CHAPTER 4

Risk Analysis in Asian Emerging Markets using Canonical Vine Copula and Extreme Value Theory

This chapter is developed from the original article "Risk analysis in Asian emerging markets using canonical vine copula and extreme value theory". The contents are extracted from the original article, which was published in "Special Issue (2014) on Copula Mathematics and Econometrics" of the Thai Journal of Mathematics, pp. 59-72 and this article can be found in the appendix B. The methodology used in this study was explained in Chapter 2.

4.1 Introduction

Volatility implies uncertainty that has implications for investment decisions. Hence, investors can find opportunities in gaining benefit with the situation when they implement efficient information and tools. In the context of risk modeling, Engle (1982) and Bollerslev (1986) proposed the econometric modeling of volatility that assumes the conditional on variance, namely, GARCH, which is taking into account the conditional heteroskedasticity inherent in time series. The GARCH models are able to yield VaR and $CVaR$ estimates. The recent financial situation has experienced extreme risk or crises in the last two decades, such as the Asian financial crisis in 1997, the U.S. Subprime crisis in 2007, and the EU debt crisis in 2009. Such studies conducted by McNeil and Frey (2000), Bali (2003) and Marimoutou et al. (2009) have applied EVT for an alternative effective framework to estimate the tail of a distribution. In the EVT based method, the GARCH models can estimate the volatility of the return series. EVT is used to capture the tail of the standardized residual distribution of the GARCH models before estimating VaR. Bali and Neftci (2003), Bystrom (2005), Fernandez (2005) and Chan and Gray (2006) also found that the

GARCH-EVT model had a more accurate estimation of VaR than that obtained from the parametric families.

Bollerslev (1990) and Engle (2002) improved the GARCH models to estimate the conditional linear dependence of volatility in pattern of multivariate random series and assumed multivariate normality. Subsequently, Lee et al. (2006), Chiang et al. (2007), Syllignakis et al. (2011), Ayusuk (2012) and Hwang et al. (2013) applied this method in topics that were related to the international diversification from the perspective of market dependence. Gupta and Guidi (2012) suggested that the conditional correlations between India and Asian markets have increased especially during the periods of international crisis. If dependence is not limited to linear correlation, then the usual correlation of returns may not provide sufficient information. Sklar (1959) proposed the copulas function to describe the joint dependence of random uniform marginal distribution. Copulas are flexible to analyze the dependence structure more than Gaussian or t distribution. According to the studies of copula in financial, Embrechts et al. (2002) introduced copula in finance to relax the assumption of dependence structures between random returns. Patton (2009) explained an overview of copula based models in financial applications. Bedford and Cook (2001,2002) and Kurowicka and Cooke (2006) developed graphical model to determine the copula networks which can be called pair copulas, then Aas et al.(2009) induced D and C-vine copula for inferential statistics. Subsequently, Nikoloulopoulos et al. (2012), Zhang (2014) and Sriboonchitta et al. (2014a) applied vine copula in the empirical studies for the international diversification of stock markets.

According to Asian markets, Wang (2014) suggested that East Asian markets are less responsive to the shocks in the USA after the global financial crisis. The Chinese economy has been rapidly becoming one of the important role in the Asian market. Jayasuriya (2011), Zhou et al. (2012) and Glick and Hutchison (2013) found evidence that the Chinese market had an impact on the Asian market. To take advantage of the portfolio allocation for international diversification and making it an alternative choice for investment, we focused on portfolio diversification based on risk analysis in the application of the Asian emerging markets.

In this chapter, we focus on two aims. For the primary aim, we used conditional EVT or GARCH-EVT with canonical vine copula to study the dependence across Asian emerging markets. As for the secondary aim, we will compute the market risk and the international portfolio performance using VaR and $CVaR$ technique. The remainder of this chapter is organized as follows. We give more details about copula, EVT and portfolio optimization technique in methodology section. We exhibit the data selection, descriptive statistics and the results. In the last section a conclusion has been provided.

4.2 Methodology

Using a three stage approaches, we estimated AR-GARCH model for the conditional volatility in stage one. To create the GARCH residuals, this study used generalized Pareto distribution (GPD) to capture the standardized residuals in the extreme tails and Gaussian distribution in the interior. The vine copula is used for the analysis of the dependence structures between markets in stage two by using . Finally, We used simulation procedure to generate the dependent return series for calculating VaR and $CVaR$.

4.2.1 The GARCH-EVT model

We use the simplified AR-GARCH model with mean equation as a first autoregressive process and the conditional variance equation as a GJR-GARCH (1,1) for modeling asymmetric volatility collecting.

$$r_{it} = \beta_{io} + \beta_{i1} r_{it-1} + \varepsilon_{it} = \beta_{i0} + \beta_{i1} r_{it-1} + \sigma_{it} z_{it} \tag{4.1}$$

$$\sigma_{it}^2 = \mu_i + \alpha_i \varepsilon_{it-1}^2 + \theta_i \sigma_{it-1}^2 + \gamma_i \varepsilon_{it-1}^2 I_{it-1} \text{ and } z_{it} \sim iid \tag{4.2}$$

where $I_{it-1} = 0$ if $\varepsilon_{it-1} \geq 0$, $I_{it-1} = 1$ if $\varepsilon_{it-1} < 0$, $r_{it} = [r_{1t}, r_{2t}, r_{3t}, r_{4t}, r_{5t}]'$ is a 5×1 market return vector at time t, β_{io}, β_{i1}, μ_i, α_i, θ_i, γ_i are parameters, $\varepsilon_{it} = \sigma_{it} z_{it}$ is return residuals and z_{it} is standardized residuals and it must satisfies independently and identically distributed, then the marginal distribution of standardized residuals can define as Gaussian distribution is $g(z) = \varphi(z)$ for general situations and GPD to select the extreme situations that are peaks over threshold (POT).

$$g(z) = 1 - \frac{k}{n}\left(1 + \eta\left(\frac{z-u}{\vartheta}\right)\right)^{-\eta^{-1}}, \tag{4.3}$$

for $z > u$ that given a threshold u, where n is the number of observation, and k is the number of observations that excess over the threshold u, ϑ is the scale parameter , η is the shape parameter that can estimated by maximum likelihood. For $\eta = 0$, this distribution close to the Gumbel distribution, for $\eta < 0$, the distribution close to the Weibull distribution, for $\eta > 0$, the distribution belongs to the heavy-tailed distribution.

4.2.2 The Vine Copula

Sklar (1959) proposed the copula theory, which is a function that links univariate marginal to their multivariate distribution and it can also be the models for the dependence between random variables by copulas. Let $\boldsymbol{x} = [x_1, \ldots, x_n]'$ be random variables for $i = 1, \ldots, n$, the continuous marginal distributions are $F_1(x_1), \ldots, F_n(x_n)$ and $F(x_1, \ldots, x_n)$ be a multivariate distribution, then a n-dimensional copula $C(\cdot): U[0,1]^n \longrightarrow [0,1]$ can defined by

$$C(u_1, \ldots, u_n) = F(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)), \tag{4.4}$$

then we can write

$$F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)). \tag{4.5}$$

Let F is continuous and strictly increasing. The probability density function of $\boldsymbol{x}$ can be defined as

$$f_{1\cdots n}(\boldsymbol{x}) = \prod_{i=1}^{n} f_i(x_i) \cdot c_{1\cdots n}(F_i(x_1), \ldots, F_n(x_n)). \tag{4.6}$$

We can write left hand side (4.6) as

$$f_{1\cdots n}(\boldsymbol{x}) = f_1(x_1) f_{2|1}(x_2|x_1) \cdots f_{i|1\cdots n-1}(x_n|x_1, \ldots, x_{n-1}), \tag{4.7}$$

where $c_{1,\ldots,n}$ is the copula density and f_i , $i = 1, \ldots, n$ are the corresponding marginal pdf. Bedford and Cooke (2001, 2002), who introduced canonical (C) and drawable (D) vines. Chollete et al. (2009), Sriboonchitta et al. (2014b) suggested that C-vine copula dominate alternative dependence structures.

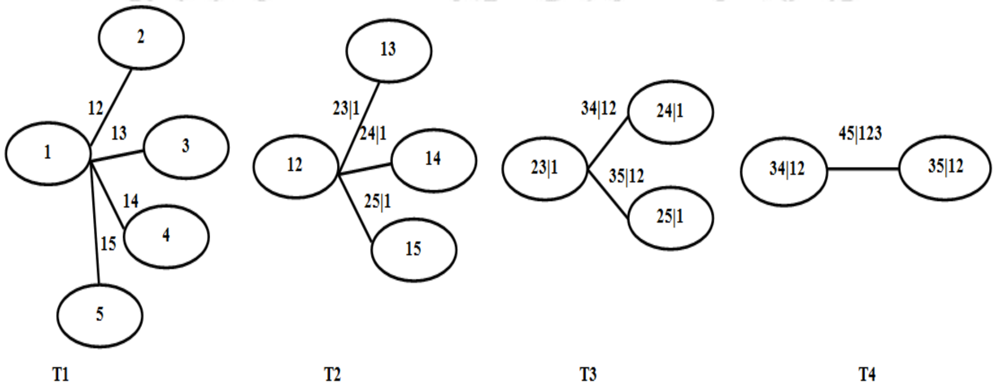

Figure 4.1 Five dimensional canonical vine construction

For this study we used a five-dimensional variable which has 240 options to design the possible pair copula constructions. Let us consider the five dimensional using C-vines, the density function which can be expressed as the following:

$$
\begin{aligned}
f(x_1, x_2, x_3, x_4, x_5) = & \prod_{i=1}^{5} f_i(x_i) \cdot c_{12}(F_1, F_2) \cdot c_{13}(F_1, F_3) \cdot c_{14}(F_1, F_4) \\
& \cdot c_{15}(F_1, F_5) \cdot c_{23|1}\left(F_{2|1}, F_{3|1}\right) \cdot c_{24|1}\left(F_{2|1}, F_{4|1}\right) \\
& \cdot c_{25|1}\left(F_{2|1}, F_{5|1}\right) \cdot c_{34|12}\left(F_{3|12}, F_{4|12}\right) \\
& \cdot c_{35|12}\left(F_{3|12}, F_{5|12}\right) \cdot c_{45|123}\left(F_{4|123}, F_{5|123}\right)
\end{aligned} \tag{4.8}
$$

Aas and Berg (2011) showed that the conditional distribution functions are computed by using partial derivatives of the bivariate copulas at the previous level as the following:

$$
\begin{aligned}
F(x_2|x_1) &= \partial C_{12}(F_1, F_2)/\partial F_1 \\
F(x_3|x_1) &= \partial C_{13}(F_1, F_3)/\partial F_1 \\
F(x_4|x_1) &= \partial C_{14}(F_1, F_4)/\partial F_1 \\
F(x_5|x_1) &= \partial C_{15}(F_1, F_5)/\partial F_1 \\
F(x_3|x_1, x_2) &= \partial C_{23|1}\left(F_{2|1}, F_{3|1}\right)/\partial F_{1|2} \\
F(x_4|x_1, x_2) &= \partial C_{24|1}\left(F_{2|1}, F_{4|1}\right)/\partial F_{1|2} \\
F(x_5|x_1, x_2) &= \partial C_{25|1}\left(F_{2|1}, F_{5|1}\right)/\partial F_{1|2} \\
F(x_4|x_1, x_2, x_3) &= \partial c_{34|12}\left(F_{3|12}, F_{4|12}\right)/\partial F_{12|3} \\
F(x_5|x_1, x_2, x_3) &= \partial c_{35|12}\left(F_{3|12}, F_{5|12}\right)/\partial F_{12|3} \\
F(x_5|x_1, x_2, x_3, x_4) &= \partial c_{45|123}\left(F_{4|123}, F_{5|123}\right)/\partial F_{123|4}
\end{aligned} \tag{4.9}
$$

4.2.3 Value at Risk, Conditional Value at Risk and Portfolio Optimization

In this section, we present the portfolio analysis determined by the risk measure. In classical work, Markowitz (1952) provided a quantitative procedure for measuring risk and return that used mean returns and variances to derive an efficient frontier where an investor could either maximize the expected return for a given variance as well as minimize the variance for a given expected return. Over the past decade, the VaR is a very popular model [see Duffie and Pan (1997), RiskMetrics (1996), Gourieroux et al.(2000) and etc.] to measure risk; it means the maximum amount of loss that are not exceed on a given confidence level (q) over a time horizon. We can perform the following equation 4.10

$$VaR_q(w) = min\{\gamma \in \mathbb{R}: P[f(w,r) \le \gamma] \ge q\}. \tag{4.10}$$

Let $q \in (0,1)$ is the confidence level, the probability of $f(w,r) = -w^T r$ not exceeding a given threshold γ, An alternative method which is defined risk by the expected loss of VaR, it called the conditional value at risk (or expected short fall) that calculated by

$$CVaR_q(w) = \gamma + \frac{1}{1-q} \int_{f(w,r) \ge VaR_q(w)} f(w,r)p(r)dr. \tag{4.11}$$

Rockafellar and Uryasev (2000) introduced the optimization portfolio problem using $CVaR$ minimizing, which is able to be simplified as the following formulas:

$$\left\{ \begin{array}{c} Minimize \quad CVaR_q(w) \\ st.\ w'r = r_p \quad and \quad e'w = 1 \end{array} \right\} \tag{4.12}$$

4.3 Data

We collected daily data from January of 2008 to December of 2013 from the DataStream. With regards to the literature review, we considered a portfolio problem from five attractive markets in Asian emerging countries, which are China (the Shanghai composite index: SC), India (the Bombay stock exchange: BE), Korea (Korea exchange: KE) , Taiwan (the Taiwan stock exchange: TE) and Thailand (the stock exchange of Thailand: SET). The stock return series are generated by $r_{it} = \log(p_{it}) - \log(p_{it-1})$

Table 4.1 Descriptive measures for return series

	SC	BE	KE	TE	SET
Min	-0.080437	-0.116044	-0.148764	-0.067351	-0.110902
Max	0.090343	0.159900	0.202302	0.065246	0.075487
Mean	-0.000588	0.000022	0.000021	0.000022	0.000278
S.D.	0.016770	0.016805	0.025203	0.013533	0.014206
Skewness	-0.179702	0.271395	0.544274	-0.293334	-0.662498
Kurtosis	6.960423	11.98158	15.06218	6.173628	9.822351
Jarque-	1023.306	5239.008	9491.497	674.0081	3125.418
Bera	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ADF	-39.94339	-37.57615	-42.45914	-37.55748	-38.26316
statistics	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Table 4.1 reports the descriptive statistics for daily returns. The mean daily return is mostly positive, mostly negative skewness and the non-normality of all distribution which rejected the null hypothesis by the high Jarque-Bera statistics. The ADF test confirmed that all return series are stationary at level.

4.4 Empirical Results

Table 4.2 shows the estimated parameters for mean and variance equations of AR(1) GJR-GARCH with Gaussian kernel and generalized Pareto distribution, that is called the "semi-parametric" distribution. Figure 4.2 presents the in-sample conditional volatility that is calculated by equation 4.2 and it is noticed that the five markets were highly volatile during the global financial crisis. For the diagnostics test in Table 4.3, the standardized residuals are non-normality distribution from the high Jarque-Bera statistics. Meanwhile, the standardized residuals satisfy the i.i.d. assumption because the Q-statistics accept the null hypothesis which implied that each series are not serially correlated. These findings confirm that the GARCH model should apply EVT on the standardized residuals. We transform the standardized residuals into uniform [0,1] based on empirical processes, then we also use Kolmogorov-Smirnov (KS) statistics to confirm that the data are uniformly distributed.

Table 4.2 Parameter Estimates for AR(1)-GJR GARCH-EVT models

	SC	BE	KE	TE	SET
Mean equation					
β_{io}	-0.000134	4.62e-06	0.000371	0.00040783	0.001011
	[0.000301]	[0.000274]	[0.000382]	[0.000241]	[0.000255]
β_{i1}	-0.018851	0.03723	-0.039035	0.041258	0.0095189
	[0.023660]	[0.02703]	[0.026614]	[0.026476]	0.027025
Variance equation					
μ_i	3.80e-07	2.29e-06	3.65e-06	5.01e-07	4.27e-06
	[3.48e-07]	[6.82e-07]	1.24e-06]	[2.94e-07]	[1.15e-06]
α_i	0.9795	0.9139	0.9324	0.95462	0.86876
	[0.00479]	[0.012078]	[0.011657]	[0.008196]	[0.018241]
θ_i	0.0061551	0.006155	0.000000	0.000000	0.047426
	[0.007331]	[0.007331]	[0.012735]	[0.009873]	[0.018295]
γ_i	0.025594	0.025594	0.11359	0.078569	0.11959
	[0.009471]	[0.009471]	[0.021114]]	[0.014977]	[0.028061]
Extreme value threshold					
u_r	0.0177	0.0182	0.0230	0.0143	0.0145
η_r	-0.0006	0.1155	-0.1087	0.0373	0.0397
ϑ_r	0.5559	0.5173	0.5886	0.4166	0.4737
u_l	-0.0194	-0.0181	-0.0248	-0.0163	-0.0166
η_l	0.0147	-0.059	-0.0198	-0.0177	-0.0438
ϑ_l	0.7031	0.5953	0.6851	0.6471	0.6845

Note: In parentheses are standard errors of the coefficient estimates.

Table 4.3 Diagnostic statistics

	SC	BE	KE	TE	SET
Jarque-Bera	1288.063	926.236	1912.282	1215.095	414.347
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Q(3)	4.1238	1.8630	2.3566	0.7779	4.1495
	[0.248]	[0.601]	[0.652]	[0.855]	[0.246]
Q(6)	6.0856	4.3268	2.5839	4.6500	7.1647
	[0.414]	[0.633]	[0.859]	[0.589]	[0.306]
KS test	0.0315	0.0222	0.0146	0.0370	0.0305
	[0.0922]	[0.4277]	[0.8935]	[0.1215]	[0.112]

Note: In parentheses are the p-value of the test statistics.

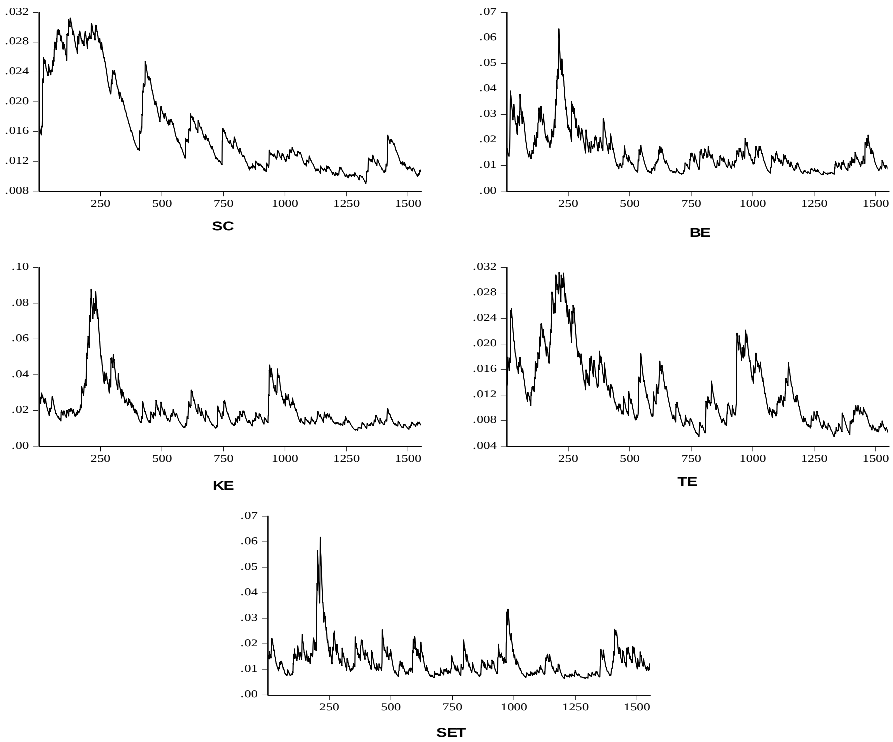

Figure 4.2 Estimated conditional volatility of SC, BE, KE, TE, SET in sample period

Table 4.4 Estimated parameters for five dimensional C- vine copula decomposition

Copula Family	Margin	$\bar{\theta}_0$	$\bar{\theta}_1$	Kendall's tau	AIC
Gaussian	C_{12}	0.265650 [0.023600]	-	0.171173	-103.17465
Gaussian	C_{13}	0.239088 [0.024060]	-	0.153697	-82.96829
BB1	C_{14}	0.355066 [0.050364]	1.100088 [0.027343]	0.228032	-222.21381
BB7	C_{15}	1.119008 [0.030929]	0.259820 [0.039628]	0.164719	-116.32319
t	$C_{23\|1}$	0.313743 [0.023594]	19.780978 [10.003315]	0.203166	-146.38576
BB1	$C_{24\|1}$	0.111484 [0.041479]	1.142413 [0.026810]	0.170877	-120.53664
t	$C_{25\|1}$	0.431295 [0.020672]	15.560217 [6.296301]	0.283887	-295.95646
Frank	$C_{34\|12}$	1.624721 [0.158124]	-	0.175962	-103.67280
Frank	$C_{35\|12}$	0.938040 [0.158465]	-	0.1033231	-33.06613
Gaussian	$C_{45\|123}$	0.207901 [0.023890]	-	0.1333264	-65.88185
Total					-1290.18

Note: 1 = SC , 2 = BE, 3 = KE, 4 = TE , 5 = SET and in parentheses are standard errors of the coefficient estimates.

To model the dependence structures of Asian emerging markets, we considered using canonical vine model from CDVine package to analysis. Table 4.4 shows the dependence structures between the markets. We selected the best fitting copula family by the Akaike information criterion (AIC). The result shows that the copula families, which are Gaussian and t copula in the linear sense as $C_{12}, C_{13}, C_{45|123}, C_{23|1}$ and $C_{25|1}$ for other copula, are far away from normality and difficult to compare with the results. From this information, we used the copula parameter to approximate the rank correlation as the Kendall's tau coefficient. This coefficient has a range between $-1 \leq \tau \leq 1$. If markets are fully independent, then the coefficient takes value close to zero. The result shows that there are highest conditional dependencies between the markets in India and Thailand, China and Taiwan.

Table 4.5 VaR and $CVaR$ estimation in each market

	SC	BE	KE	TE	SET
			N = 10000		
$VaR_{0.99}$	0.028919	0.026666	0.026291	0.026943	0.028389
$VaR_{0.95}$	0.016419	0.015650	0.015684	0.015883	0.016463
$CVaR_{0.99}$	0.035704	0.034472	0.033570	0.033629	0.034859
$CVaR_{0.95}$	0.023938	0.022782	0.022298	0.022747	0.023549
			N = 20000		
$VaR_{0.99}$	0.027537	0.026911	0.027432	0.026676	0.026897
$VaR_{0.95}$	0.016219	0.015914	0.016185	0.015950	0.016202
$CVaR_{0.99}$	0.034268	0.033979	0.034437	0.034330	0.033916
$CVaR_{0.95}$	0.023273	0.022688	0.023099	0.022632	0.022907

Note: N = number of simulated data

Table 4.6 Optimal portfolio with $CVaR$ minimization

Confidence level	SC	BE	KE	TE	SET	$CVaR$
			N = 10000			
99%	0.1857	0.2171	0.2034	0.1841	0.2097	0.0136
95%	0.1850	0.2099	0.2082	0.1982	0.1986	0.0097
			N = 20000			
99%	0.1829	0.2104	0.2036	0.2107	0.1924	0.0132
95%	0.1982	0.2032	0.1947	0.2044	0.1995	0.0096

Note: N = number of simulated data

Given the copula parameters in Table 4.4, we used algorithms belong to Aas et al. (2009) in CDVine package to generate 10,000 and 20,000 dependent uniform random variables over a holding period of one day. After generating the data, we converse the uniform series into the returns of each market and then we computed the risk measure with the daily data. Table 4.5 reports the value at risk and conditional value at risk for different confidence interval. The VaR are used to measure the maximum possibility loss of the market value over a holding period of one day. The VaR is computed as 0.028919 at 99% confidence interval, which implies the daily loss will not exceed 2.8919% in the Chinese market. Simultaneously, the $CVaR$ is estimated as 0.035704 at 99% confidence interval, which implies that the expected loss at 3.5704% would be exceed the VaR at 99% confidence interval. The Chinese market has the highest VaR and $CVaR$ based on comparison with the overall market. Table 4.6 shows the results of portfolio optimization in Asian emerging markets. The optimal weights suggest that investors should focus on Indian and Taiwan markets more than the other markets that are involved in the big picture.

4.5 Conclusions

We examined an empirical study of China, India, Korea, Taiwan and Thai in Asian emerging stock markets from the period of 2008 to 2013 that covered the global financial crisis. Methodologically, we applied conditional EVT to capture the tails of the standardized residuals in each market return which are over the threshold when the tails have high risk. Then, we used C-vine copula to analyze the dependence of diversification measures. Empirically, the results show that the five stock markets have a positive dependence. The two highest dependences are the Indian and Thai markets as well as the Chinese and Taiwanese markets, respectively. Moreover, we have extended C-vine copula by applying the Monte Carlo simulation to generate series for measuring the VaR and $CVaR$ in each markets. The results suggested that the Chinese market has the highest risk. For performing portfolio optimization, the results suggested that investors should pay attention to the Indian and Taiwanese markets.