CHAPTER 3
Robust stability criteria for uncertain neutral systems with time-varying delays and nonlinear perturbations

In this chapter, we consider the robust stability criteria for uncertain neutral systems with interval time-varying state delay, bounded differential neutral delay and time-varying nonlinear perturbations. The constraint on the derivative of time-varying state delay is not required which allows the state delay to be a fast time-varying function. Based on the Lyapunov-Krasovskii stability theory, we derive new delay-dependent stability conditions in terms of LMIs.

We consider the following neutral system with time-varying delays:

$$\begin{aligned}
\dot{x}(t) - C\dot{x}(t-d(t)) &= A(t)x(t) + B(t)x(t-\tau(t)) + D_1(t)f_1(t,x(t)) \\
&\quad + D_2(t)f_2(t,x(t-\tau(t))) \\
x(t_0+\theta) &= \phi(\theta), \quad \theta \in [-h, 0],
\end{aligned} \tag{3.1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $d(t)$ is a neutral delay, $\tau(t)$ is a time-varying continuous function which satisfies

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M, \quad 0 \leq d(t) \leq d, \quad \dot{d}(t) \leq \delta, \tag{3.2}$$

where $\tau_m, \tau_M, d, \delta$ are constants and $h = \max\{d, \tau_M\}$, the initial condition function $\phi(t)$ denotes a continuous vector-valued initial function of $t \in [-h, 0]$, $f_1(t, x(t))$ and $f_2(t, x(t-\tau(t)))$ are unknown nonlinear perturbations satisfying $f_1(t, 0) = 0$, $f_2(t, 0) = 0$ and

$$\begin{aligned}
&f_1^T(t,x(t))f_1(t,x(t)) \leq \alpha^2 x^T(t)x(t), \\
&f_2^T(t,x(t-\tau(t)))f_2(t,x(t-\tau(t))) \leq \beta^2 x^T(t-\tau(t))x(t-\tau(t)),
\end{aligned} \tag{3.3}$$

where α and β are positive real numbers. The uncertain matrices $A(t)$, $B(t)$, $D_1(t)$ and $D_2(t)$ satisfy

$$\begin{aligned}
&A(t) = A + \Delta A(t), \quad B(t) = B + \Delta B(t), \\
&D_1(t) = D_1 + \Delta D_1(t), \quad D_2(t) = D_2 + \Delta D_2(t),
\end{aligned}$$

where $A,\ B,\ D_1,\ D_2 \in \mathcal{R}^{n\times n}$ are constant matrices with appropriate dimension and $\Delta A(t)$, $\Delta B(t)$, $\Delta D_1(t)$ and $\Delta D_2(t)$ are unknown real matrices of appropriate dimensions representing the systems time-varying parameter uncertainties which satisfy

$$\begin{aligned} &\Delta A(t) = G_1 F(t) E_A, \quad \Delta B(t) = G_2 F(t) E_B, \\ &\Delta D_1(t) = G_3 F(t) E_{D_1}, \quad \Delta D_2(t) = G_4 F(t) E_{D_2}, \end{aligned} \tag{3.4}$$

where G_1, G_2, G_3, G_4, E_A, E_B, E_{D_1} and E_{D_2} are known real constant matrices of appropriate dimension. $F(t)$ is unknown time-varying matrix satisfying

$$F^T(t)F(t) \leq I. \tag{3.5}$$

For simplicity, we denote $f_1(t, x(t))$, $f_2(t, x(t-\tau(t)))$, by f_1, f_2, respectively.

Let $\tau_e = \frac{1}{2}(\tau_M + \tau_m)$ and $\rho = \frac{1}{2}(\tau_M - \tau_m)$. Then $\tau(t)$ can be expressed as

$$\tau(t) = \tau_e + \rho\xi(t), \tag{3.6}$$

where

$$\xi(t) = \begin{cases} \frac{2\tau(t)-(\tau_M+\tau_m)}{\tau_M-\tau_m}, & \tau_M > \tau_m, \\ 0, & \tau_M = \tau_m. \end{cases}$$

Obviously $|\xi(t)| \leq 1$ for this case, $\tau(t)$ is a function belonging to the interval $[\tau_e - \rho, \tau_e + \rho]$, where ρ can be taken as the range of variation of the time-varying delay $\tau(t)$. Using the fact that

$$x(t-\tau_e) - x(t-\tau(t)) = \int_{t-\tau(t)}^{t-\tau_e} \dot{x}(s)ds$$

system (3.1) can be rewritten as

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t-d(t)) \ &= \ A(t)x(t) + B(t)x(t-\tau_e) - B(t)\int_{t-\tau(t)}^{t-\tau_e} \dot{x}(s)ds \\ &\quad + D_1(t)f_1 + D_2(t)f_2. \end{aligned} \tag{3.7}$$

3.1 Stability criteria for neutral systems

Now we present a new delay-dependent condition for the stability of system (3.7).

Assumption 3.1.1 *All the eigenvalues of matrix C are inside the unit circle.*

First, we consider the system without uncertainty, namely $\Delta A(t) = \Delta B(t) = \Delta D_1(t) = \Delta D_2(t) = 0$. Then we have the stability (3.7) criteria as the following.

Theorem 3.1.2 *Under Assumption 3.1.1, nominal system of (3.7) with time-varying delays satisfying (3.2) is asymptotically stable if there exist positive definite matrices $P,\ Q,\ Q_1,\ R,\ S,\ W$, matrices $K_1, K_2, L_i,\ M_i,\ i = 1, 2, ..., 7$ of appropriate dimensions and $\delta_1,\ \delta_2 > 0$ such that*

$$\Sigma_1 = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \tau_e L_1^T & \rho M_1 \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \tau_e L_2^T & \rho M_2 \\ * & * & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \tau_e L_3^T & \rho M_3 \\ * & * & * & \phi_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \tau_e L_4^T & \rho(K_1^T B + M_4) \\ * & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} & \tau_e L_5^T & \rho(K_2^T B + M_5) \\ * & * & * & * & * & \phi_{66} & 0 & \tau_e L_6^T & \rho M_6 \\ * & * & * & * & * & * & \phi_{77} & \tau_e L_7^T & \rho M_7 \\ * & * & * & * & * & * & * & -\tau_e R & 0 \\ * & * & * & * & * & * & * & * & -\rho S \end{bmatrix} < 0, \quad (3.8)$$

where

$$\begin{aligned}
\phi_{11} &= Q + L_1 + L_1^T + \varepsilon_1 \alpha^2 I, \\
\phi_{12} &= M_1^T + L_2, \\
\phi_{13} &= -L_1^T + L_3 + M_1, \\
\phi_{14} &= P + A^T K_1 + L_4, \\
\phi_{15} &= A^T K_2 + L_5, \\
\phi_{16} &= L_6, \\
\phi_{17} &= L_7, \\
\phi_{22} &= M_2^T + M_2 + \varepsilon_2 \beta^2 I - W, \\
\phi_{23} &= -L_2^T - M_2^T + M_3 + W, \\
\phi_{24} &= M_4, \\
\phi_{25} &= M_5, \\
\phi_{26} &= M_6, \\
\phi_{27} &= M_7, \\
\phi_{33} &= -Q - L_3^T - L_3 - M_3^T - M_3 - W,
\end{aligned}$$

$$
\begin{aligned}
\phi_{34} &= B^T K_1 - L_4 - M_4, \\
\phi_{35} &= B^T K_2 - L_5 - M_5, \\
\phi_{36} &= -L_6 - M_6, \\
\phi_{37} &= -L_7 - M_7, \\
\phi_{44} &= Q_1 + \tau_e R + \rho S - K_1^T - K_1 + \rho^2 W, \\
\phi_{45} &= K_1^T C - K_2, \\
\phi_{46} &= K_1^T D_1, \\
\phi_{47} &= K_1^T D_2, \\
\phi_{55} &= -(1-\delta) Q_1 + K_2^T C + C^T K_2, \\
\phi_{56} &= K_2^T D_1, \\
\phi_{57} &= K_2^T D_2, \\
\phi_{66} &= -\delta_1 I, \\
\phi_{77} &= -\delta_2 I.
\end{aligned}
$$

Proof. We prove the Theorem 3.1.2 is true for three cases, namely, $\tau_m \leq \tau(t) < \tau_e$; $\tau(t) = \tau_e$; and $\tau_e < \tau(t) \leq \tau_M$.

Case I: $\tau_m \leq \tau(t) < \tau_e$. Choose a Lyapunov-Krasovskii functional candidate as

$$
\begin{aligned}
V_1(x_t) &= x^T(t) P x(t) + \int_{t-\tau_e}^{t} x^T(s) Q x(s) ds + \int_{t-d(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds \\
&\quad \int_{-\tau_e}^{0} \int_{t+s}^{t} \dot{x}^T(\theta) R \dot{x}(\theta) d\theta ds + \int_{-\tau_e}^{-\tau_m} \int_{t+s}^{t} \dot{x}^T(\theta) S \dot{x}(\theta) d\theta ds \\
&\quad + \rho \int_{-\tau_e}^{-\tau_m} \int_{t+s}^{t} \dot{x}^T(\theta) W \dot{x}(\theta) d\theta ds, \qquad (3.9)
\end{aligned}
$$

where P, Q, Q_1, R, S and W are positive definite matrices. Taking the derivative of $V_1(x_t)$ with respect to t along the trajectory of (3.7) yields

$$
\begin{aligned}
\dot{V}_1(x_t) &= 2x^T(t) P \dot{x}(t) + x^T(t) Q x(t) - x^T(t-\tau_e) Q x(t-\tau_e) \\
&\quad + \dot{x}^T(t) Q_1 \dot{x}(t) - (1-\dot{d}(t)) \dot{x}^T(t-d(t)) Q_1 \dot{x}^T(t-d(t)) \\
&\quad - \int_{t-\tau_e}^{t} \dot{x}^T(s) R \dot{x}(s) ds - \int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s) S \dot{x}(s) ds \\
&\quad - \rho \int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s) W \dot{x}(s) ds + \dot{x}^T(t)(\tau_e R + \rho^2 W + \rho S) \dot{x}(t),
\end{aligned}
$$

$$
\begin{aligned}
\leq\ & 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau_e)Qx(t-\tau_e) \\
& + \dot{x}^T(t)Q_1\dot{x}(t) - (1-\delta)\dot{x}^T(t-d(t))Q_1\dot{x}^T(t-d(t)) \\
& - \int_{t-\tau_e}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)S\dot{x}(s)ds \\
& + \dot{x}^T(t)(\tau_e R + \rho^2 W + \rho S)\dot{x}(t) - x^T(t-\tau(t))Wx(t-\tau(t)), \\
& - x^T(t-\tau_e)Wx(t-\tau_e) + 2x^T(t-\tau_e)Wx(t-\tau(t)).
\end{aligned}
$$

Since $\dot{d}(t) < \delta$, we have

$$
\begin{aligned}
-\int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s)S\dot{x}(s)ds &\leq -\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)S\dot{x}(s)ds, \\
-\rho\int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s)W\dot{x}(s)ds &\leq -\rho\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)W\dot{x}(s)ds.
\end{aligned}
$$

Based on Lemma 2.5.6, we obtain

$$
\begin{aligned}
-\rho\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)W\dot{x}(s)ds \quad & \leq -(\tau_e - \tau(t))\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)W\dot{x}(s)ds \\
& \leq -(\tau_e - \tau(t))(\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)ds)^T W(\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)ds) \\
& \leq -x^T(t-\tau_e)Wx(t-\tau_e) + 2x^T(t-\tau_e)Wx(t-\tau(t)) \\
& \quad - x^T(t-\tau(t))Wx(t-\tau(t)).
\end{aligned}
$$

We get from the following equalities :

$$
\begin{aligned}
& 2\Big[\dot{x}^T(t)K_1^T + \dot{x}^T(t-d(t))K_2^T\Big] \times \Big[Ax(t) + Bx(t-\tau_e) - B\int_{t-\tau(t)}^{t-\tau_e} \dot{x}(s)ds \\
& + D_1 f_1 + D_2 f_2 + C\dot{x}(t-d(t)) - \dot{x}(t)\Big] = 0, \qquad (3.10) \\
& 2\Big[x^T(t)L_1^T + x^T(t-\tau(t))L_2^T + x^T(t-\tau_e)L_3^T + \dot{x}^T(t)L_4^T + \dot{x}^T(t-d(t))L_5^T \\
& + f_1^T L_6^T + f_2^T L_7^T\Big] \times \Big[x(t) - x(t-\tau_e) - \int_{t-\tau_e}^{t} \dot{x}(s)ds\Big] = 0, \qquad (3.11) \\
& 2\Big[x^T(t)M_1^T + x^T(t-\tau(t))M_2^T + x^T(t-\tau_e)M_3^T + \dot{x}^T(t)M_4^T + \dot{x}^T(t-d(t))M_5^T \\
& + f_1^T M_6^T + f_2^T M_7^T\Big] \times \Big[x(t-\tau(t)) - x(t-\tau_e) - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}(s)ds\Big] = 0, \qquad (3.12)
\end{aligned}
$$

where K_1, K_2 and L_i, $M_i, i = 1, 2, ..., 7$ are some matrices of appropriate dimensions. Next, from (3.3), for any scalars $\delta_1 > 0$ and $\delta_2 > 0$, we obtain

$$
\begin{aligned}
& \delta_1\big[\alpha^2 x^T(t)x(t) - f_1^T f_1\big] \geq 0, \\
& \delta_2\big[\beta^2 x^T(t-\tau(t))x(t-\tau(t)) - f_2^T f_2\big] \geq 0. \qquad (3.13)
\end{aligned}
$$

By adding the terms on left of (3.10)-(3.13) to $\dot{V}_1(x_t)$, we may express $\dot{V}_1(x_t)$ as

$$\begin{aligned}
\dot{V}_1(x_t) \leq\ & 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau_e)Qx(t-\tau_e) + \dot{x}^T(t)Q_1\dot{x}(t) \\
& -(1-\delta)\dot{x}^T(t-d(t))Q_1\dot{x}(t-d(t)) + \dot{x}^T(t)(\tau_e R + \rho S + \rho^2 W)\dot{x}(t) \\
& - \int_{t-\tau_e}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)S\dot{x}(s)ds + 2\Big[\dot{x}^T(t)K_1^T \\
& + \dot{x}^T(t-d(t))K_2^T\Big]\Big[Ax(t) + Bx(t-\tau_e) + B\int_{t-\tau_e}^{t-\tau(t)} \dot{x}(s)ds + D_1 f_1 \\
& + D_2 f_2 + C\dot{x}(t-d(t)) - \dot{x}(t)\Big] + 2\Big[x^T(t)L_1^T + x^T(t-\tau(t))L_2^T \\
& + x^T(t-\tau_e)L_3^T + \dot{x}^T(t)L_4^T + \dot{x}^T(t-d(t))L_5^T + f_1^T L_6^T + f_2^T L_7^T\Big] \\
& \times\Big[x(t) - x(t-\tau_e) - \int_{t-\tau_e}^{t} \dot{x}(s)ds\Big] + 2\Big[x^T(t)M_1^T + x^T(t-\tau(t))M_2^T \\
& + x^T(t-\tau_e)M_3^T + \dot{x}^T(t)M_4^T + \dot{x}^T(t-d(t))M_5^T + f_1^T M_6^T + f_2^T M_7^T\Big] \\
& \times\Big[x(t-\tau(t)) - x(t-\tau_e) - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}(s)ds\Big] + \delta_1\big[\alpha^2 x^T(t)x(t) \\
& - f_1^T f_1\big] - x^T(t-\tau_e)Wx(t-\tau_e) + 2x^T(t-\tau_e)Wx(t-\tau(t)) \qquad (3.14) \\
& - x^T(t-\tau(t))Wx(t-\tau(t)) + \delta_2\big[\beta^2 x^T(t-\tau(t))x(t-\tau(t)) - f_2^T f_2\big] \\
=\ & \frac{1}{\tau_e}\int_{t-\tau_e}^{t} \omega^T(t,s)\phi_1\omega(t,s)ds + \frac{1}{\tau_e - \tau(t)}\int_{t-\tau_e}^{t-\tau(t)} \omega^T(t,s)\phi_2\omega(t,s)ds,
\end{aligned}$$

where

$\omega^T(t,s) = [x^T(t) \;\; x^T(t-\tau(t)) \;\; x^T(t-\tau_e) \;\; \dot{x}^T(t) \;\; \dot{x}^T(t-d(t)) \;\; f_1^T \;\; f_2^T \;\; -\dot{x}^T(s)]$,

$$\phi_1 = \begin{bmatrix}
\phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \tau_e L_1^T \\
* & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \tau_e L_2^T \\
* & * & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \tau_e L_3^T \\
* & * & * & \phi_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \tau_e L_4^T \\
* & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} & \tau_e L_5^T \\
* & * & * & * & * & \phi_{66} & 0 & \tau_e L_6^T \\
* & * & * & * & * & * & \phi_{77} & \tau_e L_7^T \\
* & * & * & * & * & * & * & -\tau_e R
\end{bmatrix} + Z,$$

$$\phi_2 = \begin{bmatrix} -Z_{11} & -Z_{12} & -Z_{13} & -Z_{14} & -Z_{15} & -Z_{16} & -Z_{17} & (\tau_e-\tau(t))M_1 \\ * & -Z_{22} & -Z_{23} & -Z_{24} & -Z_{25} & -Z_{26} & -Z_{27} & (\tau_e-\tau(t))M_2 \\ * & * & -Z_{33} & -Z_{34} & -Z_{35} & -Z_{36} & -Z_{37} & (\tau_e-\tau(t))M_3 \\ * & * & * & -Z_{44} & -Z_{45} & -Z_{46} & -Z_{47} & \varphi_1 \\ * & * & * & * & -Z_{55} & -Z_{56} & -Z_{57} & \varphi_2 \\ * & * & * & * & * & -Z_{66} & -Z_{67} & (\tau_e-\tau(t))M_6 \\ * & * & * & * & * & * & -Z_{77} & (\tau_e-\tau(t))M_7 \\ * & * & * & * & * & * & * & -|\tau_e-\tau(t)|S \end{bmatrix},$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} & Z_{17} & 0 \\ * & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} & Z_{27} & 0 \\ * & * & Z_{33} & Z_{34} & Z_{35} & Z_{36} & Z_{37} & 0 \\ * & * & * & Z_{44} & Z_{45} & Z_{46} & Z_{47} & 0 \\ * & * & * & * & Z_{55} & Z_{56} & Z_{57} & 0 \\ * & * & * & * & * & Z_{66} & Z_{67} & 0 \\ * & * & * & * & * & * & Z_{77} & 0 \\ * & * & * & * & * & * & * & 0 \end{bmatrix},$$

$\varphi_i = (\tau_e - \tau(t))(-K_i^T B + M_i)$, $i = 1, 2$, $Z_{ii} > 0$, $i = 1, 2, ..., 7$, Z_{ij}, $i = 1, 2, ..., 7$, $j = i+1, ..., 7$ are some parameter matrices of appropriate dimensions. From (3.14) if $\phi_1 < 0$ and $\phi_2 < 0$, then $\dot{V}_1(x_t) \leq -\lambda_1 \|x(t)\|^2$ for some $\lambda_1 > 0$. Pre and post-multiplying both sides of ϕ_2 by $\mathrm{diag}\{I, I, I, I, I, I, I, \mathrm{sgn}(\tau(t) - \tau_e)I\}$, we get that if

$$\begin{aligned} \varphi_2 &= \begin{bmatrix} -Z_{11} & -Z_{12} & -Z_{13} & -Z_{14} & -Z_{15} & -Z_{16} & -Z_{17} & \rho M_1 \\ * & -Z_{22} & -Z_{23} & -Z_{24} & -Z_{25} & -Z_{26} & -Z_{27} & \rho M_2 \\ * & * & -Z_{33} & -Z_{34} & -Z_{35} & -Z_{36} & -Z_{37} & \rho M_3 \\ * & * & * & -Z_{44} & -Z_{45} & -Z_{46} & -Z_{47} & \rho(-K_1^T B + M_4) \\ * & * & * & * & -Z_{55} & -Z_{56} & -Z_{57} & \rho(-K_2^T B + M_5) \\ * & * & * & * & * & -Z_{66} & -Z_{67} & \rho M_6 \\ * & * & * & * & * & * & -Z_{77} & \rho M_7 \\ * & * & * & * & * & * & * & -\rho S \end{bmatrix} \\ &< 0, \end{aligned} \tag{3.15}$$

then $\phi_2 < 0$: which follows from $\phi_2 \leq \varphi_2$ and Lemma 2.5.9. (3.8) holds if and only if $\phi_1 < 0$ and (3.15) simultaneously hold. Then (3.8) hold if and only if there exists a symmetric matrix Z, $\phi_1 < 0$ and (3.15) simultaneously hold. Therefore, nominal system of (3.7) is asymptotically stable.

Case II : $\tau(t) = \tau_e$ For this case, we choose a Lyapunov-Krasovskii functional candidate as

$$
\begin{aligned}
V_2(x_t) &= x^T(t)Px(t) + \int_{t-\tau_e}^{t} x^T(s)Qx(s)ds + \int_{t-d(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds \\
&\quad + \int_{-\tau_e}^{0}\int_{t+s}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta ds,
\end{aligned}
$$

where $P, \; Q, \; Q_1$ and R positive definite matrices are the same as those in $V_1(x_t)$.

Case III : $\tau_e < \tau(t) < \tau_M$ For this case, we choose the Lyapunov-Krasovskii functional candidate as

$$
\begin{aligned}
V_3(x_t) &= x^T(t)Px(t) + \int_{t-\tau_e}^{t} x^T(s)Qx(s)ds + \int_{t-d(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds \\
&\quad \int_{-\tau_e}^{0}\int_{t+s}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta ds + \int_{-\tau_M}^{-\tau_e}\int_{t+s}^{t} \dot{x}^T(\theta)S\dot{x}(\theta)d\theta ds \\
&\quad +\rho\int_{-\tau_M}^{-\tau_e}\int_{t+s}^{t} \dot{x}^T(\theta)W\dot{x}(\theta)d\theta ds,
\end{aligned}
$$

where P, Q, Q_1, R, S and $W$are positive definite matrices the same as those in $V_1(x_t)$.

By similar arguments used in proof of Theorem 3.1.2, we conclude that the nominal system of (3.7) is asymptotically stable. The proof is complete. □

Based on Theorem 3.1.2, we can perform the robust stability analysis for system (3.7) with uncertainties (3.4) and (3.5).

Theorem 3.1.3 *Under Assumption 3.1.1, system (3.7) with time-varying delays satisfying (3.2) and uncertainties (3.4) and (3.5) is asymptotically stable if there exist positive definite matrices* $P, \; Q, \; Q_1, \; R, \; S, \; W$ *and* $K_1, \; K_2, \; L_i, \; M_i, i = 1, 2, ..., 7$ *of appropriate dimensions and scalars* $\epsilon_i > 0, i = 1, 2, ..., 10$ *such that*

$$
\begin{aligned}
\mathcal{M} &= \begin{bmatrix}
M_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \tau_e L_1^T & \rho M_1 \\
* & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \tau_e L_2^T & \rho M_2 \\
* & * & M_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \tau_e L_3^T & \rho M_3 \\
* & * & * & M_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \tau_e L_4^T & \rho(K_1^T B + M_4) \\
* & * & * & * & M_{55} & \phi_{56} & \phi_{57} & \tau_e L_5^T & \rho(K_2^T B + M_5) \\
* & * & * & * & * & M_{66} & 0 & \tau_e L_6^T & \rho M_6 \\
* & * & * & * & * & * & M_{77} & \tau_e L_7^T & \rho M_7 \\
* & * & * & * & * & * & * & -\tau_e R & 0 \\
* & * & * & * & * & * & * & * & M_{99}
\end{bmatrix} \\
&< 0, \qquad (3.16)
\end{aligned}
$$

$$\mathcal{M}_1 = \begin{bmatrix} \mathcal{M}_{111} & K_1^T G_1 & K_1^T G_2 & K_1^T G_2 & K_1^T G_3 & K_1^T G_4 \\ G_1^T K_1 & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ G_2^T K_1 & 0 & -\epsilon_2 I & 0 & 0 & 0 \\ G_2^T K_1 & 0 & 0 & -\epsilon_3 I & 0 & 0 \\ G_3^T K_1 & 0 & 0 & 0 & -\epsilon_4 I & 0 \\ G_4^T K_1 & 0 & 0 & 0 & 0 & -\epsilon_5 I \end{bmatrix} < 0, \tag{3.17}$$

$$\mathcal{M}_2 = \begin{bmatrix} \mathcal{M}_{211} & K_2^T G_1 & K_2^T G_2 & K_2^T G_2 & K_2^T G_3 & K_2^T G_4 \\ G_1^T K_2 & -\epsilon_6 I & 0 & 0 & 0 & 0 \\ G_2^T K_2 & 0 & -\epsilon_7 I & 0 & 0 & 0 \\ G_2^T K_2 & 0 & 0 & -\epsilon_8 I & 0 & 0 \\ G_3^T K_2 & 0 & 0 & 0 & -\epsilon_9 I & 0 \\ G_4^T K_2 & 0 & 0 & 0 & 0 & -\epsilon_{10} I \end{bmatrix} < 0 \tag{3.18}$$

where

$$\begin{aligned} \mathcal{M}_{111} &= -0.1K_1^T - 0.1K_1 \\ \mathcal{M}_{211} &= -0.1Q_1 + 0.1K_2^T C + 0.1C^T K_2 \\ M_{11} &= \phi_{11} + \epsilon_1 E_A^T E_A + \epsilon_6 E_A^T E_A, \\ M_{33} &= \phi_{33} + \epsilon_2 E_B^T E_B + \epsilon_7 E_B^T E_B, \\ M_{44} &= Q_1 + \tau_e R + \rho S - 0.9K_1^T - 0.9K_1 + \rho^2 W, \\ M_{55} &= -0.9Q_1 + \delta Q_1 + 0.9K_2^T C + 0.9C^T K_2, \\ M_{66} &= \phi_{66} + \epsilon_4 E_{D_1}^T E_{D_1} + \epsilon_9 \rho^2 E_B^T E_B, \\ M_{77} &= \phi_{77} + \epsilon_5 E_{D_2}^T E_{D_2} + \epsilon_{10} \rho^2 E_B^T E_B, \\ M_{99} &= -\rho S + \epsilon_3 \rho^2 E_B^T E_B + \epsilon_8 \rho^2 E_B^T E_B. \end{aligned}$$

Proof. By choosing Lyapunov-Krasovskii functional as in Theorem 3.1.2, we may proof this Theorem by using a similar arguments as in the proof of Theorem 3.1.2 by replacing A, B, D_1 and D_2 in (3.14) with $A + GF(t)E_A$, $B + GF(t)E_B$, $D_1 + GF(t)E_{D_1}$ and$D_2 + GF(t)E_{D_2}$, respectively. For *Case I :*

$$\begin{aligned} \dot{V}_1(x_t) \leq\ & 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau_e)Qx(t-\tau_e) + \dot{x}^T(t)Q_1\dot{x}(t) \\ & -(1-\delta)\dot{x}^T(t-d(t))Q_1\dot{x}(t-d(t)) + \dot{x}^T(t)(\tau_e R + \rho S + \rho^2 W)\dot{x}(t) \\ & -\int_{t-\tau_e}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)S\dot{x}(s)ds + 2\Big[\dot{x}^T(t)K_1^T \end{aligned}$$

$$
\begin{aligned}
&+\dot{x}^T(t-d(t))K_2^T\Big]\Big[(A+G_1F(t)E_A)x(t)+(B+G_2F(t)E_B)x(t-\tau_e)\\
&+(B+G_2F(t)E_B)\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds+(D_1+G_3F(t)E_{D_1})f_1+(D_2\\
&+G_4F(t)E_{D_2})f_2+C\dot{x}(t-d(t))-\dot{x}(t)\Big]+2\Big[x^T(t)L_1^T+x^T(t-\tau(t))L_2^T\\
&+x^T(t-\tau_e)L_3^T+\dot{x}^T(t)L_4^T+\dot{x}^T(t-d(t))L_5^T+f_1^TL_6^T+f_2^TL_7^T\Big]\times[x(t)\\
&-x(t-\tau_e)-\int_{t-\tau_e}^{t}\dot{x}(s)ds]+2\Big[x^T(t)M_1^T+x^T(t-\tau(t))M_2^T+x^T(t-\tau_e)M_3^T\\
&+\dot{x}^T(t)M_4^T+\dot{x}^T(t-d(t))M_5^T+f_1^TM_6^T+f_2^TM_7^T\Big]\Big[x(t-\tau(t))-x(t-\tau_e)\\
&-\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds\Big]+\delta_1\big[\alpha^2x^T(t)x(t)-f_1^Tf_1\big]-x^T(t-\tau_e)Wx(t-\tau_e)\\
&+2x^T(t-\tau_e)Wx(t-\tau(t))-x^T(t-\tau(t))Wx(t-\tau(t))\\
&+\delta_2\big[\beta^2x^T(t-\tau(t))x(t-\tau(t))-f_2^Tf_2\big]. \qquad (3.19)
\end{aligned}
$$

By applying Lemma 2.5.5 and Lemma 2.5.11, the following estimations hold

$$
\begin{aligned}
2\dot{x}^T(t)K_1^T(A+G_1F(t)E_A)x(t) &\leq 2\dot{x}^T(t)K_1^TAx(t)\\
&\quad+\epsilon_1^{-1}\dot{x}^T(t)K_1^TG_1G_1^TK_1\dot{x}(t)\\
&\quad+\epsilon_1x^T(t)E_A^TE_Ax(t), \qquad (3.20)\\
2\dot{x}^T(t)K_1^T(B+G_2F(t)E_B)x(t-\tau_e) &\leq 2\dot{x}^T(t)K_1^TBx(t-\tau_e) \qquad (3.21)\\
&\quad+\epsilon_2^{-1}\dot{x}^T(t)K_1^TG_2G_2^TK_1\dot{x}(t)\\
&\quad+\epsilon_2x^T(t-\tau_e)E_B^TE_Bx(t-\tau_e),\\
2\dot{x}^T(t)K_1^T(B+G_2F(t)E_B)\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds &\leq 2\dot{x}^T(t)K_1^TB\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds \quad (3.22)\\
&\quad+\epsilon_3^{-1}\dot{x}^T(t)K_1^TG_2G_2^TK_4\dot{x}^T(t)\\
&\quad+\epsilon_3\rho\Big(\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)E_B^TE_B\dot{x}(s)ds\Big),\\
2\dot{x}^T(t)K_1^T(D_1+G_3F(t)E_{D_1})f_1 &\leq 2\dot{x}^T(t)K_1^TD_1f_1\\
&\quad+\epsilon_4^{-1}\dot{x}^T(t)K_1^TG_3G_3^TK_1\dot{x}(t)\\
&\quad+\epsilon_4f_1^TE_{D_1}^TE_{D_1}f_1, \qquad (3.23)\\
2\dot{x}^T(t)K_1^T(D_2+G_4F(t)E_{D_2})f_2 &\leq 2\dot{x}^T(t)K_1^TD_2f_2\\
&\quad+\epsilon_5^{-1}\dot{x}^T(t)K_1^TG_4G_4^TK_1\dot{x}(t)\\
&\quad+\epsilon_5f_2^TE_{D_2}^TE_{D_2}f_2, \qquad (3.24)
\end{aligned}
$$

$$
\begin{aligned}
2\dot{x}^T(t-d(t))K_2^T(A+G_1F(t)E_A)x(t) &\leq 2\dot{x}^T(t-d(t))K_2^TAx(t) \\
&+\epsilon_6^{-1}\dot{x}^T(t-d(t))K_2^TG_1G_1^TK_2 \\
&\times\dot{x}(t-d(t))+\epsilon_6x^T(t-d(t)) \\
&\times E_A^TE_Ax(t), \qquad (3.25) \\
2\dot{x}^T(t-d(t))K_2^T(B+G_2F(t)E_B)x(t-\tau_e) &\leq 2\dot{x}^T(t-d(t))K_1^TBx(t-\tau_e) \\
&+\epsilon_7^{-1}\dot{x}^T(t-d(t))K_1^TG_2G_2^TK_1 \\
&\times\dot{x}(t-d(t))+\epsilon_7x^T(t-\tau_e) \\
&\times E_B^TE_Bx(t-\tau_e), \qquad (3.26) \\
2\dot{x}^T(t-d(t))K_2^T(D_1+G_3F(t)E_{D_1})f_1 &\leq 2\dot{x}^T(t-d(t))K_2^TD_1f_1 \qquad (3.27) \\
&+\epsilon_9^{-1}\dot{x}^T(t-d(t))K_2^TG_3G_3^TK_2 \\
&\times\dot{x}(t-d(t))+\epsilon_9f_1^TE_{D_1}^TE_{D_1}f_1, \\
2\dot{x}^T(t-d(t))K_2^T(D_2+G_4F(t)E_{D_2})f_2 &\leq 2\dot{x}^T(t-d(t))K_2^TD_2f_2 \qquad (3.28) \\
&+\epsilon_{10}^{-1}\dot{x}^T(t-d(t))K_2^TG_4G_4^TK_2 \\
&\times\dot{x}(t-d(t))+\epsilon_{10}f_2^TE_{D_2}^TE_{D_2}f_2, \\
2\dot{x}^T(t-d(t))K_2^T(B+G_2F(t)E_B)\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds &\leq 2\dot{x}^T(t-d(t))K_2^TB \\
&\times\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds \\
&+\epsilon_8^{-1}\dot{x}^T(t-d(t))K_2^TG_2 \\
&\times G_2^TK_2\dot{x}^T(t-d(t)) \\
&+\epsilon_8\rho(\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)E_B^T \\
&\times E_B\dot{x}(s)ds). \qquad (3.29)
\end{aligned}
$$

Therefore, from (3.19)-(3.29), it follows that

$$
\dot{V}_1(x_t)\leq\omega^T(t,s)\mathcal{M}\omega(t,s)+\dot{x}^T(t)\Omega_1\dot{x}(t)+\dot{x}^T(t-d(t))\Omega_2\dot{x}(t-d(t)) \qquad (3.30)
$$

where

$$
\begin{aligned}
\Omega_1 &= -0.1K_1^T-0.1K_1+\epsilon_1^{-1}K_1^TG_1G_1^TK_1+\epsilon_2^{-1}K_1^TG_2G_2^TK_1 \\
&\quad +\epsilon_3^{-1}K_1^TG_2G_2^TK_1+\epsilon_4^{-1}K_1^TG_3G_3^TK_1+\epsilon_5^{-1}K_1^TG_4G_4^TK_1, \\
\Omega_2 &= -0.1Q_1^T+0.1K_2^TC+0.1C^TK_2+\epsilon_6^{-1}K_2^TG_1G_1^TK_2+\epsilon_7^{-1}K_2^TG_2G_2^TK_2 \\
&\quad +\epsilon_8^{-1}K_2^TG_2G_2^TK_2+\epsilon_9^{-1}K_2^TG_3G_3^TK_2+\epsilon_{10}^{-1}K_2^TG_4G_4^TK_2.
\end{aligned}
$$

Applying Lemma 2.5.1, the inequalities $\Omega_1<0$ and $\Omega_2<0$ are equivalent to $\mathcal{M}_1<0$ and $\mathcal{M}_2<0$, respectively. Therefore, system (3.7) is robust asymptotically stable if the conditions (3.16)-(3.18) hold.

By using arguments similar to the proof of Case I for Case II and Case III , we may conclude that the close-loop system (3.7) is asymptotically stable.$\square$

Remark 3.1.4 The method in this thesis has no restrictions on the derivatives of the state delay, while traditional design methods is required the derivative of them such as [5,12,14,22,25–27,42,53,59,67,70,73]. It is worth pointing out that our proposed method can be used with fast time-varying delays.

Remark 3.1.5 Since, we need the negative terms along the diagonal entires of LMIs (3.17) and (3.18), so the significance of splitting the term K_1 and Q_1 are needed. Observe that in LMIs 3.18 and 3.19, we required $\mathcal{M}_{111}$ and $\mathcal{M}_{211}$ to be negative definite. Therefore in 3.31, we split $-K_1 - K_1^T$ into $(-0.1K_1 - 0.1K_1^T)+$ $(-0.9K_1^T - 0.9K_1^T)$ and $-Q_1 = -0.1Q_1 - 0.9Q_1$.

3.2 Numerical examples

In this section, we provide numerical examples to show the effectiveness of our theoretical results.

Example 3.2.1 Consider the following uncertain neutral system with time-varying delay and nonlinear uncertainties which is studied in [59, 70]:

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t-d) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - \tau(t)) \\ &\quad +(D_1 + \Delta D_1(t))f_1(t, x(t)) \\ &\quad +(D_2 + \Delta D_2(t))f_2(t, x(t - \tau(t))), \end{aligned} \tag{3.31}$$

where

$$A = \begin{bmatrix} -1.2 & -0.1 \\ -0.1 & -1 \end{bmatrix},\ B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix},\ C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},\ D_2 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},$$

and

$$\begin{aligned} &\Delta A(t) = GF(t)E_A,\ \Delta B(t) = GF(t)E_B,\ \Delta D_1(t) = GF(t)E_{D_1}, \\ &\Delta D_2(t) = GF(t)E_{D_2},\ F^T(t)F(t) \le I,\ G = \gamma I,\ E_A = I,\ E_B = I, \\ &E_{D_1} = I,\ E_{D_2} = I. \end{aligned}$$

Table 3.1: Maximum allowable upper bounds τ_M for $\gamma = 0.1$.

Methods	$\alpha = 0, \quad \beta = 0.1$		$\alpha = 0.1, \quad \beta = 0.1$	
	Restriction on $\dot{\tau}(t)$	Maximum allowable τ_M	Restriction on $\dot{\tau}(t)$	Maximum allowable τ_M
[70]	$\dot{\tau}(t) \leq 0.5$	0.704	$\dot{\tau}(t) \leq 0.5$	0.689
[59]	$\dot{\tau}(t) \leq 0.5$	1.060	$\dot{\tau}(t) \leq 0.5$	1.040
Theorem3.1.3	no restriction on $\dot{\tau}(t)$	1.0762	no restriction on $\dot{\tau}(t)$	1.0638

Table 3.2: Maximum allowable upper bounds τ_M for $\gamma = 0.5$.

Methods	$\alpha = 0, \quad \beta = 0.1$		$\alpha = 0.1, \quad \beta = 0.1$	
	Restriction on $\dot{\tau}(t)$	Maximum allowable τ_M	Restriction on $\dot{\tau}(t)$	Maximum allowable τ_M
[70]	$\dot{\tau}(t) \leq 0.5$	0.259	$\dot{\tau}(t) \leq 0.5$	0.240
[59]	$\dot{\tau}(t) \leq 0.5$	0.450	$\dot{\tau}(t) \leq 0.5$	0.429
Theorem3.1.3	no restriction on $\dot{\tau}(t)$	0.5333	no restriction on $\dot{\tau}(t)$	0.5207

It is assumed that the nonlinear perturbations satisfy

$$\|f_1(t, x(t))\| \leq \alpha\|x(t)\|, \quad \|f_2(t, x(t-\tau(t)))\| \leq \beta\|x(t-\tau(t))\|, \ \alpha > 0, \ \beta > 0.$$

Applying Theorem 3.1.3, the maximum allowable value of τ_M are given in the table 3.1 when $\gamma = 0.1$ and in the table 3.2 for $\gamma = 0.5$. The results obtained in [59,70] may not be used for the case when $\tau_m \neq 0$. Moreover, the differentiability of the time delay $\tau(t)$ is not required in Theorem 3.1.3. Table 3.1 and Table 3.2. Show that our results are less conservative than those in [59, 70].

Moreover, it should be pointed out that if we let $\tau_m = 0.1$ and $\tau_M = 0.85$, then from Theorem 3.1.3, the solutions of LMI (3.16)-(3.18) are given as follows:

$$\begin{aligned}
P &= \begin{bmatrix} 2.4597 & 0.3509 \\ 0.3509 & 1.9870 \end{bmatrix}, Q = \begin{bmatrix} 0.9955 & 0.1967 \\ 0.1967 & 1.0236 \end{bmatrix}, \\
Q_1 &= \begin{bmatrix} 0.5615 & -0.0545 \\ -0.0545 & 0.2292 \end{bmatrix}, S = \begin{bmatrix} 1.4211 & 0.1224 \\ 0.1224 & 1.1393 \end{bmatrix}, \\
R &= \begin{bmatrix} 1.8164 & 0.2718 \\ 0.2718 & 1.7699 \end{bmatrix}, W = \begin{bmatrix} 0.4131 & 0.0520 \\ 0.0520 & 0.3768 \end{bmatrix}, \\
L_1 &= \begin{bmatrix} -1.8399 & -0.0430 \\ -0.1215 & -1.9400 \end{bmatrix}, L_2 = \begin{bmatrix} 1.5341 & 1.0797 \\ -1.1061 & 1.2226 \end{bmatrix}, \\
L_3 &= \begin{bmatrix} -0.1656 & -0.0202 \\ -0.1437 & -0.3157 \end{bmatrix}, L_4 = \begin{bmatrix} -1.3144 & -0.4567 \\ 0.1806 & -1.3580 \end{bmatrix}, \\
L_5 &= \begin{bmatrix} -0.1514 & -0.0592 \\ -0.0410 & 0.0423 \end{bmatrix}, L_6 = \begin{bmatrix} -0.0047 & 0.0064 \\ -0.0036 & -0.0045 \end{bmatrix}, \\
L_7 &= \begin{bmatrix} -0.0047 & 0.0064 \\ -0.0037 & -0.0046 \end{bmatrix}, K_1 = \begin{bmatrix} 1.7652 & 0.2207 \\ 0.0928 & 1.4950 \end{bmatrix}, \\
K_2 &= \begin{bmatrix} -0.2816 & 0.0203 \\ 0.0408 & -0.1346 \end{bmatrix}, M_1 = \begin{bmatrix} -1.4428 & -0.0867 \\ -0.0535 & -1.3096 \end{bmatrix}, \\
M_2 &= \begin{bmatrix} -1.2780 & -1.0258 \\ 1.1266 & -0.9849 \end{bmatrix}, M_3 = \begin{bmatrix} 0.2312 & -0.1717 \\ 0.0660 & 0.4381 \end{bmatrix}, \\
M_4 &= \begin{bmatrix} 0.3926 & -1.0687 \\ 0.9108 & 0.5805 \end{bmatrix}, M_5 = \begin{bmatrix} 0.3601 & 0.1664 \\ -0.1928 & 0.1095 \end{bmatrix}, \\
M_6 &= \begin{bmatrix} 0.0032 & -0.0046 \\ 0.0110 & 0.0081 \end{bmatrix}, M_7 = \begin{bmatrix} 0.0032 & -0.0047 \\ 0.0111 & 0.0081 \end{bmatrix},
\end{aligned}$$

$\delta_1 = 2.0156,\ \ \delta_2 = 1.9922,\ \ \epsilon_1 = 0.4383,\ \ \epsilon_2 = 0.4197,\ \ \epsilon_3 = 0.5513,\ \ \epsilon_4 = 0.9140,$

$\epsilon_5 = 0.9042,\ \ \epsilon_6 = 0.3288,\ \ \epsilon_7 = 0.3143,\ \ \epsilon_8 = 0.4376,\ \ \epsilon_9 = 0.9021,\ \ \epsilon_{10} = 0.9003.$

Figure 3.1. shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the system (3.31) with time-varying delay $\tau(t) = 0.1 + 0.75|\sin 10t|$, $d = 1$, $\phi(t) = [\sin t, \cos t)]$, $\forall t \in [-1, 0]$, $f_1(t, x(t)) = [0.1 \sin |x_1(t)|, 0.1 \cos |x_2(t)|]^T$, $f_2(t, x(t - \tau(t))) = [0.1e^{-\sin^2 x_1(t-\tau(t))}, 0.1e^{-\cos^2 x_2(t-\tau(t))}]^T$ and $F(t) = \mathrm{diag}\{\sin^2(t), \sin^2(t)\}$. Since the time-delay $\tau(t)$ is not differentiable, the stability criterion in [59, 70] can not be applied to this case because it is only applicable to the system with the differentiable delay.

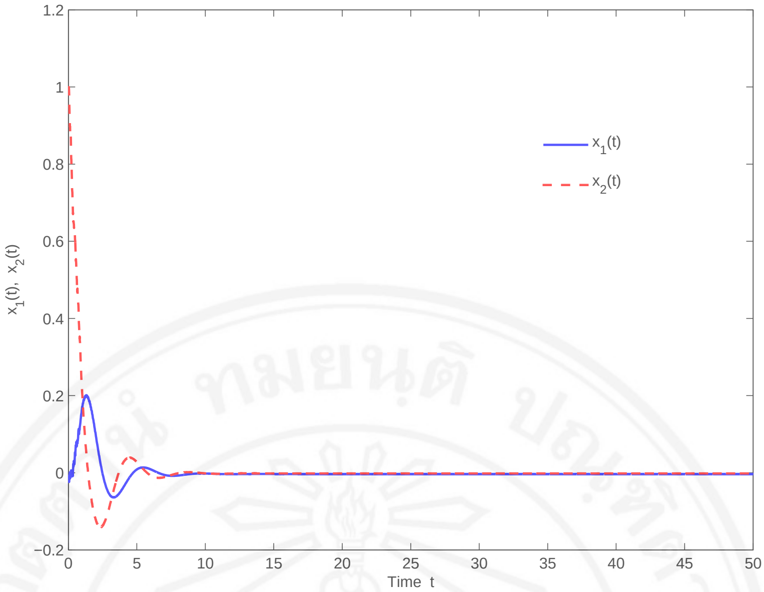

Figure 3.1: The trajectories of $x_1(t)$ and $x_2(t)$ of the system (3.31) with time-varying delay $\tau(t) = 0.1 + 0.75|\sin 10t|$.

Example 3.2.2 Consider the following uncertain neutral system with time-varying delays considered in [26, 67]:

$$\dot{x}(t) - C\dot{x}(t - d(t)) = (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - \tau(t)), \qquad (3.32)$$

where

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \ B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \ C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix},$$

$$\Delta A(t) = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}, \ \Delta B(t) = \begin{bmatrix} \gamma_3 & 0 \\ 0 & \gamma_4 \end{bmatrix},$$

$0 \leq |c| < 1$, and γ_i, $i = 1, 2, ..., 4$ are unknown parameters satisfying $|\gamma_1| \leq 1.6$, $|\gamma_2| \leq 0.05$, $|\gamma_3| < 0.1$ and $|\gamma_4| < 0.3$.

Case I. For $c = 0.1$, $\delta \leq 0$, $d = 0.1$, the maximum values of τ_M is listed in Table 3.3 for $c = 0.1$ by applying criteria in [26, 67] and in . We see that, the maximum allowable bounds for τ_M obtained from Theorem 3.1.3 are much better than those obtained in [26, 67] and in Theorem 3.1.3 .

Case II. For $c = 0.1$, $\delta = 0.1$, $d = 0.2$, the maximum allowable value τ_M obtained from Theorem 3.1.2 is lised in Table 3.4. Note that the method obtained in [26, 67] cannot be applied to systems with time-varying neutral delay. Furthermore, the

Table 3.3: Maximum allowable upper bounds τ_M of the time-varying delay for different values of the lower bounds τ_m, $d = 0.1$ and $c = 0.1$.

Methods	$\tau_m = 0$	$\tau_m = 0.5$
[67]	0.658	0.793
[26]	0.852	0.894
Theorem 3.1.3	0.873	0.951

stability criterion in [14,52] required derivative of time-varying delay but Theorem 3.2 does not have restriction on the derivative of time-varying delay. It is obvious that our obtained results are less conservative than those in [14, 26, 52, 67].

Figure 3.2. shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the system (3.32) with time-varying delay $\tau(t) = 0.3 + 0.5|\cos 10t|$, $d(t) = 0.1\sin^2 t$, $\phi(t) = [\sin t, \cos t)]$, $\forall t \in [-0.8, 0]$.

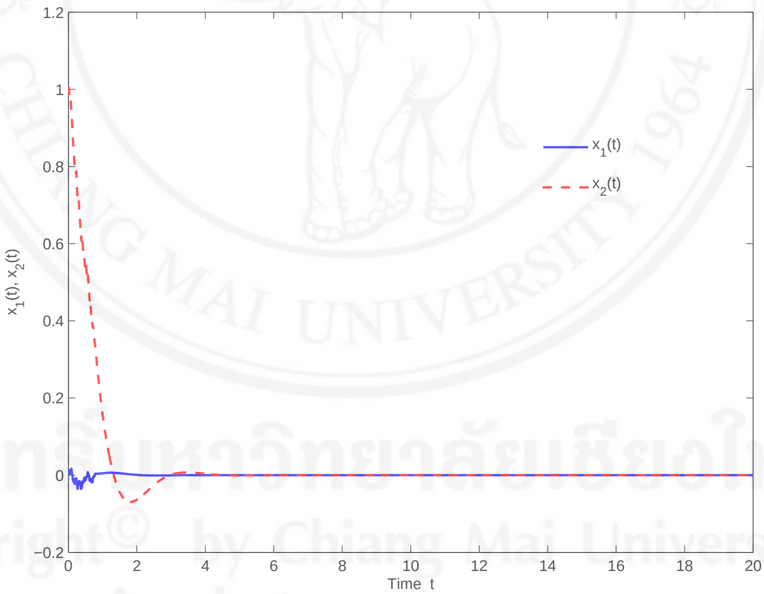

Figure 3.2: The trajectories of $x_1(t)$ and $x_2(t)$ of the system (3.32) with time-varying delay $\tau(t) = 0.3 + 0.5|\cos 10t|$.

Table 3.4: Maximum allowable upper bounds τ_M of the time-varying delay for different values of the lower bounds τ_m, $c = 0.1$ and $\delta = 0.1, d = 0.2$.

Methods	$\tau_m = 0, c = 0.1, \delta = 0.1$ and $d = 0.2$		
[14]	0.57 with $\dot{\tau}(t) \leq 0.5$	0.17 with $\dot{\tau}(t) \leq 0.9$	not applicable with $\dot{\tau}(t) \geq 1$
[52]	0.7890 with $\dot{\tau}(t) \leq 0.5$	0.7199 with $\dot{\tau}(t) \leq 0.9$	0.7216 with $\dot{\tau}(t) \geq 1$
Theorem 3.1.3	0.8676 with no restriction on $\dot{\tau}(t)$		
Methods	$\tau_m = 0.5, c = 0.1, \delta = 0.1$ and $d = 0.2$		
[14]	not applicable with $\dot{\tau}(t) \leq 0.5$	not applicable with $\dot{\tau}(t) \leq 0.9$	not applicable with $\dot{\tau}(t) \geq 1$
[52]	not applicable with $\dot{\tau}(t) \leq 0.5$	not applicable with $\dot{\tau}(t) \leq 0.9$	not applicable with $\dot{\tau}(t) \geq 1$
Theorem 3.1.3	0.9460 with no restriction on $\dot{\tau}(t)$		