CHAPTER 4

Conclusion

In this chapter, we conclude all main results obtained throughout the thesis. This thesis has presented iterative methods defined by admissible functions and properties for this algorithms. Here are the results:

1. Let C be a closed convex bounded subset of a uniformly convex Banach space X and $T : C \to C$ be a nonexpansive and demicompact mapping. If $G : C \times C \to C$ is an affine Lipschitzian admissible function which constant $\lambda \in (0, 1)$. Then the GK-algorithm $\{x_n\}_{n=1}^{\infty}$ given by $x_1 \in C$ and

   $$x_{n+1} = G(x_n, Tx_n), \quad n \in \mathbb{N}$$

   converges(strongly) to a fixed point of T in C.

2. Let C be a closed convex subset of a uniformly convex Banach space X and $T : C \to C$ be a continuous quasi-nonexpansive mapping with satisfies Condition I. If $\{G_n\}$ is sequentially affine Lipschitzian with a sequence $\{\alpha_n\}$ which is bounded away from 0 and 1. Then the GM-algorithm $\{x_n\}_{n=1}^{\infty}$ given by $x_1 \in C$ and

   $$x_{n+1} = G_n(x_n, Tx_n), \quad n \in \mathbb{N}$$

   converges(strongly) to a fixed point of T in C.

3. Let C be a closed convex subset of a real Hilbert space H and $T : C \to C$ be a nonexpansive mapping with $F(T) \neq \emptyset$. If $G_n : H \times H \to H$ is an admissible function which has the property (C*) for each $n \in \mathbb{N}$ and $\{G_n\}$ is sequentially affine Lipschitzian with $\{\alpha_n\}$ satisfying the following conditions:

   a) $\lim_{n\to\infty} \alpha_n = 0$,

   b) $\Sigma_{n=0}^{\infty} \alpha_n = \infty$,

   c) $\lim_{n\to\infty} \frac{\alpha_n - \alpha_{n-1}}{\alpha_n} = 0$.

   Then the GH-algorithm $\{x_n\}_{n=1}^{\infty}$ given by $x_1 \in C$, $u \in C$ and

   $$x_{n+1} = G_n(u, Tx_n), \quad n \in \mathbb{N},$$

converges(strongly) to a fixed point of T in C.

4. Let C be a closed convex bounded subset of a Hilbert space H and $T : C \to C$ be a nonexpansive mapping. If $G_n^1, G_n^2 : H \times H \to H$ are admissible function for all $n \in \mathbb{N}$ and $\{G_n^1\}, \{G_n^2\}$ are sequentially affine Lipschitzian with $\{\alpha_n\}$ and $\{\beta_n\}$ respectively, and suppose that

$$\limsup_{n\to\infty} \alpha_n < 1 \text{ and } 0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1.$$

Then the GI-algorithm $\{x_n\}_{n=1}^{\infty}$ given by $x_1 \in X$ and

$$\begin{cases} y_n = G_n^2(x_n, Tx_n), \\ x_{n+1} = G_n^1(x_n, Ty_n), \;\; n \in \mathbb{N}, \end{cases}$$

converges weakly to a fixed point of T.

5. Let X be a uniformly convex Banach space that satisfies Opials condition, C be a nonempty closed convex subset of X and $T : C \to C$ be a nonexpansive mapping with $F(T) \neq \emptyset$. Let $G_n^1, G_n^2 : X \times X \to X$ be admissible functions for $n \in \mathbb{N}$ and $\{G_n^1\}, \{G_n^2\}$ are sequentially affine Lipschitzian with $\{\alpha_n\}$ and $\{\beta_n\}$ respectively. Suppose that $\limsup_{n\to\infty} \alpha_n < 1$ and $0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1$. Then the GS-algorithm $\{x_n\}_{n=1}^{\infty}$ given by $x_1 \in X$ and

$$\begin{cases} y_n = G_n^2(x_n, Tx_n), \\ x_{n+1} = G_n^1(Tx_n, Ty_n), \;\; n \in \mathbb{N}, \end{cases}$$

converges weakly to a fixed point of T.

6. Let C be a closed convex subset of a uniformly convex Banach space X. Let $S, T : C \to C$ be two nonexpansive mappings such that one of the mappings T and S satisfies Condition I and $F(S) \cap F(T) \neq \emptyset$. If $G_n^1, G_n^2 : H \times H \to H$ are admissible function and $\{G_n^1\}, \{G_n^2\}$ are sequentially affine Lipschitzian with $\{\alpha_n\}$ and $\{\beta_n\}$ respectively. Suppose that

$$0 < \liminf_{n\to\infty} \alpha_n \leq \limsup_{n\to\infty} \alpha_n < 1 \text{ and } 0 < \liminf_{n\to\infty} \beta_n \leq \limsup_{n\to\infty} \beta_n < 1.$$

Then the GC-algorithm $\{x_n\}_{n=1}^{\infty}$ given by $x_1 \in X$ and

$$\begin{cases} y_n = G_n^2(x_n, Tx_n), \\ x_{n+1} = G_n^1(y_n, Sy_n), \;\; n \in \mathbb{N}, \end{cases}$$

converges(strongly) to a common fixed point of S and T.

7. Let C be a closed convex subset of a uniformly convex Banach space X that satisfies Opials condition and let $S, T : C \to C$ be two nonexpansive mappings with $F(S) \cap F(T) \neq \emptyset$. If $\{x_n\}$ be a sequence as in Theorem 3.2.8, then $\{x_n\}_{n=1}^{\infty}$ converges weakly to a common fixed point of S and T.