CHAPTER 1

Introduction

Let X be a nonempty set. Let $T(X)$ denote the set of all transformations from X into itself. Then $T(X)$ is a semigroup under the composition of maps and it is called the full transformation semigroup on X.

The semigroup we consider is $S(X,Y)$ (under the composition) consists of all mapping in $T(X)$ which leave $Y \subseteq X$ invariant. K. D. Magill [7] introduced and studied the semigroup $S(X,Y)$ in 1966. In fact, if $Y = X$, then $S(X,Y) = T(X)$. So we may regard $S(X,Y)$ as a generalization of $T(X)$. It is known that $T(X)$ is a regular semigroup, that is, for every $\alpha \in T(X)$, $\alpha = \alpha\beta\alpha$ for some $\beta \in T(X)$. Furthermore, we recall that: an element $\alpha \in S(X,Y)$ is left regular [right regular] if $\alpha = \beta\alpha^2$ [$\alpha = \alpha^2\beta$] for some $\beta \in S(X,Y)$ and it is intra-regular if $\alpha = \lambda\alpha^2\mu$ for some $\lambda, \mu \in S(X,Y)$.

In 2005, S. Nenthein, P. Youngkhong and Y. Kemprasit [8] showed that $S(X,Y)$ is a regular semigroup if and only if $X = Y$ or Y contains exactly one element, and $Reg\ S(X,Y) = \{\alpha \in S(X,Y) : X\alpha \cap Y = Y\alpha\}$ is the set of all regular elements of $S(X,Y)$. Moreover, they counted the numbers of regular elements of $S(X,Y)$ when X is a finite set. The numbers are given in terms of $|X|$ and $|Y|$.

As far back in 2011, P. Honyam and J. Sanwong [4] characterized when $S(X,Y)$ is isomorphic to $T(Z)$ for some set Z and prove that every semigroup A can be embedded in $S(A^1, A)$. Then they described Green's relations and ideals on $S(X,Y)$ and applied these results to obtain its group $\mathcal{H}$-classes and ideals.

Later in 2013, W. Choomanee, P. Honyam and J. Sanwong [1] studied left regular, right regular and intra-regular elements of $S(X,Y)$ and consider the relationships between these elements. Moreover, they counted the number of left regular elements of $S(X,Y)$ when X is a finite set.

The purpose of this research are:

1. To prove the existence and uniqueness of maximal and minimal ideals of $S(X,Y)$.
2. To present maximal and minimal congruences on $S(X,Y)$ when X is a finite set.

We devide this thesis into three chapters. Chapter 1 is an introduction to the research problems. Chapter 2 deals with some preliminaries and some useful results those will be used in later chapter. Chapter 3 is the main results of this research work consisting of the following three sections:

1. Some properties of $K(Z)$.
2. The lattice of ideals of $S(X,Y)$.
3. Minimal and maximal congruences on $S(X,Y)$.