CHAPTER 4

Conclusion

This thesis has presented new results on $S(X,Y)$, a semigroup of transformations on X which leave a subset Y of X invariant. We studied the relationships between $K(Z)$ and $S(r,s,t)$, we proved that if $\beta \in S(X,Y)$ with $|X\beta| = r$, $|Y\beta| = s$ and $|X\beta \backslash Y| = t$, then $K(\beta) = S(r+1, s+1, t+1)$. In addition, we have $K(\beta_1, \beta_2, ..., \beta_n) = K(\beta_1) \cup K(\beta_2) \cup ... \cup K(\beta_n)$ for all $\beta_1, \beta_2, ..., \beta_n \in S(X,Y)$. So we see that the ideals of $S(X,Y)$ are precisely the set $K(\alpha_1) \cup ... \cup K(\alpha_n)$, where $\alpha_1, ..., \alpha_n \in S(X,Y)$.

We determined maximal and minimal ideals of $S(X,Y)$ and proved that if $|Y| = 1$, then $J_1 \cup J_{2,1,1}$ is the unique minimal ideal of $S(X,Y)$, and showed that if $|Y| > 1$, then $J_1 \cup J_{2,1,0}$ is the unique minimal ideal of $S(X,Y)$. We also showed that if S is a semigroup with identity 1 and contains a proper ideal, then S has a maximal ideal. Moreover, we proved that if S is a semigroup with identity 1 and S has a maximal ideal, then it is unique. Further, we proved that if $|X| > 1$, then $S(X,Y) \setminus G(X,Y)$ is the unique maximal ideal of $S(X,Y)$.

In addition, we studied the maximal and minimal congruences on $S(X,Y)$ when X is a finite set. We proved that $\rho = (S \setminus G \times S \setminus G) \cup (G \times G)$ is a maximal congruence on $S(X,Y)$. We also showed that the Rees congruence $\rho_{J_1 \cup J_2}$ is a minimal congruence on $S(X,Y)$ if $|X| = 2, |Y| = 1$ or $|X| > 2, |Y| = 1$. Moreover, if $|Y| > 2$, then the Rees congruence ρ_{J_1} is a minimal congruence on $S(X,Y)$.