APPENDIX A

Basis Wave Functions

In my calculation there are ten two-electron angular momentum basis in the LS representation expanded over $|\ell_1\ell_2 L M_L S M_S\rangle$ basis set with $M_J = 0$. The spin quantum numbers of two electrons are omiited because all electrons have the same spin 1/2. From Eq. (2.23) we obtain

$$
\begin{aligned}
& {}^{2S+1}L_{\mathrm{J}} = |\ell_1\ell_2 L M_L S M_S\rangle \\
1: \quad & {}^1S_0 = |000000\rangle \\
2: \quad & {}^3S_1 = |000010\rangle \\
3: \quad & {}^1P_1 = |011000\rangle \\
4: \quad & {}^3P_0 = \frac{1}{\sqrt{3}}|01111{-}1\rangle - \frac{1}{\sqrt{3}}|011010\rangle + \frac{1}{\sqrt{3}}|011{-}111\rangle \\
5: \quad & {}^3P_1 = \frac{1}{\sqrt{2}}|01111{-}1\rangle - \frac{1}{\sqrt{2}}|011{-}111\rangle \\
6: \quad & {}^3P_2 = \frac{1}{\sqrt{6}}|01111{-}1\rangle + \frac{2}{\sqrt{6}}|011010\rangle + \frac{1}{\sqrt{6}}|011{-}111\rangle \\
7: \quad & {}^1D_2 = |022000\rangle \\
8: \quad & {}^3D_1 = \sqrt{\frac{3}{10}}|02211{-}1\rangle - \sqrt{\frac{4}{10}}|022010\rangle + \sqrt{\frac{3}{10}}|022{-}111\rangle \\
9: \quad & {}^3D_2 = \frac{1}{\sqrt{2}}|02211{-}1\rangle - \frac{1}{\sqrt{2}}|022{-}111\rangle \\
10: \quad & {}^3D_3 = \frac{1}{\sqrt{5}}|02211{-}1\rangle + \sqrt{\frac{3}{5}}|022010\rangle + \frac{1}{\sqrt{5}}|022{-}111\rangle
\end{aligned}
\tag{A.1}
$$

Diagonal matrix elements are

$$
\begin{aligned}
D_{11} &= \frac{1}{R} & D_{66} &= \frac{1}{R} + \frac{1}{5}\frac{r^2}{R^3} \\
D_{22} &= \frac{1}{R} & D_{77} &= \frac{1}{R} + \frac{2}{7}\frac{r^2}{R^3} + \frac{2}{7}\frac{r^4}{R^5} \\
D_{33} &= \frac{1}{R} + \frac{2}{5}\frac{r^2}{R^3} & D_{88} &= \frac{1}{R} + \frac{1}{5}\frac{r^2}{R^3} \\
D_{44} &= \frac{1}{R} & D_{99} &= \frac{1}{R} + \frac{1}{7}\frac{r^2}{R^3} - \frac{4}{21}\frac{r^4}{R^5} \\
D_{55} &= \frac{1}{R} - \frac{1}{5}\frac{r^2}{R^3} & D_{1010} &= \frac{1}{R} + \frac{8}{35}\frac{r^2}{R^3} + \frac{2}{21}\frac{r^4}{R^5}
\end{aligned}
\tag{A.2}
$$

off-diagonal matrix elements are

$$
\begin{aligned}
D_{13} &= \frac{1}{\sqrt{3}}\left(\frac{r}{R^2} + \frac{\alpha_d}{r^2R^2}\right) & D_{46} &= -\frac{\sqrt{2}}{5}\frac{r^2}{R^3} \\
D_{17} &= \frac{1}{\sqrt{5}}\frac{r^2}{R^3} & D_{48} &= -\frac{\sqrt{2}}{3}\left(\frac{r}{R^2} + \frac{\alpha_d}{r^2R^2}\right) \\
D_{24} &= -\frac{1}{3}\left(\frac{r}{R^2} + \frac{\alpha_d}{r^2R^2}\right) & D_{410} &= -\frac{\sqrt{3}}{7}\frac{r^3}{R^4} \\
D_{26} &= \frac{\sqrt{2}}{3}\left(\frac{r}{R^2} + \frac{\alpha_d}{r^2R^2}\right) & D_{59} &= \frac{1}{\sqrt{5}}\left(\frac{r}{R^2} + \frac{\alpha_d}{r^2R^2}\right) - \frac{3}{7\sqrt{5}}\frac{r^3}{R^4} \\
D_{28} &= -\frac{\sqrt{2}}{5}\frac{r^2}{R^3} & D_{68} &= -\frac{7}{105}\left(\frac{r}{R^2} + \frac{\alpha_d}{r^2R^2}\right) - \frac{27}{105}\frac{r^3}{R^4} \\
D_{210} &= \frac{\sqrt{3}}{5}\frac{r^2}{R^3} & D_{610} &= \frac{\sqrt{6}}{5}\left(\frac{r}{R^2} + \frac{\alpha_d}{r^2R^2}\right) + \frac{2\sqrt{6}}{35}\frac{r^3}{R^4} \\
D_{37} &= \frac{2}{\sqrt{15}}\left(\frac{r}{R^2} + \frac{\alpha_d}{r^2R^2}\right) + \frac{3}{7}\sqrt{\frac{3}{5}}\frac{r^3}{R^4} & D_{810} &= -\sqrt{\frac{2}{3}}\left(\frac{10}{35}\frac{r^3}{R^4} + \frac{3}{35}\frac{r^2}{R^3}\right)
\end{aligned}
\tag{A.3}
$$

APPENDIX B

JJ-LS Transformation

The tranformation from jj representation into LS representation can be written as

$$|\ell_1\ell_2 LSJM_J\rangle = \sum_{j_1,j_2}^{J} C_{j_1,j_2}^{J,M_J} |\ell_1\ell_2 j_1 j_2 JM_J\rangle \tag{B.1}$$

where the coefficients C_{j_1,j_2}^{J,M_J} can be calculated from the following procedure. Starting from the expansion

$$|\ell_1\ell_2 LSJM_J\rangle = \sum_{M_L,M_S}^{M_J} C_{M_L M_S}^{J,M_J} |\ell_1\ell_2 LM_L SM_S\rangle\,, \tag{B.2}$$

where the basis vector $|\ell_1\ell_2 LM_L SM_S\rangle$ can be further expanded in uncoupld electron spinor as

$$|\ell_1\ell_2 LM_L SM_S\rangle = \left(\sum_{m_{\ell_1},m_{\ell_2}}^{M_L} C_{m_{\ell_1}m_{\ell_2}}^{L,M_L} |\ell_1 m_{\ell_1}\ell_2 m_{\ell_2}\rangle\right)\left(\sum_{m_{s_1},m_{s_2}}^{M_S} C_{m_{s_1}m_{s_2}}^{S,M_S} |m_{s_1} m_{s_2}\rangle\right) \tag{B.3}$$

Applying the same procedure with jj representation,

$$|\ell_1\ell_2 j_1 j_2 JM_J\rangle = \sum_{m_{j_1},m_{j_2}} C_{m_{j_1}m_{j_2}}^{J,M_J} |\ell_1\ell_2 j_1 m_{j_1} j_2 m_{j_2}\rangle \tag{B.4}$$

where

$$|\ell_1\ell_2 j_1 m_{j_1} j_2 m_{j_2}\rangle = \left(\sum_{m_{\ell_1},m_{s_1}} C_{m_{\ell_1}m_{s_1}}^{j_1,m_{j_1}} |\ell_1 m_{\ell_1} m_{s_1}\rangle\right)\left(\sum_{m_{\ell_2},m_{s_2}} C_{m_{\ell_2}m_{s_2}}^{j_2,m_{j_2}} |\ell_2 m_{\ell_2} m_{s_2}\rangle\right) \tag{B.5}$$

Due to the valence electron of the neutral atom is in s orbital, the quantum number ℓ_1 is alway zero. The jj representation naturally is used as appropriate basis for Dirac wave

function. Redefining the basis vector $|\ell_1\ell_2 j_1 j_2 J M_J\rangle$ by

$$|\kappa J M_J\rangle \equiv |\ell_1\ell_2 j_1 j_2 J M_J\rangle\,, \tag{B.6}$$

where κ is the relativistic quantum number used to define eigenstates of Dirac equation. In addition, the fundamental basis set $|\ell_1\ell_2 m_{\ell_1} m_{\ell_2} m_{s_1} m_{s_2}\rangle$ is defined as

$$\begin{aligned}
|\phi_1\rangle &= |0000\uparrow\downarrow\rangle & |\phi_6\rangle &= |010{-}1\uparrow\uparrow\rangle \\
|\phi_2\rangle &= |0000\downarrow\uparrow\rangle & |\phi_7\rangle &= |0201\downarrow\downarrow\rangle \\
|\phi_3\rangle &= |0101\downarrow\downarrow\rangle & |\phi_8\rangle &= |0200\uparrow\downarrow\rangle \\
|\phi_4\rangle &= |0100\uparrow\downarrow\rangle & |\phi_9\rangle &= |0200\downarrow\uparrow\rangle \\
|\phi_5\rangle &= |0100\downarrow\uparrow\rangle & |\phi_{10}\rangle &= |020{-}1\uparrow\uparrow\rangle
\end{aligned} \tag{B.7}$$

where the symbols $\uparrow$ and $\downarrow$ denote $m_s = 1/2$ and $m_s = -1/2$ respectively. Notice that all states in Eq. (B.7) corresponds with zero projection of total anular momoentum $M_J = 0$ and can be used to expand all states listed in Eq. (A.1).

$$\begin{pmatrix} |^1S_0\rangle \\ |^3S_1\rangle \\ |^1P_1\rangle \\ |^3P_0\rangle \\ |^3P_1\rangle \\ |^3P_2\rangle \\ |^1D_2\rangle \\ |^3D_1\rangle \\ |^3D_2\rangle \\ |^3D_3\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{-1}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{3}{10}} & \frac{-1}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & \sqrt{\frac{3}{10}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{5}} & \sqrt{\frac{3}{10}} & \sqrt{\frac{3}{10}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} |\phi_1\rangle \\ |\phi_2\rangle \\ |\phi_3\rangle \\ |\phi_4\rangle \\ |\phi_5\rangle \\ |\phi_6\rangle \\ |\phi_7\rangle \\ |\phi_8\rangle \\ |\phi_9\rangle \\ |\phi_{10}\rangle \end{pmatrix} \tag{B.8}$$

and for jj representation $|\kappa J\rangle$ we obtain

$$\begin{pmatrix} |-10\rangle \\ |-11\rangle \\ |10\rangle \\ |11\rangle \\ |-21\rangle \\ |-22\rangle \\ |21\rangle \\ |22\rangle \\ |-32\rangle \\ |-33\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{10}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & -\sqrt{\frac{3}{10}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{3}{10}} & \frac{1}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & -\sqrt{\frac{3}{10}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{\sqrt{5}} & \sqrt{\frac{3}{10}} & -\sqrt{\frac{3}{10}} & \frac{1}{\sqrt{5}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{5}} & \sqrt{\frac{3}{10}} & \sqrt{\frac{3}{10}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} |\phi_1\rangle \\ |\phi_2\rangle \\ |\phi_3\rangle \\ |\phi_4\rangle \\ |\phi_5\rangle \\ |\phi_6\rangle \\ |\phi_7\rangle \\ |\phi_8\rangle \\ |\phi_9\rangle \\ |\phi_{10}\rangle \end{pmatrix} \tag{B.9}$$

Hence I can write LS representation in terms of jj representation

$$\begin{pmatrix} |^1S_0\rangle \\ |^3S_1\rangle \\ |^1P_1\rangle \\ |^3P_0\rangle \\ |^3P_1\rangle \\ |^3P_2\rangle \\ |^1D_2\rangle \\ |^3D_1\rangle \\ |^3D_2\rangle \\ |^3D_3\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{2}{3}} & \frac{-1}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{2}{5}} & \sqrt{\frac{3}{5}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{3}{5}} & -\sqrt{\frac{2}{5}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |-10\rangle \\ |-11\rangle \\ |10\rangle \\ |11\rangle \\ |-21\rangle \\ |-22\rangle \\ |21\rangle \\ |22\rangle \\ |-32\rangle \\ |-33\rangle \end{pmatrix} \tag{B.10}$$

APPENDIX C

Quantum Dynamic of four-level system in magneto-optical trap

The physical system we study using the described formulation in the section 2.5 is a stationary four-level Rb-85 atom at center of MOT in the presence of cooling and repumping laser fields. The schematic of energy levels and detuning of fields with respect to transition is shown in Fig.(C.1). At the center of trap, the magnetic field is very small in which the Zeeman effect is negligible compared to the Rabi frequencies driven by the cooling and repumping laser fields. This approximation corresponds with an atom interacting with isotropic light polarization and it allows us to apply effective dipole transitions from $S_{1/2}, F$ to $P_{3/2}, F'$ without concerning sub-magnetic levels m_F. The field-free Hamiltonian of four-level atom is

$$\hat{H}_0 = \begin{pmatrix} \hbar\omega_1 & 0 & 0 & 0 \\ 0 & \hbar\omega_2 & 0 & 0 \\ 0 & 0 & \hbar\omega_3 & 0 \\ 0 & 0 & 0 & \hbar\omega_4 \end{pmatrix} \tag{C.1}$$

and the optical interaction with cooing beam and repumping beam are represented in interaction picture as

$$\hat{H}_I = \begin{pmatrix} 0 & \langle 1| V |2\rangle & 0 & 0 \\ 0 & \hbar\omega_2 & 0 & 0 \\ 0 & 0 & \hbar\omega_3 & 0 \\ 0 & 0 & 0 & \hbar\omega_4 \end{pmatrix} \tag{C.2}$$

By applying Eq.(2.69) to this system, the obtained optical Bloch equations are the system of first order coupled differential equations. Under a particular initial condition, the equations can be solved numerically.

In Figure C.2(a)-(b), we present the numerical solutions of optical Bloch equations

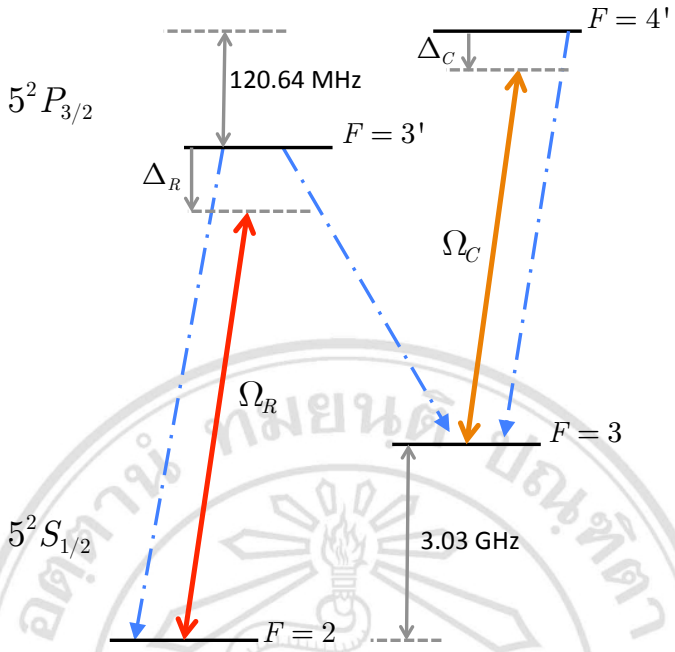

Figure C.1: The D2 line energy levels of rubidium-85. Hyperfine splitting of different F states of excited state $5^2P_{3/2}$ and ground state $5^2S_{1/2}$ are 120.6 MHz and 3.03 GHz respectively. The cooling light and repumping light have the detuning of Δ_C and Δ_R from resonance frequencies $F = 3 \rightarrow F = 4'$ and $F = 2 \rightarrow F = 3'$. Ω_C and Ω_R denote on-resonance Rabi frequencies of the two transitions. The spontaneous decay channels from $F = 3'$ and $F = 4'$ are shown by dash-doted blue line.

under two different initial conditions. The diagonal elements of density matrix are plotted as function of time and it denotes the evolution of probability of finding atoms occupying each hyperfine energy level. Figure C.2(a) shows the result where atom is initially in the lower ground state $F = 2$ and the populations reach the steady state at 2μs. This means at steady state of MOT the population of upper ground state $F = 3$ is dominate while there is around 20 percents atom occupying the excited state $F = 4'$. The result shown in Figure C.2(b) corresponds with the initial condition in which the population at the beginning equally distributes between the two ground states. Although these two initial conditions give the same steady state solutions, the main feature of the second condition is the beat-like fluctuation of populations in $F = 3$ and $F = 3'$. This beat oscillation happens due to the interference between two transitions $2 \rightarrow 3'$ and $3 \rightarrow 4'$ that have slightly different Rabi oscillation frequencies. Figure C.3 shows the time evolution of

coherence term relating to phase relation between $F = 3$ and $F = 3'$. According to the first condition, Figure C.3(a), the real part and imaginary part of the coherence term oscillate with equal amplitude and hence its trajectory reveals a helix curve. In the other hand, the coherence of the second initial condition, Figure C.3(b), has the behavior of perturbed-helical motion.

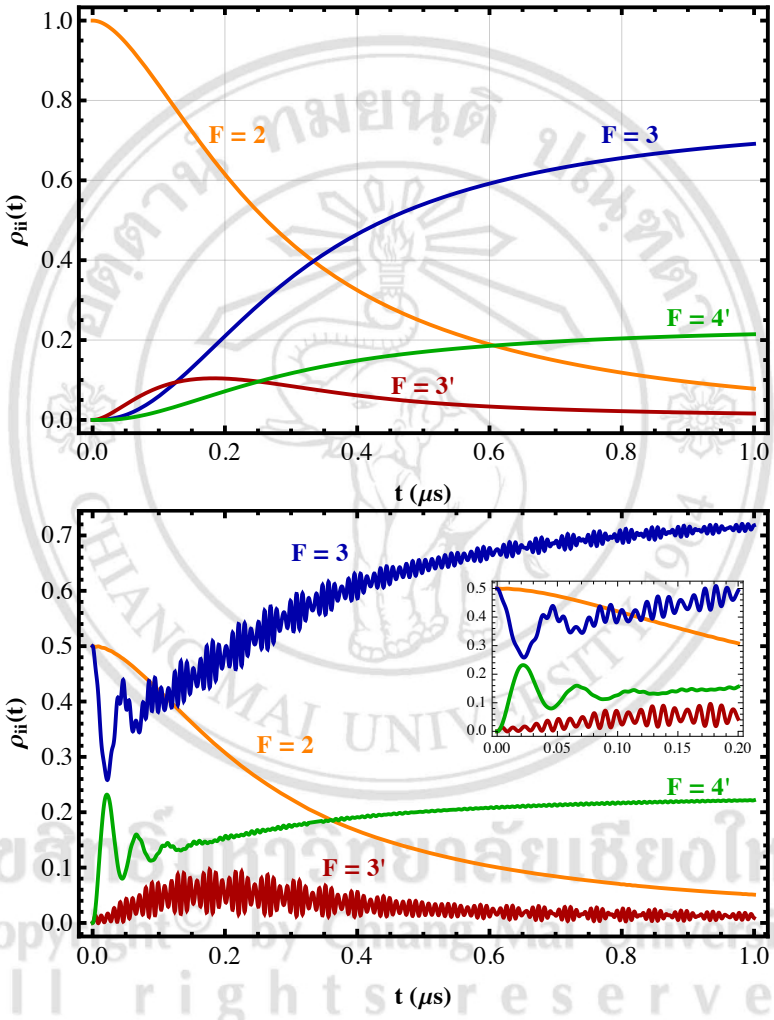

Figure C.2: Dynamic of populations under the presence of the cooling field and the repumping field. (a) atom is initially prepared in the ground state $F = 2$ and (b) the system starts from superposition of the two ground states, $F = 2$ and $F = 3$, with equal probability. The frequency of repumping field is on resonance while the detuning of cooling beam is of 2π(-14)MHz. The Rabi frequencies Ω_C and Ω_R are 114 MHz and 11.3 MHz respectively.

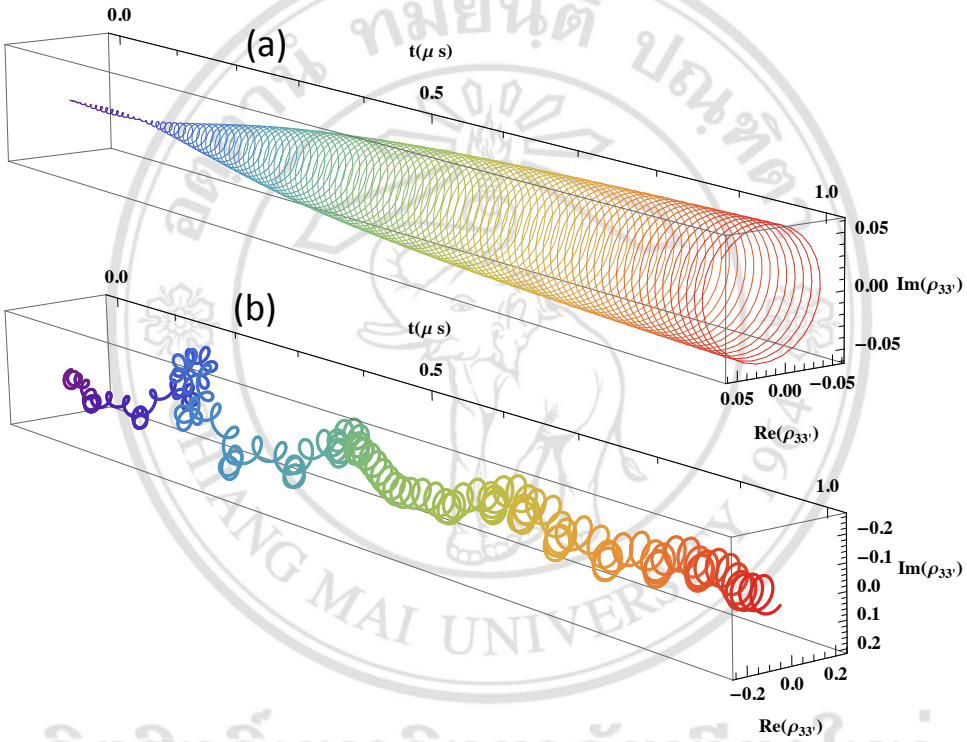

Figure C.3: The real and imaginary parts of coherence term $\rho_{33'}(t)$ as function of time: (a) atom is initially setup in lower ground state and (b) atom initially evolve from equal probability over two ground states.

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