CHAPTER 5

Copula Based Volatility Models and Extreme Value Theory for Portfolio Simulation with an Application to Asean Stock Markets

This chapter is developed from the original article namely, "Copula based volatility models and extreme value theory for portfolio simulation with an application to Asean stock markets". The contents are extracted from the original article, which was accepted in "Causal Inference in Econometrics" Studies in Computational Intelligence, pp. 223-235 and this article can be found in the appendix C. The methodology used in this study was explained in Chapter 2.

5.1 Introduction

For asset allocation models, the risk-return characteristics are the most important issue for investors to consider. The conventional portfolio theory uses standard deviation and linear correlation coefficient to measure portfolio risk under multivariate normal distribution. To construct the optimal portfolio, this theory uses the risk return framework to allocate assets by minimizing the risk of the portfolio subject to the portfolio return being greater or equal to the risk free rate.

The Value at Risk (VaR) is one of the most important and popular tool to measure the financial risk. It measures the maximum amount of loss that is not exceeded on a given confidence interval. An alternatively risk measure is the condtional VaR ($CVaR$), which is used to estimate the expected loss from VaR. Rockafellar and Uryasev (2000) showed a representation of $CVaR$ based approach to optimize portfolios. Moreover, Artzner et al. (1999) and Rockafellar and Uryasev (2002) explained that VaR is not coherence whereas $CVaR$ satisfies the properties of the risk of a diversified portfolio, which are the sub-additive and convex properties. For these reasons, $CVaR$ has the advantages over VaR.

The most widely used econometric approach to volatility modeling is the family of autoregressive conditional heteroscedasticity (ARCH), which is introduced by Engle (1982). It assumes that the conditional variance takes into account the conditional heteroskedasticity inherent in time with the assumption of normally distributed innovations. Bollerslev (1986) then improved the ARCH to generalized ARCH (GARCH) model, which can yields VaR and $CVaR$ as well.

In recent years, the EVT has been utilized to analyze financial data. It is a statistical tool to examine the extreme deviations from the median of probability distribution. It is very popular and useful for modeling in rare events. Hence, the EVT can be an alternative for an effective framework to estimate the tail of financial series when there are extreme financial events, such as the Asian financial crisis, Subprime crisis and European debt crisis. Embrechts et al. (1999) provided examples for applications of EVT in finance and insurance. Bali (2003), Wang et al. (2010), Ren and Giles (2010) and Jesús et al.[16] applied EVT to calculate VaR for risk management.

The EVT based method combines ideas from the GARCH models with the tail of the innovations distribution using EVT to estimate VaR and $CVaR$. Exemplary works by McNeil and Frey (2000) introduced EVT based method (or conditional EVT models) to forecast VaR. Karmakar (2013) applied this method to estimate VaR in different percentiles for negative and positive BSE India returns. Furio´ and Climent (2013) found that GARCH-EVT model is more accurate than the GARCH models assuming Gaussian or Students t distribution innovations for VaR simulation analysis. Meanwhile, Allen et al. (2013) used both unconditional and conditional EVT models to forecast VaR. Marimoutou et al. (2009) found that this model performs better than other methods without EVT, such as conventional GARCH, historical simulation and filtered historical simulation.

To study the dependence among stock markets using traditional methods, Pearsons correlation has been the most commonly used in empirical works. However, Pearsons correlation used to measure the degree of linear dependence between multivariate normally distributed data. More precisely, Copulas can relax the dependence structure beyond normal distribution. Moreover, the copula is flexible as it can be used to analyze linear, nonlinear or tail dependence. In the context of the copula in financial studies, Embrechts et al. (2002) introduced copula in finance to relax the assumption of dependence structures between random returns. Patton (2009) explained an overview of copula based models for financial applications. In the study of multivariate copulas, Kole et al. (2007) and Wang et al. (2010) found that the multivariate t copula is the best measure of the dependence structure between multiple assets because it can capture the dependence both in the center and the tails. Aas et al. (2009) introduced the flexible way to set the pair copula construction, namely, D and C-vine copula. The recent study such as Low et al. (2013), Hernandez (2014), Ayusuk and Sriboonchitta (2014), Mensi et al. (2015) have applied vine copula with applications to portfolio management.

There are researchers on the effects of the subprime crisis. Hemche et al. (2014) found the dynamic linkages between the US and developed stock markets (as France, Mexico, Italy and the UK) with strong comovements in times of financial crisis. The correlation between the US and other markets (as China, Japan, Tunisia, Egypt and Morocco) were weak and thus they suggested that the investors should also invest in some emerging countries. Moralesa and Callaghan (2012) and Wang (2014) suggested that the US stock markets are less generating effects into the Asian stock markets. In 2015, Asean Economic Community (AEC) is set to be implemented. There will be free trade of goods, services, skilled labor and investment capital following the liberalization and most countries in AEC are still emerging economies. Hence, to take advantage of the portfolio allocation for international stock market, this study focused on VaR and $CVaR$ based on the econometric approaches with the application on the Asean stock markets during and post subprime crisis.

In this chapter, the primary objective is to compare the econometric approaches to portfolio simulation. These econometric approaches include the multivariate t copula GARCH-EVT, C-vine copula GARCH-EVT and D-vine copula GARCH-EVT. The secondary objectives to measure the dependence among Asean stock markets.

The remainder of this chapter is organized as follows. In methodology section, we provide details about the GARCH model, EVT, copulas and the portfolio simulation procedure. In data section discuss the data by descriptive statistics. The result section shows the empirical work. In the final section, we present concluding remarks.

5.2 Methodology

5.2.1 Marginal Models

Generally, data on market returns present conditional heteroscedasticity. Hence, this study focuses on the marginal returns through the autoregressive conditional heteroskedasticity model. To capture the asymmetry property under the sense that shocks not have the exact same impact on volatility in between negative and positive shocks, we used the GJR GARCH model that was proposed by Glosten et al. (1993).

$$r_t = \beta_0 + \beta_1 r_{t-1} + \varepsilon_t = \beta_0 + \beta_1 r_{t-1} + \sigma_t z_t, \tag{5.1}$$

$$\sigma_t^2 = \mu + \alpha\varepsilon_{t-1}^2 + \theta\sigma_{t-1}^2 + \gamma\varepsilon_{t-1}^2 I_{t-1}, \tag{5.2}$$

where r_t is a market return at time t, $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, $I_{t-1} = 0$ if otherwise, $\mu,\ \alpha,\ \theta, \gamma$ are parameters. For stationarity and positivity, the GJR GARCH model has the following properties: $\alpha > 0, \theta > 0, \gamma > 0$, $\alpha + \gamma > 0$ and $\alpha + \theta + \frac{\gamma}{2} < 1$, $\varepsilon_t = \sigma_t z_t$ is residual returns, σ_t is the volatility of the returns, z_t is standardized residuals that must satisfy independently and identically distributed. Traditionally, the standardized residuals follow a normal distribution.

5.2.2 The Distributions of Standardized Residuals

In this study, we focus on EVT, which is an appropriate approach to define the behavior of extreme tail observations. We apply the semi parametric approach to generate the standardized residuals of the GJR GARCH model. To capture the extreme tails, we use the generalized Pareto distribution (GPD) to select the extreme tails that are peaks over the threshold. To capture the interior distribution, we define by using the Gaussian kernel distribution ($\varphi(z)$). The distribution is given by

$$F(z) = \begin{cases} \frac{k_{u^L}}{n}\left(1+\eta^L\left(\frac{u^L-z}{\vartheta^L}\right)\right)^{-(\eta^L)^{-1}}, & z < u^L \\ \varphi(z), \quad u^L < z < u^R & \\ 1-\frac{k_{u^R}}{n}\left(1+\eta^R\left(\frac{u^R-z}{\vartheta^R}\right)\right)^{-(\eta^R)^{-1}}, & z > u^R \end{cases} \tag{5.3}$$

where , u^L, u^R are lower and upper thresholds, z is the standardized residuals that excess over the thresholds k_{u^L} and k_{u^R} are the number of observations that excess over lower and upper thresholds, n is the number of observation, ϑ^L and ϑ^R are the scale parameters, η^L and η^R are the shape parameters.

5.2.3 Copula Approach

A copula is a function that connects univariate marginals to construct the multivariate distribution with uniformly distributed marginals $U(0,1) \rightarrow [0,1]$. It also can be used to portray the dependency of random variables in each event. This study used the copula approach for describing the dependence between international markets. Originally, Sklar (1959) introduced the important theorem for copula function as follows.

Theorem 5.1 Let $x_1, \dots, x_n$ are random variables for $i = 1, \dots, n$. $F_1(x_1), \dots, F_n(x_n)$ are the continuous marginal distributions and $F(x_1, \dots, x_n)$ be a multivariate distribution. Then, n -dimensional copulas $C(\cdot): [0,1]^n \longrightarrow [0,1]$ can be defined by

$$F(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n)). \tag{5.4}$$

Inversely, equation 5.4 can be written as

$$F(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n)), \tag{5.5}$$

where $F_i^{-1}(u_i)$ are the inverse distribution function of the marginals and $u_i \in [1,0]$. We can determine the copula density $c(u_1, \dots, u_n)$ by using n order partial derivative as follows:

$$c(u_1, \dots, u_n) = \frac{\partial^n C(u_1, \dots, u_n)}{\partial u_1 \dots \partial u_n} \tag{5.6}$$

According to the joint density function $f(x_1, \dots, x_n)$, it can be defined by n order partial derivative of a multivariate distribution $F(x_1, \dots, x_n)$ as follows:

$$f(x_1, \dots, x_n) = \frac{\partial^n F(x_1, \dots, x_n)}{\partial x_1 \dots \partial x_n} \tag{5.7}$$

$$f(x_1, \dots, x_n) = \prod_{i=1}^{n} f_i(x_i) \cdot \frac{\partial^n C(u_1, \dots, u_n)}{\partial u_1 \dots \partial u_n} \tag{5.8}$$

$$f(x_1, \dots, x_n) = \prod_{i=1}^{n} f_i(x_i) \cdot c(u_1, \dots, u_n) \tag{5.9}$$

Equation 5.9 shows that the joint density function is the combination between the copula density and the product of marginal densities. In the study of copulas, Mashal and Zeevi (2002), Breymann et al. (2003), Kole et al. (2007) and Wang et al. (2010) suggested that t copula is the better measure of the dependency structure for multiple assets. Hence, this study considered t copula for measuring the market dependence. We can define a multivariate t copula for n dimensional as follows:

$$C^t(u_{1t},\dots,u_{nt}) = t_{v,\Sigma}\left(t_v^{-1}(u_{1t}),\dots\dots,t_v^{-1}(u_{nt})\right), \tag{5.10}$$

where $t_{v,\Sigma}$ is the distribution function of multivariate t copula, Σ is a correlation matrix and v is the degree of freedom. Moreover, this study also applied C and D-vine structures with t copula to determine the market dependence. The two vine copulas were introduced by Aas et al. (2009). In n dimensions, $\frac{n(n-1)}{2}$ is the number of pair copula, $n-1$ is the number of trees in vine copulas and $n!$ is the number of possible tree structures. To select the tree structures, this study determines the appropriate ordering of the tree structures by choosing the maximum of absolute empirical Kendall' s tau values for all bivariate copula. C and D-vine density functions can be defined by

$$f(x_1,\dots,x_n) = \prod_{i=1}^{n} f_i(x_i)\cdot\prod_{j=1}^{n-1}\prod_{j=1}^{n-j} c_{j,j+k|1,\dots,j-1}\left(F(x_j|\boldsymbol{v_1}),F(x_{j+k}|\boldsymbol{v_1})\right), \tag{5.11}$$

$$f(x_1,\dots,x_n) = \prod_{i=1}^{n} f_i(x_i)\cdot\prod_{j=1}^{n-1}\prod_{k=1}^{n-j} c_{k,k+j|k+1,\dots,k+j-1}\left(F(x_k|\boldsymbol{v_2}),F(x_{k+j}|\boldsymbol{v_2})\right), \tag{5.12}$$

where $\boldsymbol{v_1} = x_1,\dots,x_{j-1}$ and $\boldsymbol{v_2} = x_{k+1},\dots,x_{k+j-1}$, j is the tree in vine copulas, k is the edge in each tree, $c_{k,k+j|k+1,\dots,k+j-1}$ in equation 5.11 and $c_{k,k+j|k+1,\dots,k+j-1}$ in equation 5.12 are bivariate copula densities. In order to compute the conditional distribution functions $F(x|\boldsymbol{v})$ in equation 5.11 and 5.12 by following Joe (1996) as in equation 5.13

$$F(x|\boldsymbol{v}) = \frac{\partial C_{xv_j|\boldsymbol{v}_{-j}}\left(F\left(v_j|\boldsymbol{v}_{-j}\right), F\left(v_j|\boldsymbol{v}_{-j}\right)\right)}{\partial F\left(v_j|\boldsymbol{v}_{-j}\right)}, \tag{5.13}$$

where the vector $\boldsymbol{v}_{-j}$ is the vector v_j that excludes the component v_j. $C_{xv_j|\boldsymbol{v}_{-j}}$ is the bivariate copula distribution between x and v_j that is taken conditional on $\boldsymbol{v}_{-j}$. The estimated dependence parameters of various copulas are obtained by maximum likelihood (see Aas et al. (2009)).

5.2.4 Portfolio Simulation

We forecast one-day-ahead for VaR, $CVaR$ based on t copula GARCH-EVT at 95% and 99% confidence level with the procedures as follows:

(1) We estimate the parameters of the GARCH model for each market return series. We obtain the standardized residuals over the threshold follow the generalized Pareto distribution (GPD), because GPD can capture the upper and lower tails. Additionally, we also use the Gaussian kernel estimation for the interior part.

(2) We transform each standardized residuals (z_t) of each univariate distribution to approximate i.i.d. uniform data(u_t) on $[0,1]$ by using empirical distribution functions and then fit t copula for estimating its parameter.

(3) Given the parameters of copula function, we simulate the uniform series 100,000 dimensional time series and obtain the standardized residuals by using the inverse functions of the estimated marginals.

(4) We converse the standardized residuals from step (3) into the returns at $t+1$, calculate the empirical one-day-ahead VaR, $CVaR$ at 95% and 99% confidence level, and optimize the portfolio based on $CVaR$ minimization problem at 99% confidence level (or $Min_{w\epsilon W}\ CVaR$) by following the procedure of Rockafellar and Uryasev (2000,2002)

5.3 Data

We used the daily data of five main stock market indices in Asean countries from DataStream: The indices composed of SET index (Thailand:TH), Straits Times index (Singapore:SP), KLSE Composite index (Malaysia:MS), JSX Composite index (Indonesia:ID) and PSE Composite(the Philippines:PP). We defined the market returns by $r_t = \log(p_t) - log(p_{t-1})$. Following Horta et al. (2014) and Lee et al. (2014), this study focuses on subprime crises period and then we divide it into sub periods: the subprime crisis period (1 August 2007 to 29 December 2009) and the post subprime crisis period (4 January 2010 to 29 December 2014).

Table 5.1 Descriptive measures for Asean markets

Index	TH	SP	MS	ID	PP
			A: crisis period		
Mean	-0.000221	-0.000341	-9.38E-05	0.000210	-0.000220
Max	0.086167	0.102705	0.057165	0.190719	0.083854
Min	-0.085892	-0.129279	-0.102374	-0.257802	-0.136399
S.D.	0.018798	0.020622	0.012513	0.026211	0.019945
Skewness	-0.104720	-0.034376	-0.873542	-1.404863	-0.687729
Kurtosis	6.331662	8.140576	13.30447	27.81636	9.093367
Jarque-Bera	243.3070	577.0605	2384.952	13618.47	851.9586
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ADF	-20.55669	-21.93688	-21.54766	-22.41699	-21.35143
statistics	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
			B: after-crisis period		
Mean	0.000667	0.000141	0.000304	0.000651	0.000818
Max	0.057515	0.029001	0.047228	0.070136	0.055419
Min	-0.058119	-0.037693	-0.026757	-0.092997	-0.069885
S.D.	0.011965	0.008567	0.006207	0.012800	0.011732
Skewness	-0.367571	-0.423877	0.108370	-0.822461	-0.500209
Kurtosis	6.294566	4.813557	8.424547	10.33282	6.756750
Jarque-Bera	509.4336	179.1765	1317.675	2524.951	675.7222
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
ADF	-30.01431	-30.38082	-29.13288	-22.97933	-22.97933
statistics	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]

Note: The values reported in parentheses are p-value for testing the null hypothesis

Table 5.1 shows summary statistics. We found that almost all markets of the average yield (mean of market return) are negative during the subprime crisis. SP has the most negative returns. After the subprime crisis, the average yield has a positive sign in every market and the standard deviation (SD) is less than a period of the subprime. The Jarque-Bera rejects the null hypothesis which indicated that returns of the markets are not following the normality assumption. The ADF test approved the stationary property of all markets.

5.4 Empirical Results

Table 5.2 shows GJR GARCH parameter estimation. The mean equation is in the simplest form of first autoregressive $(AR(1))$. The Q-statistics confirm that the marginals mostly accept the null hypotheses which suggested that there are no serial correlations and satisfy an i.i.d. assumption for almost all the markets. Then, we transform standardized residuals into the uniform$(U[0,1])$ by using the empirical distribution functions. The Kolmogorov-Smirnov test (KS-test) is used to test the null hypothesis that the transformed data are uniformly distributed, because all data series support the null hypothesis and use this result to carry out the copula procedure. Jarque-Bera statistics suggested that the standardized residuals of are non-normality distribution. These findings from statistical testing confirm that the GJR GARCH model can apply EVT to handle on the standardized residuals.

Table 5.2 Parameter Estimates for AR(1)-GJR GARCH-EVT models

	TH	SP	MS	ID	PP
			A: crisis period		
Mean equation					
β_o	0.000549	0.000135	0.000199	0.000541	0.000142
	[0.000665]	[0.000651]	[0.000392]	[0.000688]	[0.000678]
β_1	0.009793	0.001870	0.065287	0.11054	0.086547
	[0.045931]	[0.046813]	[0.044449]	[0.046162]	[0.046992]
Variance equation					
μ	1.69e-005	3.22e-006	1.17e-005	0.000137	7.79e-005
	[8.32e-006]	[3.16e-006]	[5.24e-006]	[3.16e-005]	[2.59e-005]
α	0.83871	0.91095	0.80145	0.38876	0.57183
	[0.051076]	[0.023375]	[0.059008]	[0.083337]	[0.097427]
θ	0.045528	0.035349	0.018407	0.000000	0.063385
	[0.036726]	[0.020904]	[0.028609]	[0.040953]	[0.048436]
γ	0.13214	0.1004	0.23656	0.88391	0.30257
	[0.064542]	[0.040892]	[0.095621]	[0.25269]	[0.12236]

Table 5.2 (continued)

	TH	SP	MS	ID	PP
Q(2)	9.0621	0.8644	0.8474	4.3689	0.3434
	[0.0108]	[0.6491]	[0.6546]	[0.1125]	[0.8423]
Q(6)	16.4338	13.7795	3.5713	6.9328	5.8217
	[0.0116]	[0.0322]	[0.7345]	[0.3271]	[0.4435]
KS-*statistics*	0.0027	0.003	0.003	0.0036	0.0032
	[0.4771]	[0.3156]	[0.3438]	[0.1599]	[0.2709]
Jarque-Bera	82.0629	85.3962	789.3312	1316.8	231.8684
	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
		B: after crisis period			
Mean equation					
β_o	0.001186	0.000277	0.000306	0.000988	0.000791
	[0.000282]	[0.000210]	[0.000146]	[0.000289]	[0.000293]
β_1	0.027503	0.010368	0.083798	0.004194	0.085756
	[0.033591]	[0.032322]	[0.028665]	[0.03082]	[0.032237]
Variance equation					
μ	5.95e-006	8.02e-007	1.97e-006	6.29e-006	8.96e-006
	[1.68e-006]	[3.40e-007]	[6.75e-007]	[1.99e-006]	[2.63e-006]
α	0.83591	0.92912	0.85629	0.87223	0.81495
	[0.029077]	[0.015984]	[0.03229]	[0.027457]	[0.036924]
θ	0.035536	0.014864	0.024771	0.019357	0.020671
	[0.024681]	[0.018464]	[0.021801]	[0.025768]	[0.025987]
γ	0.16857	0.085624	0.14827	0.125	0.18828
	[0.041524]	[0.0243]	[0.043269]	[0.04037]	[0.048925]
Q(2)	3.0135	0.2932	2.4969	4.1248	0.7148
	[0.2216]	[0.8636]	[0.2870]	[0.1271]	[0.6995]
Q(6)	6.5855	7.2224	3.3907	24.2223	8.8284
	[0.3609]	[0.3008]	[0.7585]	[0.0005]	[0.1835]
KS-*statistics*	0.0161	0.0164	0.0227	0.0208	0.0154
	[0.945]	[0.9339]	[0.6376]	[0.7443]	[0.9651]

Table 5.2 (continued)

	TH	SP	MS	ID	PP
Jarque-Bera	114.0268	40.9454	1319.4	968.1900	160.1151
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]

Note: In parentheses are standard errors of the coefficient estimates

In parentheses of Q-statistics, KS-statistics and Jarque-Bera-statistics are p-value for testing the null hypothesis

Table 5.3 shows parameter estimation of extreme value theory, we use the GPD in our study where ϑ , η are the scale parameter and the shape parameter and we fixed the threshold value u at 10% level of confidence. Figure 5.1 is a sample of the CDF by using semi-parametric form of Singapore market, in the subprime period, obviously, the valued of upper tail was higher than the after-crisis period.

Table 5.3 GPD estimation of each market's residuals

	TH	SP	MS	ID	PP
			A: crisis period		
u_r	1.1663	1.2225	1.1225	1.1281	1.1864
ϑ_r	0.4929	0.7693	0.5947	0.5580	0.4916
η_r	0.1237	-0.0712	0.1764	0.1827	0.1447
u_l	-1.3113	-1.3290	-1.1794	-1.2537	-1.2979
ϑ_l	0.4835	0.4938	0.3622	0.6470	0.6831
η_l	0.0875	0.0625	0.4554	0.2680	-0.0168
			B: after-crisis period		
u_r	1.2222	1.2172	1.0998	1.0367	1.1718
ϑ_r	0.5245	0.4665	0.4004	0.4673	0.4097
η_r	0.0178	-0.0380	0.3282	0.0805	0.1942
u_l	-1.3012	-1.2407	-1.1444	-1.1542	-1.2279
ϑ_l	0.6973	0.7277	0.7322	0.6031	0.5931
η_l	-0.0715	-0.1496	0.0092	0.1908	0.0282

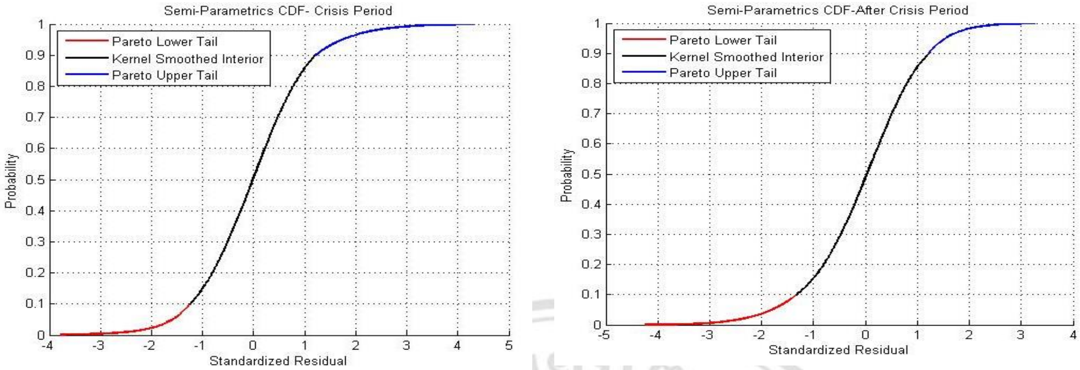

Figure 5.1 Semi-parametric CDFs of Singapone residuals

Table 5.4 The matrixes of the Kendall's rank correlation from the multivariate t copula

	TH	SP	MS	ID	PP
		A: crisis period			
TH	1				
SP	0.4410	1			
MS	0.3524	0.4289	1		
ID	0.3970	0.4866	0.3741	1	
PP	0.2306	0.2808	0.3498	0.2721	1
		B: after-crisis period			
TH	1				
SP	0.3435	1			
MS	0.2520	0.3291	1		
ID	0.3206	0.3831	0.3550	1	
PP	0.2274	0.2574	0.2662	0.2890	1

Table 5.4 shows the values of Kendall's rank correlation, which were computed by using the parameter of the multivariate t copula function from equation 5.10. The results show that five markets have a monotonic relationship because of the Kendall's tau is more than zero. During the crisis, the highest relationship is SP & ID, SP & TH and SP & MS, respectively. While, PP & ID has the weakest relationship. After the

crisis, the strongest relationship is still SP & ID, ID & MS and SP & MS, respectively. While, TH & PP has the weakest relationship.

Table 5.5 The matrixes of the Kendall's rank correlation from C and D vine copula

A: crisis period				*B: after-crisis period*			
C-vine copula		*D-vine copula*		*C-vine copula*		*D-vine copula*	
τ_{51}	0.2501	τ_{41}	0.4024	τ_{51}	0.2386	τ_{31}	0.2654
τ_{52}	0.3101	τ_{43}	0.3668	τ_{52}	0.2681	τ_{41}	0.3262
τ_{53}	0.3685	τ_{51}	0.2500	τ_{53}	0.2760	τ_{52}	0.2681
τ_{54}	0.2749	τ_{52}	0.3101	τ_{54}	0.2965	τ_{53}	0.2760
$\tau_{31\|5}$	0.2851	$\tau_{21\|5}$	0.3876	$\tau_{31\|5}$	0.1931	$\tau_{32\|5}$	0.2676
$\tau_{32\|5}$	0.3307	$\tau_{31\|4}$	0.2094	$\tau_{32\|5}$	0.2676	$\tau_{43\|1}$	0.2836
$\tau_{43\|5}$	0.2826	$\tau_{54\|1}$	0.1817	$\tau_{43\|5}$	0.2865	$\tau_{51\|3}$	0.1522
$\tau_{21\|53}$	0.3100	$\tau_{42\|51}$	0.3007	$\tau_{21\|53}$	0.2447	$\tau_{21\|53}$	0.2448
$\tau_{41\|53}$	0.2655	$\tau_{53\|14}$	0.2548	$\tau_{41\|53}$	0.2057	$\tau_{54\|31}$	0.1488
$\tau_{42\|531}$	0.2627	$\tau_{32\|514}$	0.1576	$\tau_{42\|531}$	0.1892	$\tau_{42\|531}$	0.1886

Note: 1 = TH, 2 = SP, 3 = MS, 4 = ID, 5 = PP

Table 5.5 shows the values of Kendall's tau that compute by using the parameter of vine copula function follow equation 5.11 and 5.12. In table 5.5, we found that five markets have a positive dependence. The highest relationship is ID & TH by D-vine copula in during and after the crisis period.

Table 5.6 Portfolio risk of the equally weighted market strategy

	the multivariate t copula GARCH EVT	C-vine copula GARCH-EVT	D-vine copula GARCH-EVT
	crisis period		
$VaR_{0.95}$	-0.0146	-0.0150	- 0.0150
$VaR_{0.99}$	-0.0250	-0.0255	- 0.0253
$CVaR_{0.95}$	-0.0214	-0.0220	-0.0220
$CVaR_{0.99}$	-0.0342	-0.0353	-0.0353
	after-crisis period		
$VaR_{0.95}$	-0.0135	-0.0135	-0.0138
$VaR_{0.99}$	-0.0219	-0.0219	-0.0224
$CVaR_{0.95}$	-0.0188	-0.0188	-0.0192
$CVaR_{0.99}$	-0.0276	-0.0275	-0.0281

Table 5.6 shows the simulation results of one step ahead forecasting in portfolio risk using the multivariate t copula GARCH-EVT, C-vine copula GARCH-EVT and D-vine copula GARCH-EVT under the same strategies for all markets. The simulation results found that VaR and $CVaR$ values during the crisis are higher than after crisis at 0.95 and 0.99 significant levels. In the crisis period, the multivariate t copula GARCH-EVT gives the values of VaR higher than both vine copula GARCH-EVT and D-vine copula GARCH-EVT gives the values of VaR and $CVaR$ higher then C-vine copula GARCH-EVT. Then we also found that the computational of VaR and $CVaR$ using D-vine copula GARCH-EVT gives higher value than the multivariate t copula GARCH-EVT and the C-vine copula GARCH-EVT after the crisis period.

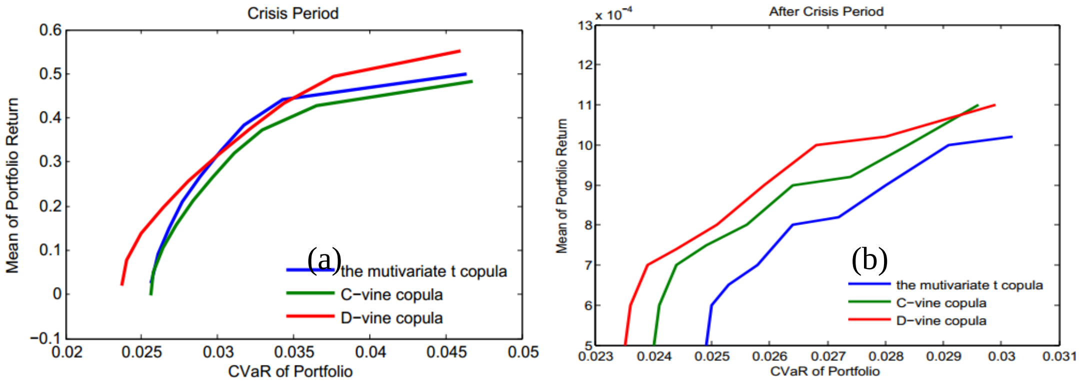

Figure 5.2 Efficient frontier from minimizing CVaR at 99% confidence level

Figure 5.2 shows the efficient frontier of Asean portfolio by minimizing portfolio risk. A subfigures (a) is the efficient frontier during the crisis period and a subfigure (b) is the efficient frontier after the crisis period. From the figure 2, we can conclude that at the same level of $CVaR$, D-vine copula GARCH-EVT generates portfolio return higher than the multivariate t copula GARCH-EVT and C-vine copula GARCH-EVT during the crisis period. After the crisis period, D-vine copula GARCH-EVT gives portfolio return higher than C-vine copula GARCH-EVT and the multivariate t copula GARCH-EVT at the same level of $CVaR$. Finally, We calculate the optimal weights of the portfolio at the efficient frontier as Table 5.7. All three approaches generate the return and $CVaR$ of the portfolio in the crisis period higher than the post crisis period.

Table 5.7 The optimal portfolio weights in efficient frontier for Asean markets

Portfolios	TH	SP	MS	ID	PP	Return	$CVaR_{0.99}$
A. crisis period by the multivariate t copula GARCH-EVT							
1	0.0483	0.0483	0.6923	0.1032	0.1080	0.0343	0.0257
2	0.1129	0.1129	0.4703	0.1306	0.1734	0.1507	0.0268
3	0.1752	0.1752	0.2400	0.1801	0.2295	0.2671	0.0289
4	0.2646	0.2646	0.0000	0.0000	0.4709	0.4417	0.0343

Table 5.7 (continued)

Portfolios	TH	SP	MS	ID	PP	Return	$CVaR_{0.99}$
B. crisis period by C-vine copula GARCH-EVT							
1	0.1506	0.0000	0.5981	0.0000	0.2513	0.0506	0.0258
2	0.1982	0.0589	0.3768	0.0000	0.3661	0.1584	0.0273
3	0.2372	0.1306	0.1612	0.0000	0.4711	0.2661	0.0297
4	0.0000	0.5310	0.0000	0.0000	0.4690	0.4277	0.0365
C. crisis period by D-vine copula GARCH-EVT							
1	0.2902	0.1128	0.0000	0.0000	0.5969	0.0781	0.0240
2	0.2700	0.2532	0.0784	0.0000	0.3984	0.1966	0.0264
3	0.2566	0.3569	0.1833	0.0000	0.2032	0.3152	0.0300
4	0.0083	0.5311	0.4606	0.0000	0.0000	0.5524	0.0376
D. after crisis period by the multivariate t copula GARCH-EVT							
1	0.0000	0.0000	0.5158	0.3826	0.1016	0.0006	0.0250
2	0.0000	0.0000	0.3575	0.5084	0.1341	0.0007	0.0258
3	0.0000	0.0000	0.1997	0.6349	0.1654	0.0008	0.0272
4	0.0000	0.0000	0.0000	0.8604	0.1396	0.0010	0.0302
E. after crisis period by C-vine copula GARCH-EVT							
1	0.3534	0.0642	0.5117	0.0707	0.0000	0.0006	0.0241
2	0.4165	0.1553	0.3626	0.0656	0.0000	0.0007	0.0249
3	0.4930	0.2324	0.2099	0.0648	0.0000	0.0009	0.0264
4	0.6017	0.3682	0.0000	0.0301	0.0000	0.0011	0.0296
F. after crisis period by D-vine copula GARCH-EVT							
1	0.4440	0.0423	0.3284	0.0000	0.1853	0.0006	0.0236
2	0.2887	0.1340	0.3893	0.0000	0.1880	0.0007	0.0244
3	0.1309	0.2273	0.4455	0.0000	0.1964	0.0009	0.0259
4	0.0000	0.4152	0.5826	0.0000	0.0022	0.0011	0.0299

5.4 Conclusions

In this study, we adopt copula based volatility models to measure the dependence between Asean stock markets and then we used a semi-parametric approach from extreme value theory to capture the tail distribution of standardized residuals from the data in the context of the subprime crisis. We examine the portfolio simulation produced by each model and emphasize comparing three models. The models consist of the multivariate t copula GARCH-EVT, C-vine copula GARCH-EVT and D-vine copula GARCH-EVT. Regarding dependence, all copulas provide evidence of positive dependence in every pair. The dependences are mostly strong between Singapore and other markets by the multivariate t copula, which may imply that Singapore market plays an important role in Asean markets. Meanwhile, the risk measure was simulated with equally weighted strategy. This result indicates that D-copula GARCH model-EVT can be estimates VaR and $CVaR$ greater than C-copula GARCH model-EVT and the multivariate t copula GARCH-EVT in the post subprime crisis period. The values of VaR and $CVaR$ during the subprime crisis are higher than those after the subprime crisis. Moreover, the results of the portfolio optimization problem using $CVaR$ objective show that D-vine copula GARCH-EVT is a more efficient tool to simulate the portfolio optimization. Finally, the optimal portfolio weights suggest that the international investors should concentrate on the Malaysian market at the high risk and return, and should invest in Thailand market at the low portfolio risk and return after the subprime crisis.