard errors of the coefficient estimates

According to the financial information and market knowledge from the above results, we use MVE to optimize the portfolio weights because this method performs better than others. The optimal weight can represent by using the partially swept matrix as:

$$m\left(P, \overrightarrow{AS_t}, \overrightarrow{CN_t}, \overrightarrow{US_t}\right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.3403 & 0.3255 & 0.3255 \\ 0.3403 & 0 & 0 & 0 \\ 0.3255 & 0 & 0 & 0 \\ 0.3255 & 0 & 0 & 0 \end{pmatrix} \quad (3.10)$$

Figure3.1 $Bel\left(AS_t, AS_{t-1}, CN_{t-1}, US_{t-1}, E_{AS_t}\right)$, we can define by the partially swept matrix from the first equation in a VAR model as $m\left(AS_t, \overrightarrow{AS}_{t-1}, \overrightarrow{CN}_{t-1}, \overrightarrow{US}_{t-1}, \vec{E}_{AS_t}\right)$ with $Var\left(e_{AS_t}\right) = 0.00006$ is variance of residual and $Cov\left(e_{AS_t}, e_{CN_t}\right) = 0.00004$, $Cov\left(e_{AS_t}, e_{US_t}\right) = 0.00004$ are covariance of residual as follows:

$$m\left(AS_t, \overrightarrow{AS}_{t-1}, \overrightarrow{CN}_{t-1}, \overrightarrow{US}_{t-1}, \vec{E}_{AS_t}\right)$$

$$= \begin{pmatrix} 0.000431 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.01081 & -0.04052 & 0.33031 & 1 & 1 & 1 \\ -0.01081 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.04052 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.33031 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0.00006 & 0.00004 & 0.00004 \\ 1 & 0 & 0 & 0 & 0.00004 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0.00004 & 0 & 0 \end{pmatrix} \quad (3.11)$$

Therefore, to analyze the portfolio performance when the markets are related by using the linear belief function. We use six step method of Liu et al. (2006) to integrate knowledge using the combination of matrix sweeping.

Table 3.4 Computational results for moment matrix in portfolio

	Portfolio	ASEAN	China	U.S.
Returns	0.000584	0.000594	0.000110	0.000551
Var-Cov	0.000051			
	0.000058	0.000077		
	0.000007	0.000011	0.000180	
	0.000031	-0.000005	-0.000003	0.000146
Sharpe ratio	0.067433			

Table3.4 presents portfolio performance using the linear belief function. The result shows risks and returns in a portfolio: the ASEAN return 0.0594 % is highest, the standard deviation of ASEAN 0.8775 % is smallest, and the portfolio return is 0.0584 % with the standard deviation 0.7141 %.

3.5 Conclusions

This study provided empirical example for ASEAN, China and the U.S. markets between January 2009 and December 2013. We use three entropy methods base on Sharnon measure, which are ME, MVE and SRE, to select the optimal weights and

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compare its performances with conventional method, which is MV. Moreover, we use the linear belief function to extend portfolio evaluation because the belief function method can allow us to add information on the conditions of the relationship between international markets. There are two main findings from this study. First, all entropy methods perform better than MV because the entropy method well handles uncertainty information from result of Sharpe ratio. Moreover, we found that MVE has higher performed than ME and SRE since MVE has added more information in constrains. Second, after integrating the information between the optimal portfolio strategy and international market relationship using linear belief function, we found that the portfolio risk is decreased and the portfolio return is increased. This implies that the relationship of international markets affects portfolio performance and this finding suggests that an investor should increase the investment proportion in the ASEAN market.

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