CHAPTER 2

Transient State Estimation Algorithm

State Estimation (SE) is a technique that uses partial measured data to estimate the state of power system. Estimation techniques have been extended from steady-state to transient phenomena call Transient State Estimation (TSE). There are two main parts of TSE which are system modeling and state estimation. The system modeling is a set of state equation used for measurement equation. State estimation is a technique used for solving the measurement equation.

For measurement equation, the system modeling part will build the combination of each component model (i.e. generator, transformer, transmission line and load) forming the power network. There are several transmission models that can represent transmission line. PI model is one of the popular models. However, longer transmission line needs to consider distributed parameters that corresponding to travelling wave theory for more accurate estimation.

2.1 The Basis of State Estimation

A general linear measurement system used in power system state estimation can be written as [1-2,4-5]

$$z = [H]x + \varepsilon \tag{2.1}$$

where z is a vector of measurement values collected from monitoring equipment,

x is a vector of state variables to be estimated,

$[H]$ is a measurement matrix,

ε is a measurement error vector.

The problem of solving SE equation, equation (2.1), can be categorized as over-determined, completely-determined or under-determined depending on the rank of $[H]$. An over-determined system consists of redundant measurements that can mitigate the effect of any bad data. In general, the method for solve SE problem are the Weighted Least Squares (WLS) [2]. It is a best estimator in the maximum likelihood sense when the errors have Gaussian nature. However, it does not exhibit an inherent capability of filtering bad data. There are essentially four different approaches to solve the WLS problem: the normal equation method, the orthogonal transformation method, the augmented matrix method, and the pseudo-inverse method. Another approach called the Weighted Least Absolute Value (WLAV) method is said to perform well in filtering bad data. It can be solved using the linear programming, the simplex method, or the interior point method. Since WLAV method is computational intensive, a technique to improve its speed is required when using in a real-time system.

2.2 Normal Equation Method

The SE problem without constraints of a power system can be solved using the weighted least squares (WLS). The solution to the state estimation problem can be formulated as a minimization of following objective function [2],

$$J(x) = \sum_{i=1}^{m} \left(z_i - H_i x \right)^2 / R_{ii} \tag{2.2}$$

where z_i is a vector of i-th measurement values, H_i is i-th measurement matrix, m is number of measurements and R is a diagonal measurement covariance matrix [2,4],

$$R = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_m^2 \end{bmatrix} \tag{2.3}$$

where σ_i is the standard deviation of the i-th measurements, m is the number of measurements. Normally, the covariance of the measurements are unknown and often assumed to be an identity matrix since the same instrumentation is used to obtain the data.

Equation (2.2) represents the summation of the squares of the measurement residuals weighted by their respective measurement error covariance. This can be rewritten as [2],

$$J(x) = \left[z - Hx\right]^T R^{-1} \left[z - Hx\right]. \tag{2.4}$$

The minimum of this objective function is when its derivative becomes zero, i.e.

$$\frac{dJ(x)}{dx} = 0. \tag{2.5}$$

The yields the normal equation is obtained by [2,4]

$$\hat{x} = G^{-1}\left[H^T R^{-1}\right] z, \tag{2.6}$$

where $\hat{x}$ is the estimated state and $G = \left[H^T R^{-1} H\right]$ is called gain matrix.

2.3 State Variable Formulation

The system network used in TSE can be expressed as [5,10]

$$\begin{aligned} \dot{x} &= [A]x + [B]u \\ y &= [C]x + [D]u \end{aligned} \tag{2.7}$$

where x is the state vector or state variable measurement data, $\dot{x}$ is the vector of state variable derivatives with respect to time and y is the vector of output variables or dependent variable measurement data. [*A*], [*B*], [*C*] and [*D*] value are coefficient matrices with proper dimensions, and u represents the vector of inputs. Moreover, the measurements in relation to the state variables are [5]

State variable measurement data:

$$x_{measured} = [1]x. \tag{2.8}$$

Derivative of measured state variables:

$$\dot{x}_{measured} = [A]x + [B]u. \tag{2.9}$$

Dependent variable measurement data:

$$y_{measured} = [C]x + [D]u. \tag{2.10}$$

Derivative of measured dependent variable:

$$\dot{y}_{measured} = [C]\dot{x} + [D]\dot{u}, \tag{2.11}$$

$$= [C][A]x + [C][B]u + [D]\dot{u}.$$

The measurement equation are built up from rows of measurement equation corresponding to the selected measurement point and selecting from equations (2.8) to (2.11). The measurement equation is then solved using the weighted least squares equation in (2.6). Once the state variables are known, dependent variables can be calculated and the complete knowledge of the system can be determined.

2.4 System Modeling

The components in a power system are the generator, transformer, transmission line and load. The selection of the component model should be done properly to obtain accurate estimation. Generally, lumped *RLC* elements are often used for short and simple transmission lines and constructed by cascaded connection of *T*, π or *L* sections [8]. If the transmission lines are sufficiently long, the traveling time will be greater than the solution time step. Figure 2.1 shows a decision tree for the selection of the appropriate transmission line model. For example, in the case of a general solution time step,Δt of 50 μs, the minimum limit for travelling time is length/c where the c is the speed of light (c = 3×10^8 m/s), therefore line over 15 km can be represented by the Bergeron model [16].

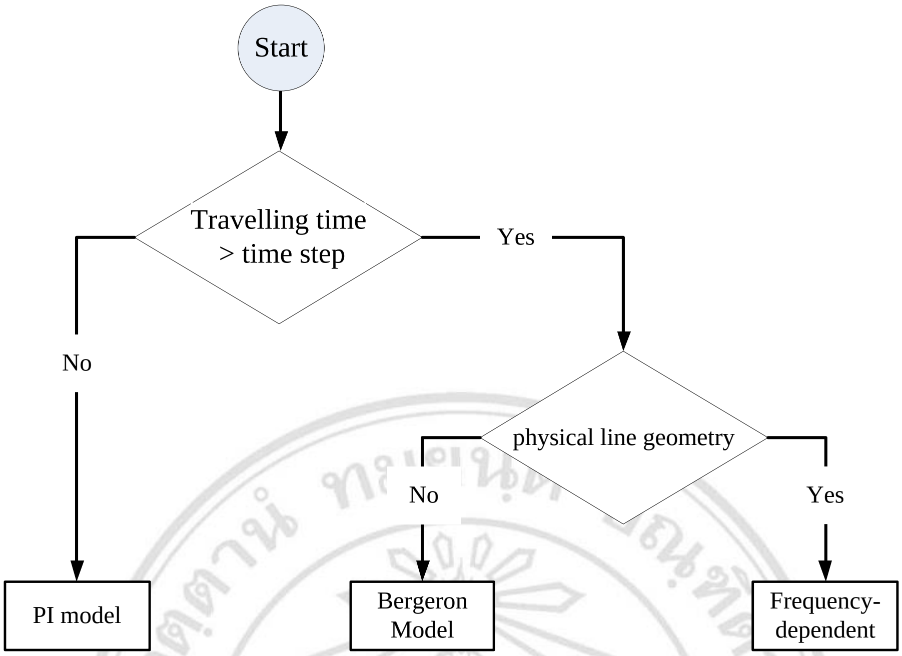

Figure 2.1 Decision tree for select the transmission line model [20].

Figure 2.2 shows propagation of a wave on a transmission line which distributed parameter where ℓ and c are line inductance and line conductance per unit length parameters, respectively.

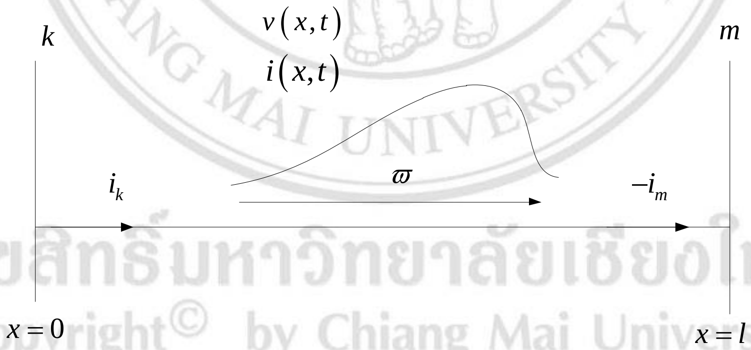

Figure 2.2 Propagation of a wave on a transmission line [20].

The wave travel along the transmission line from bus *k* to bus *m*. The telegraph equation is used to describe the relationship between the voltage and current along the transmission line, both voltage and current are function of position, *x* and time, *t*. The gradient of voltage $v(x,t)$ in equation (2.12) is related to the time derivative of the current through the cable inductance. Similarly in equation (2.13), the gradient of current $i(x,t)$ is related to the time derivative of the voltage. Therefore, the propagation equation for transmission line are

$$-\frac{\partial v(x,t)}{\partial x} = \ell \frac{\partial i(x,t)}{\partial t} , \tag{2.12}$$

$$-\frac{\partial i(x,t)}{\partial x} = c \frac{\partial v(x,t)}{\partial t} . \tag{2.13}$$

General solutions in forward and backward travelling wave are

$$v(x,t) + Z_C i(x,t) = 2Z_C f_1(x - \varpi t) , \tag{2.14}$$

$$v(x,t) - Z_C i(x,t) = -2Z_C f_2(x + \varpi t) , \tag{2.15}$$

where $f_1(x-\varpi t)$ is function of $(x-\varpi t)$ and represents a wave travelling at velocity ϖ in forward direction (from bus *k* to bus *m*). Therefore, $f_2(x+\varpi t)$ is function of $(x+\varpi t)$ and represents a wave travelling at velocity ϖ in backward direction (from bus *m* to *k* bus). The surge impedance, Z_C and the phase velocity, ϖ is given by [10,20].

$$Z_C = \sqrt{\frac{\ell}{c}} \quad > 0 , \tag{2.16}$$

$$\varpi = \frac{1}{\sqrt{\ell c}} \quad > 0 . \tag{2.17}$$

If *l* is length of transmission line, the travelling time, τ between bus *k* and *m* is

$$\tau = \frac{l}{\varpi} = l\sqrt{\ell c} . \tag{2.18}$$

The line can be modeled by an equivalent network consisted of current sources and impedance element follow the figure 2.3. The current sources I_k and I_m in equation (2.19) and (2.20) are calculated from the past history at time $(t-\tau)$ [10,20],

$$I_k(t-\tau) = -(1/Z_C)v_m(t-\tau) - i_m(t-\tau), \tag{2.19}$$

$$I_m(t-\tau) = -(1/Z_C)v_k(t-\tau) - i_k(t-\tau). \tag{2.20}$$

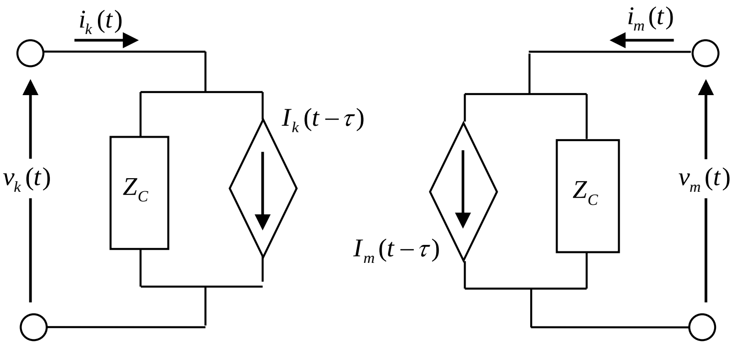

Figure 2.3 Equivalent two-port network for a lossless line [20].

The effective series resistance of the line can be represented by lumped resistors, and the effect of the line attenuation is approximated by adding half of the total line resistance, $R' = rl$ at the ends of line, where r is line resistance per unit length and l is length of transmission line, or for more accurately by adding $R'/4$ at the terminals and $R'/2$ in the middle of the line. In this case, Z_C in figure 2.3 is replaced by $Z_C + R'/4$ and the past history current can be calculated by equation (2.23) and (2.24).

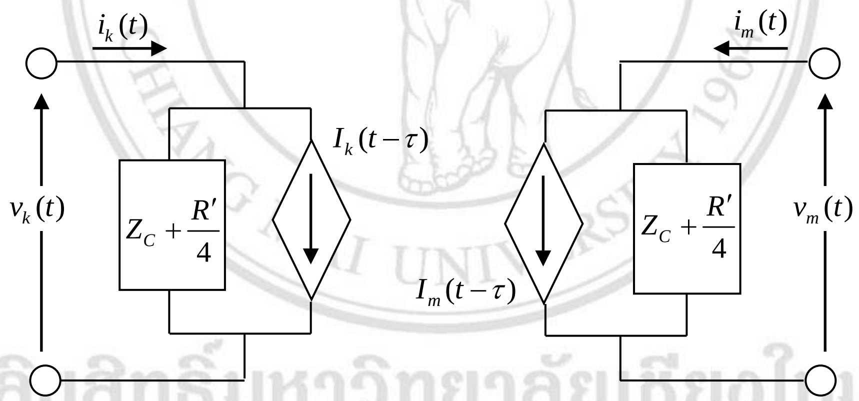

Figure 2.4 Bergeron transmission line model [20].

The Bergeron equivalent two-port network is shown in figure 2.4. The transmission line between the sending bus and receiving bus has a traveling time, τ (subscript k and m denotes the sending and the receiving-end, respectively). In this study, each Bergeron transmission line construct refer to [10]. There is transient simulation for transmission lines based on the concept of travelling waves by using state equation. It is driven by a bus voltage assumed as a voltage source (v_s). Its state variables for [10] use current pass through a series inductor L connect with source and voltage across parallel RC load as shown in figure 2.5.

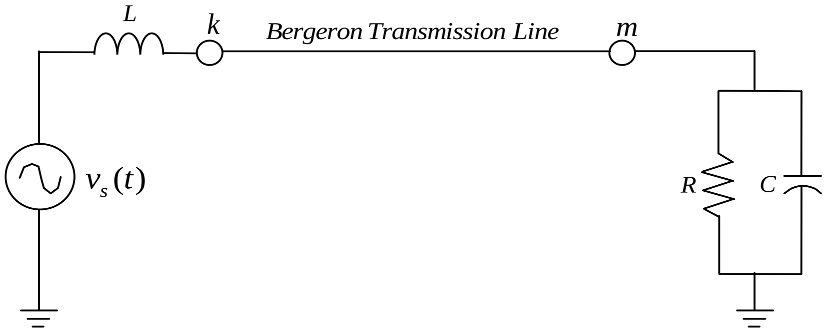

Figure 2.5 Single line view with termination [10].

The Bergeron model is used for transmission line which sufficiently long and yield the travelling wave theory models representation. The set of differential equations can express by choosing the inductor current and capacitor voltage as the state variables. The state equations for single-phase are shown below [10].

$$\begin{bmatrix} \dfrac{di_k}{dt} \\ \dfrac{dv_m}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{Z}{L} & 0 \\ 0 & -\dfrac{1}{C}\left(\dfrac{Z+R}{ZR}\right) \end{bmatrix} \begin{bmatrix} i_k \\ v_m \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} & \dfrac{Z}{L} & 0 \\ 0 & 0 & -\dfrac{1}{C} \end{bmatrix} \begin{bmatrix} v_s(t) \\ I_k(t-\tau) \\ I_m(t-\tau) \end{bmatrix}. \tag{2.21}$$

The dependent variable equations or output equations to obtain the sending-end voltages and the receiving-end currents are written as

$$\begin{bmatrix} v_k \\ i_m \end{bmatrix} = \begin{bmatrix} Z & 0 \\ 0 & \dfrac{1}{Z} \end{bmatrix} \begin{bmatrix} i_k \\ v_m \end{bmatrix} + \begin{bmatrix} 0 & -Z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_s(t) \\ I_k(t-\tau) \\ I_m(t-\tau) \end{bmatrix}, \tag{2.22}$$

where $Z = Z_C + rl/4$ and i_k , i_m represent the sending-end and the receiving-end current, and v_k, , v_m represent the sending-end and the receiving-end busbar voltages, respectively. The current sources that represent the past history terms are determined as [10, 20-21]

$$\begin{aligned} I_k(t-\tau) = {} & \frac{-Z_C}{(Z_C + R'/4)^2}[v_m(t-\tau) + (Z_C - R'/4)i_m(t-\tau)] \\ & + \frac{-R'/4}{(Z_C + R'/4)^2}[v_k(t-\tau) + (Z_C - R'/4)i_k(t-\tau)], \end{aligned} \tag{2.23}$$

$$I_m(t-\tau) = \frac{-Z_C}{(Z_C + R'/4)^2}[v_k(t-\tau) + (Z_C - R'/4)i_k(t-\tau)] + \frac{-R'/4}{(Z_C + R'/4)^2}[v_m(t-\tau) + (Z_C - R'/4)i_m(t-\tau)] \quad (2.24)$$

2.5 Transient State Estimation

State estimation techniques are used to determine the state values at unmonitored location. The complete system values including partial measurements value and the estimated value are used to develope power quality of system.

The state estimation techniques are extended from steady-state to transient phenomena, call transient state estimation. The frequently occurring transient phenomena are voltage sag which is one of the most concerned power quality issues refer to IEEE Standard 1159-2009 [22]. Voltage sag is a sudden decrease in Root-Mean-Square (RMS) voltage for durations from 0.5 cycles to 1 minute which can be categoried to three subgroups as shown in table 2.1. Voltage sag defines the magnitue voltage drop between 0.1 to 0.9 pu , reported as the remaining voltage.

Table 2.1 Categories and characteristics of power systems electromagnetic phenomena [22].

Categories	Typical Duration	Typical Magnitude
Instantaneous		
Sag	0.5-30 cycles	0.1-0.9 pu
Swell	0.5-30 cycles	1.1-1.8 pu
Momentary		
Interruption	0.5-3 seconds	< 0.1 pu
Sag	0.5-3 seconds	0.1-0.9 pu
Swell	0.5-3 seconds	1.1-1.8 pu
Temporary		
Interruption	3 sec-1 minute	< 0.1 pu
Sag	3 sec-1 minute	0.1-0.9 pu
Swell	3 sec-1 minute	1.1-1.8 pu

The transient phenomena that occur in power system can be determined by state estimation technique. It combines the component and network equation to state equation form. The state equation (repeated here) use the function to relate the system state vector, x with the set of measurement vector, z can be written as [1-2,4-5].

$$z = [H]x + \varepsilon , \tag{2.25}$$

where $[H]$ is the measurement matrix and ε is the error vector. The previous state formulation of the Bergeron transmission line in equation (2.21) that included operator d/dt can be approximated and rewritten in terms of the previous system state at $(t - \Delta t)$ as [4]

$$\left.\begin{aligned} \frac{di(t)}{dt} &\approx \frac{i(t) - i(t - \Delta t)}{\Delta t}, \\ \frac{dv(t)}{dt} &\approx \frac{v(t) - v(t - \Delta t)}{\Delta t}. \end{aligned}\right\} \tag{2.26}$$

In this study, the three-phase transmission line uses modal transformation technique which requires eigenvalue analysis [10] to transform coupled equations to decoupled equations. The effect of coupling between phases can be eliminated. The three-phase voltage and current can be calculated as three independent modes and each mode can be treated as a single-phase transmission line [21, 23-24]. Phase to modal transformation (subscript *phase* and *mode*, respectively) can be described as [24]

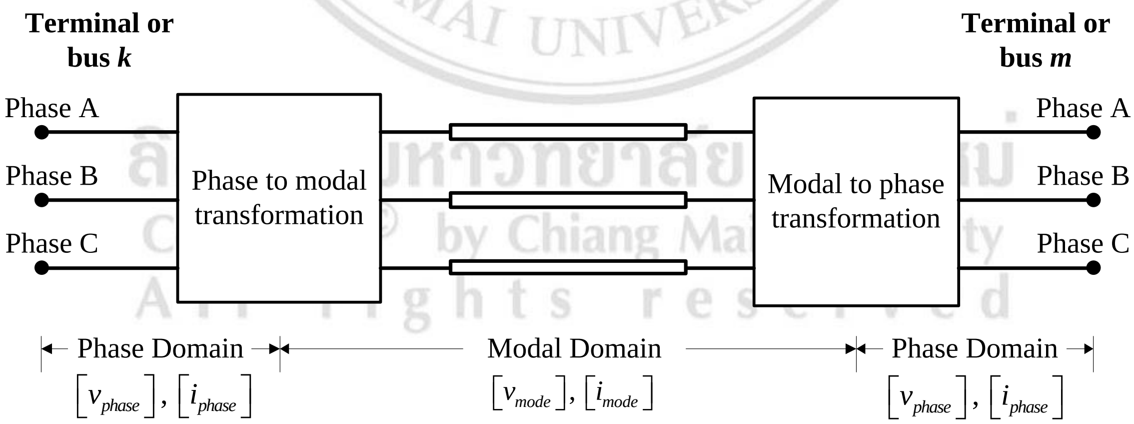

Figure 2.6 Transformation between phase and modal domain on three-phase line.

$$\left.\begin{aligned} [v_{mode}] &= [T]^{-1}[v_{phase}], \\ [i_{mode}] &= [T]^{-1}[i_{phase}]. \end{aligned}\right\} \tag{2.27}$$

For modal to phase transformation are determined as

$$\left.\begin{aligned}\left[v_{phase}\right]&=\left[T\right]\left[v_{mode}\right],\\ \left[i_{phase}\right]&=\left[T\right]\left[i_{mode}\right].\end{aligned}\right\} \tag{2.28}$$

where $\left[v_{mode}\right]$, $\left[i_{mode}\right]$ denote the voltage and current in modal domain, and $\left[v_{phase}\right]$, $\left[i_{phase}\right]$ denote the voltage and current in phase domain. Let $\left[T\right]$ is modal transformation matrix for voltage and current value and $\left[T\right]^{-1}$ is the inverse matrix of $\left[T\right]$. The modal transformation is not unique, for a transposed three-phase line, this can be determined as [24]

$$\left[T\right]=\begin{bmatrix}1 & 1 & 0\\ 1 & 0 & 1\\ 1 & -1 & -1\end{bmatrix}, \tag{2.29}$$

$$\left[T\right]^{-1}=\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 2 & -1 & -1\\ -1 & 2 & -1\end{bmatrix}. \tag{2.30}$$

Therefore, the Bergeron transmission line can be constructed as a modal domain in the state space equation. The equivalent circuit for the positive sequence network is shown in figure 2.7.

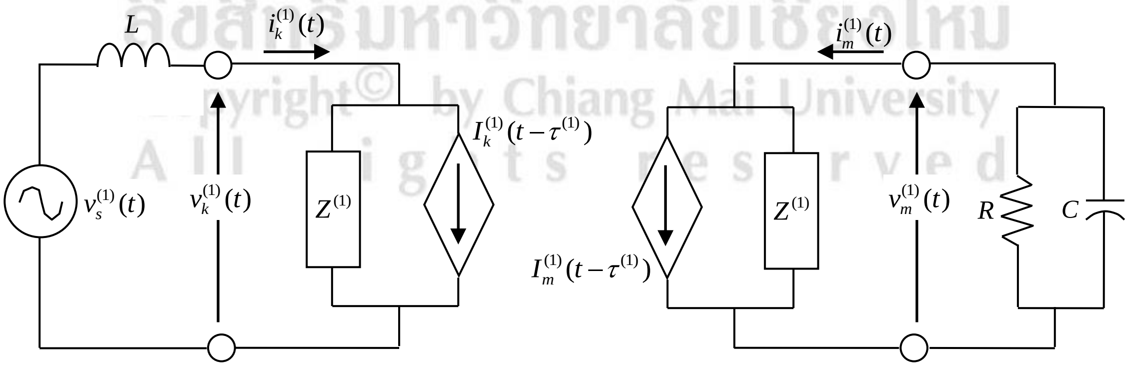

Figure 2.7 Equivalent circuit model for the positive sequence transmission line.

Consider the equivalent circuit for the positive sequence shown in figure 2.7. The equation (2.21) and (2.22) can reform to positive sequence in modal domain.

$$\begin{bmatrix} \dfrac{di_k^{(1)}}{dt} \\ \dfrac{dv_m^{(1)}}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{Z^{(1)}}{L} & 0 \\ 0 & -\dfrac{1}{C}\left(\dfrac{Z^{(1)}+R}{Z^{(1)}R}\right) \end{bmatrix} \begin{bmatrix} i_k^{(1)} \\ v_m^{(1)} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} & \dfrac{Z^{(1)}}{L} & 0 \\ 0 & 0 & -\dfrac{1}{C} \end{bmatrix} \begin{bmatrix} v_s^{(1)}(t) \\ I_k^{(1)}\left(t-\tau^{(1)}\right) \\ I_m^{(1)}\left(t-\tau^{(1)}\right) \end{bmatrix} \tag{2.31}$$

$$\begin{bmatrix} v_k^{(1)} \\ i_m^{(1)} \end{bmatrix} = \begin{bmatrix} Z^{(1)} & 0 \\ 0 & \dfrac{1}{Z^{(1)}} \end{bmatrix} \begin{bmatrix} i_k^{(1)} \\ v_m^{(1)} \end{bmatrix} + \begin{bmatrix} 0 & -Z^{(1)} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_s^{(1)}(t) \\ I_k^{(1)}\left(t-\tau^{(1)}\right) \\ I_m^{(1)}\left(t-\tau^{(1)}\right) \end{bmatrix} \tag{2.32}$$

where $Z^{(1)} = Z_C^{(1)} + r^{(1)} l / 4$, $Z_C^{(1)} = \sqrt{\dfrac{\ell^{(1)}}{c^{(1)}}}$ and $\tau^{(1)} = l\sqrt{\ell^{(1)} c^{(1)}}$.

Similar expressions are derived for the other sequence network. The equations shown in (2.31) can be represented in terms of the previous time from equation (2.26) in the modal domain as the result in equation (2.33).

$$\begin{bmatrix} i_k^{(0)} \\ i_k^{(1)} \\ i_k^{(2)} \\ v_m^{(0)} \\ v_m^{(1)} \\ v_m^{(2)} \end{bmatrix}_{t-\Delta t} = \begin{bmatrix} i_k^{(0)} \\ i_k^{(1)} \\ i_k^{(2)} \\ v_m^{(0)} \\ v_m^{(1)} \\ v_m^{(2)} \end{bmatrix}_{t} - \Delta t \begin{bmatrix} -\frac{Z^{(0)}}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{Z^{(1)}}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{Z^{(2)}}{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{C}\left(\frac{Z^{(0)}+R}{Z^{(0)}R}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{C}\left(\frac{Z^{(1)}+R}{Z^{(1)}R}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{C}\left(\frac{Z^{(2)}+R}{Z^{(2)}R}\right) \end{bmatrix} \begin{bmatrix} i_k^{(0)} \\ i_k^{(1)} \\ i_k^{(2)} \\ v_m^{(0)} \\ v_m^{(1)} \\ v_m^{(2)} \end{bmatrix}_{t}$$

$$- \Delta t \begin{bmatrix} \frac{1}{L} & 0 & 0 & \frac{Z^{(0)}}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{L} & 0 & 0 & \frac{Z^{(1)}}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{L} & 0 & 0 & \frac{Z^{(2)}}{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{C} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{C} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{C} \end{bmatrix} \begin{bmatrix} v_s^{(0)}(t) \\ v_s^{(1)}(t) \\ v_s^{(2)}(t) \\ I_k^{(0)}\left(t-\tau^{(0)}\right) \\ I_k^{(1)}\left(t-\tau^{(1)}\right) \\ I_k^{(2)}\left(t-\tau^{(2)}\right) \\ I_m^{(0)}\left(t-\tau^{(0)}\right) \\ I_m^{(1)}\left(t-\tau^{(1)}\right) \\ I_m^{(2)}\left(t-\tau^{(2)}\right) \end{bmatrix}. \qquad \text{.... (2.33)}$$

The superscript (0), (1), and (2) denote zero, positive and negative sequence parameters, respectively. Note that a series inductance in each sequence is assigned as $L^{(0)} = L^{(1)} = L^{(2)} = L$. Moreover, resistance and capacitance load for each sequence assign as $R^{(0)} = R^{(1)} = R^{(2)} = R$ and $C^{(0)} = C^{(1)} = C^{(2)} = C$, respectively. The output equations to obtain the sending-end voltages and receiving-end current in modal domain are calculated by equation (2.34).

$$\begin{bmatrix} v_k^{(0)} \\ v_k^{(1)} \\ v_k^{(2)} \\ i_m^{(0)} \\ i_m^{(1)} \\ i_m^{(2)} \end{bmatrix} = \begin{bmatrix} Z^{(0)} & 0 & 0 & 0 & 0 & 0 \\ 0 & Z^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z^{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{Z^{(0)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{Z^{(1)}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{Z^{(2)}} \end{bmatrix} \begin{bmatrix} i_k^{(0)} \\ i_k^{(1)} \\ I_k^{(2)} \\ v_m^{(0)} \\ v_m^{(1)} \\ v_m^{(2)} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & -Z^{(0)} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Z^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Z^{(2)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_s^{(0)}(t) \\ v_s^{(1)}(t) \\ v_s^{(2)}(t) \\ I_k^{(0)}\left(t-\tau^{(0)}\right) \\ I_k^{(1)}\left(t-\tau^{(1)}\right) \\ I_k^{(2)}\left(t-\tau^{(2)}\right) \\ I_m^{(0)}\left(t-\tau^{(0)}\right) \\ I_m^{(1)}\left(t-\tau^{(1)}\right) \\ I_m^{(2)}\left(t-\tau^{(2)}\right) \end{bmatrix}. \tag{2.34}$$

The past history current source for each sequence can be calculated using equation (2.23) and (2.24) which show in positive sequence as equation (2.35) and (2.36).

$$\begin{aligned} I_k^{(1)}(t-\tau^{(1)}) &= \frac{-Z_C^{(1)}}{(Z_C^{(1)}+R'^{(1)}/4)^2}[v_m^{(1)}(t-\tau^{(1)})+(Z_C^{(1)}-R'^{(1)}/4)i_m^{(1)}(t-\tau^{(1)})] \\ &+ \frac{-R'^{(1)}/4}{(Z_C^{(1)}+R'^{(1)}/4)^2}[v_k^{(1)}(t-\tau^{(1)})+(Z_C^{(1)}-R'^{(1)}/4)i_k^{(1)}(t-\tau^{(1)})], \end{aligned} \tag{2.35}$$

$$\begin{aligned} I_m^{(1)}(t-\tau^{(1)}) &= \frac{-Z_C^{(1)}}{(Z_C^{(1)}+R'^{(1)}/4)^2}[v_k^{(1)}(t-\tau^{(1)})+(Z_C^{(1)}-R'^{(1)}/4)i_k^{(1)}(t-\tau^{(1)})] \\ &+ \frac{-R'^{(1)}/4}{(Z_C^{(1)}+R'^{(1)}/4)^2}[v_m^{(1)}(t-\tau^{(1)})+(Z_C^{(1)}-R'^{(1)}/4)i_m^{(1)}(t-\tau^{(1)})], \end{aligned} \tag{2.36}$$

where $Z_C^{(1)} = \sqrt{\dfrac{\ell^{(1)}}{c^{(1)}}}$, $R'^{(1)} = r^{(1)} l$ and $\tau^{(1)} = l\sqrt{\ell^{(1)} c^{(1)}}$.

Equations (2.33) to (2.36) are formed to solve state estimation for each Bergeron transmission line. The power system is set as a group of Bergeron transmission line. The group of state equation and output equation are reformed for all Bergeron transmission line in the network.

The proposed algorithm uses the sending-end current and receiving-end voltage as the state variables [10]. Rows of measurement equations associated with the selected measurement location (value of both previous time-step, $z_{t-\Delta t}$ and present time-step, z_t) are added to the measurement matrix [*H*]. At each time step, measurements are updated by applying an estimated result, $\hat{x}$ to virtual measurements, $z_{t-\Delta t}$ for the next time step calculation. These can form a new set of measurement equations. From equation (2.6) the weighted least squares formulation which the covariance of the measurements assumed to be an identity matrix (collecting data from the same instrumentation) is used to describe the measurement system as [4-5]

$$\hat{x} = \left([H]^T [H] \right)^{-1} [H]^T z \tag{2.37}$$

After the equations are solved to obtain the sending-end currents and receiving-end voltages in the modal domain, the output variables of sending-end voltages and receiving-end currents are determined using equation (2.34) and history term current in equation (2.35) and (2.36). Finally, the current and voltage in the phase domain can be obtained by the modal to phase domain transformation in equation (2.28) and (2.29).

The flowchart of TSE with the modal transformation is presented in figure 2.8.

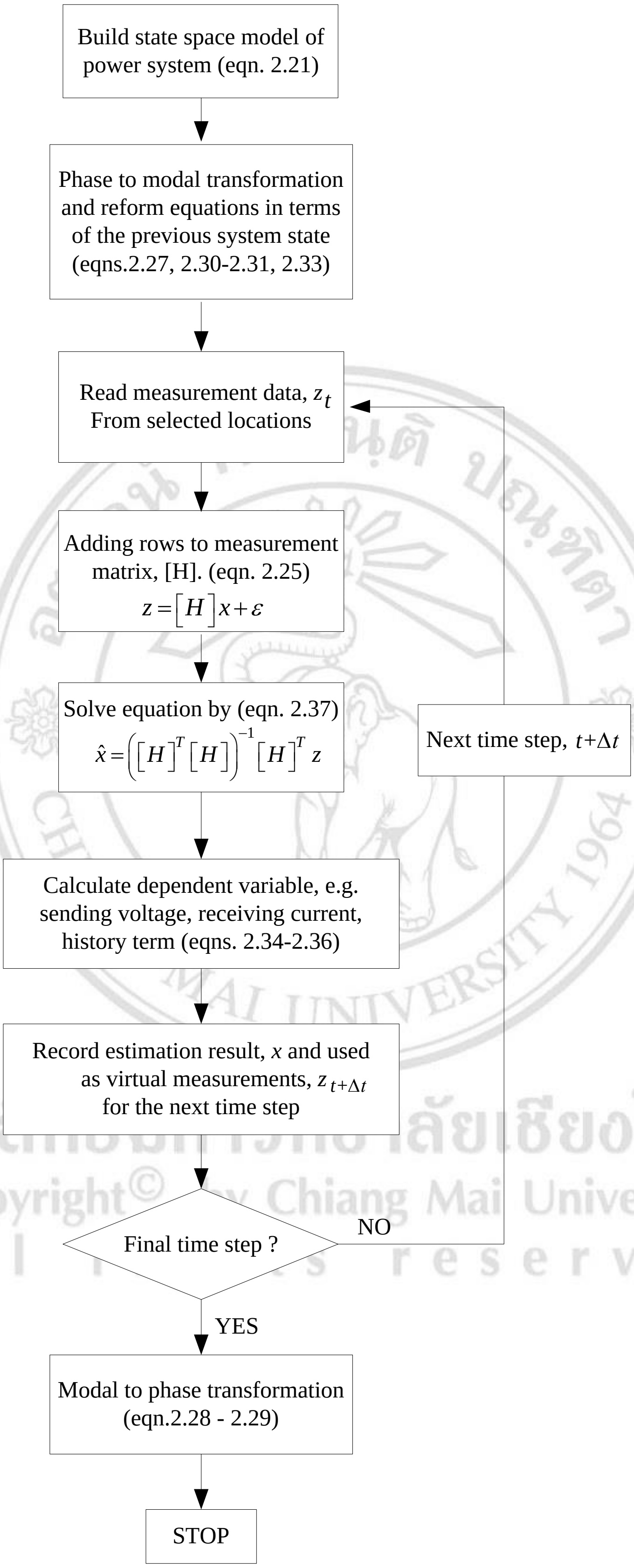

Figure 2.8 Flowchart of proposed transient state estimation.

Algorithms for predicting a parameter, signal, and estimate state value are used in many fields. Its performance needs to be evaluated in practice to serve a number of purposes, such as represents of its validity, demonstration of its performance, and comparison with other method or other estimators.

The widely used performance tool for the developed model included the percentage Root Mean Squared Error (%RMSE) and the percentage Mean Absolute Error (%MAE) calculated by [25-26]

$$\%RMSE = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}}{peak\ voltage\ at\ nominal} \times 100 , \qquad (2.38)$$

$$\%MAE = \frac{\frac{1}{n}\sum_{i=1}^{n}\left|x_i - \hat{x}_i\right|}{peak\ voltage\ at\ nominal} \times 100 , \qquad (2.39)$$

where x_i is actual value, $\hat{x}_i$ is estimate value, *n* is number of value and *peak voltage at nominal* means the peak voltage value while there is no fault occur.

Both performance evaluation methods are applied with the actual values and the estimated values at bus location without voltage measurement.

2.6 Measurement Noise

State estimation process is an important technique for monitoring the behavior of power system. It involves collecting the data set from phasor measurement units (PMU) consistent with the state variables from estimator. If the measurements data are collected with large errors, then the estimated state may be incorrect and degrades the quality of estimator. These bad data can result from erroneous measurement data, incorrect system parameters, or incorrect network topology. Cause of problem may be occurring at transducer that wired incorrectly or malfunctioning in the transducer itself so that it gives incorrect data affect to accurate readings. Erroneous measurement has been the main focus of bad data analysis. This can be classified into three parts: extreme errors, gross errors and normal measurement noise. The problem of bad data can be overcome by detection, identification and removal of bad data. This procedure usually requires the system to be over-determined (the system has redundancy measurements). A practical method for bad data analysis should have at least two features. The first one corresponds to having a good identification performance. The other feature is related with the computational efficiency. However, this study focuses on only the effect of measurement noise to estimation performance.

Generally, the state estimation is particularly useful to filter out measurement noise but still affect to the performance of algorithms. From equation (2.1), the measurement error vector is the difference between the measurements value from their true values cause by the presence of measurement noise or bad data.

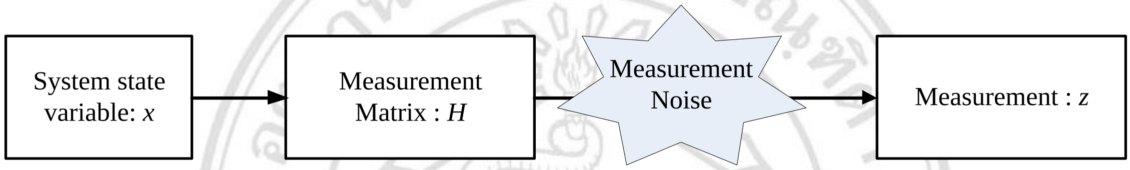

Figure 2.9 Measurement noise added to measurement data.

Measurement noise considered is normal (or Gaussian) distribution and added to all of measurement data for evaluating the proposed algorithm as shown in figure 2.9.

2.7 Nonlinear Equipment

In case of ideal transformer model, simple transformer model is represented using coupled coil model. This method can be extended to three-phase transformer by neglect the interphase mutual coupling and then consider three-phase equations independently. Many transformer models have been developed continuously. There are nonlinear components in some models use to consider the magnetic core saturation characteristics.

For PSCAD simulation program, saturation can be represented in the classical transformer models combine with a compensating current source injection across the selected winding as shown in figure 2.10.

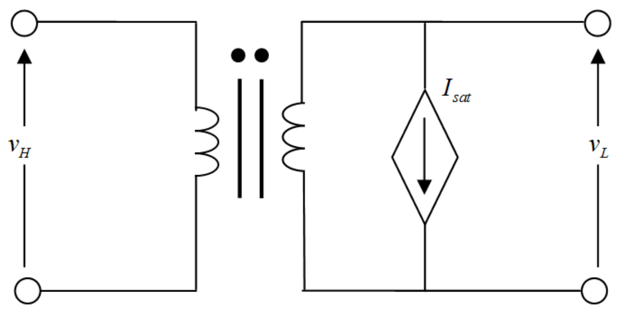

Figure 2.10 Compensating current sources for core saturation

There are three input parameters which are air core reactance, knee voltage and magnetizing current for adjust the core saturation characteristic of transformer as show in figure 2.11.

Air core reactance: This parameter adjusts the slope of the asymptotic line as dashed line in figure 2.11.

Knee voltage: The knee voltage input adjusts vertically shifts the Y-intercept of the air core reactance asymptotic line in the figure 2.11.

Magnetizing current: This parameter determines the horizontal position along the V = 1.0 p.u. voltage line, of the effective knee point. It affects the shape of curve. For instance, if magnetizing current is increase, the curve of the saturation characteristic will become less sharp.

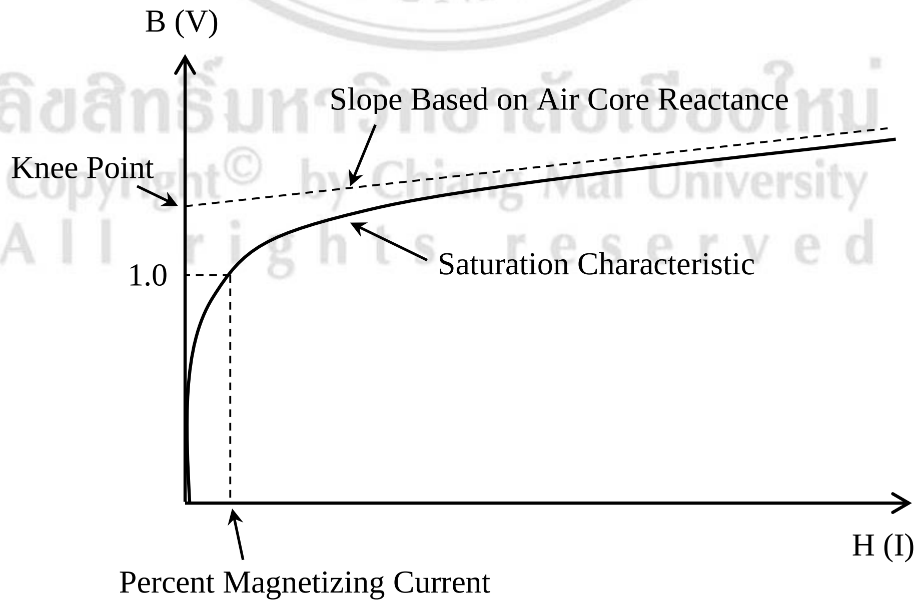

Figure 2.11 Magnetization curves of saturation properties.

The saturation of magnetic core in power transformer is nonlinear characteristic which degrade the effectiveness of the transformer. According to the relation between magnetic flux density (B) and magnetic field intensity (H) are tend to nonlinear curve which call B-H curve. In the saturation behavior, magnetic flux is produced by the magnetizing current. The magnetizing current will be slightly increasing when the amplitude of the voltage (or flux) is large enough to reach the nonlinear region of the B-H curve. Similarly, the induced voltage in secondary winding of transformer will no longer match the sinusoidal wave with primary winding. In other words, the saturation of magnetic core in transformer will distort the wave shape from primary to secondary side, and also create harmonics problem in the output of secondary side.

The nonlinear phenomenon of magnetic saturation affects the steady state and the transient behaviour of transformer. Therefore, it is necessary to consider the nonlinear characteristic with state estimation of the proposed algorithm.