CHAPTER 4

Empirical Result

4.1 Descriptive Statistic

In the first state, precious metal prices (secondary data) consisting of gold, palladium, platinum, and silver were used to calculate the natural log returns which are defined as $r_{i,j} = \ln\left(P_{i,j} / P_{i,j-1}\right)$ where $P_{i,j}$ is the i^{th} metal price at time j, $r_{i,j}$ is the i^{th} log return of metal price at time j, and i indicated the i^{th} precious metal price. The daily return of each precious metal is shown in Figure 4.1. The return plots in markets move in a similar fashion.

In table 4.1, it is clear that the mean of each precious metal variable is positive except platinum. The highest mean returns is palladium (0.000333), the lowest mean return is platinum (-0.000016), and the standard deviations in silver is highest (0.023551) and in gold lowest (0.012547). The value of skewness and kurtosis in all of the precious metal returns are not equal to zero and have excess kurtosis, respectively. Consequently, the distributions of metal returns have a fatter tail than the normal distribution. Moreover, the Jaque-Bera test, which is the normal distribution test of return series, rejects the null hypothesis, thus the return series of precious metal price is non-normal distribution.

Table 4.1 Descriptive statistics on precious metal returns

	GOLD	**PALLADIUM**	**PLATINUM**	**SILVER**
Mean	0.000276	0.000333	-0.000016	0.000089
Median	0.000306	0.000000	0.000000	0.000000
Maximum	0.068653	0.109199	0.069395	0.182786
Minimum	-0.101624	-0.178590	-0.084934	-0.186926
Std. Dev.	0.012547	0.020347	0.014856	0.023551
Skewness	-0.518193	-0.596640	-0.633697	-0.415566
Kurtosis	8.462625	9.509218	7.899701	12.026550
Jarque-Bera	2854.427	4043.631	2364.967	7586.972
Probability	0.000000	0.000000	0.000000	0.000000

Source: Calculation

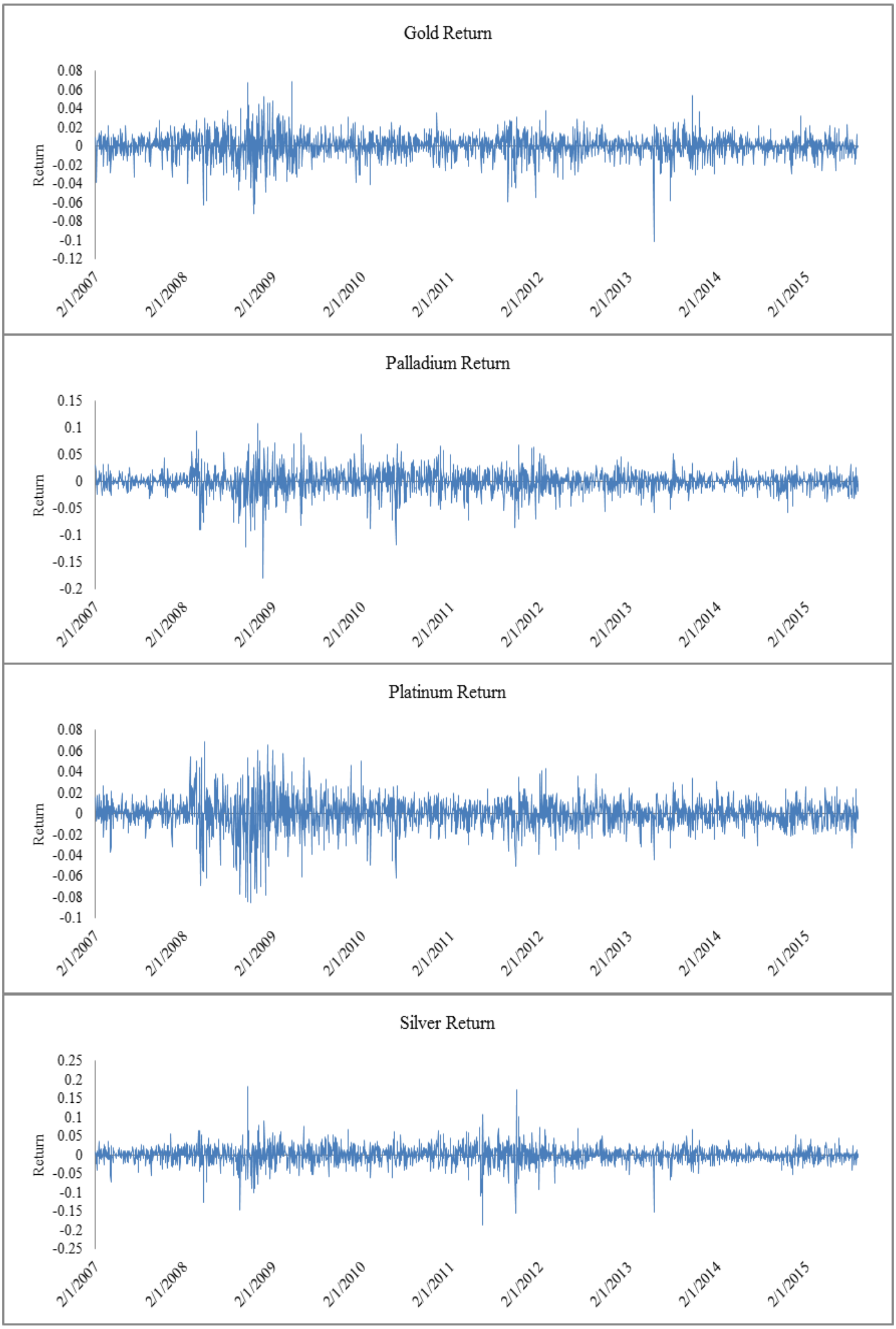

Figure 4.1 Precious metal returns

4.2 Unit Root Test Result

There are 3 approaches for the unit root test to test the stationarity of return consisting of Augmented Dickey-Fuller (ADF), Phillips and Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test shown in table 4.2. The result shows that the unit root tests for each variable have a statistical significance level of 0.01 and all statistic values are less than critical value which conforms to the hypothesis in chapter 3. This means that all of the precious metal returns are stationary characteristics. Therefore, these variables can be used to estimate ARMA-GARCH model in the next step.

Table 4.2: Augmented Dickey-Fuller test, Phillips and Perron test, and Kwiatkowski-Phillips-Schmidt-Shin test on precious metal returns

Unit Root Test	**GOLD**		**PALLADIUM**		**PLATINUM**		**SILVER**	
	Statistic	Critical Value	Statistic	Critical Value	Statistic	Critical Value	Statistic	Critical Value
ADF-test								
None	-47.273*	-2.566	-45.986*	-2.566	-44.112*	-2.566	-51.945*	-2.566
Intercept	-47.284*	-3.433	-45.987*	-3.433	-44.103*	-3.433	-51.934*	-3.433
Trend and Intercept	-47.338*	-3.962	-45.979*	-3.962	-44.124*	-3.962	-51.951*	-3.962
PP test								
None	-47.297*	-2.566	-45.986*	-2.566	-44.120*	-2.566	-51.938*	-2.566
Intercept	-47.319*	-3.433	-45.987*	-3.433	-44.110*	-3.433	-51.927*	-3.433
Trend and Intercept	-47.403*	-3.962	-45.978*	-3.962	-44.128*	-3.962	-51.952*	-3.962
KPSS test								
Intercept	0.373	0.739	0.108	0.739	0.218	0.739	0.184	0.739
Trend and Intercept	0.0364	0.216	0.098	0.216	0.062	0.216	0.059	0.216

Source: Calculation

Note: 1) * denote significant at level 99%

2) Critical value at 1% level

4.3 Marginal Distribution Result

4.3.1 Filter the Returns for Each Index

In this section, we first consider in the sample autocorrelation function (ACF) of the precious metal returns and the sample ACF of the squared precious metal returns, because some degree of autocorrelation and heteroskedasticity is exhibited in most financial return series. In figure 4.2 and figure 4.3 show the sample ACF of precious metal returns and sample ACF of squared precious metal returns.

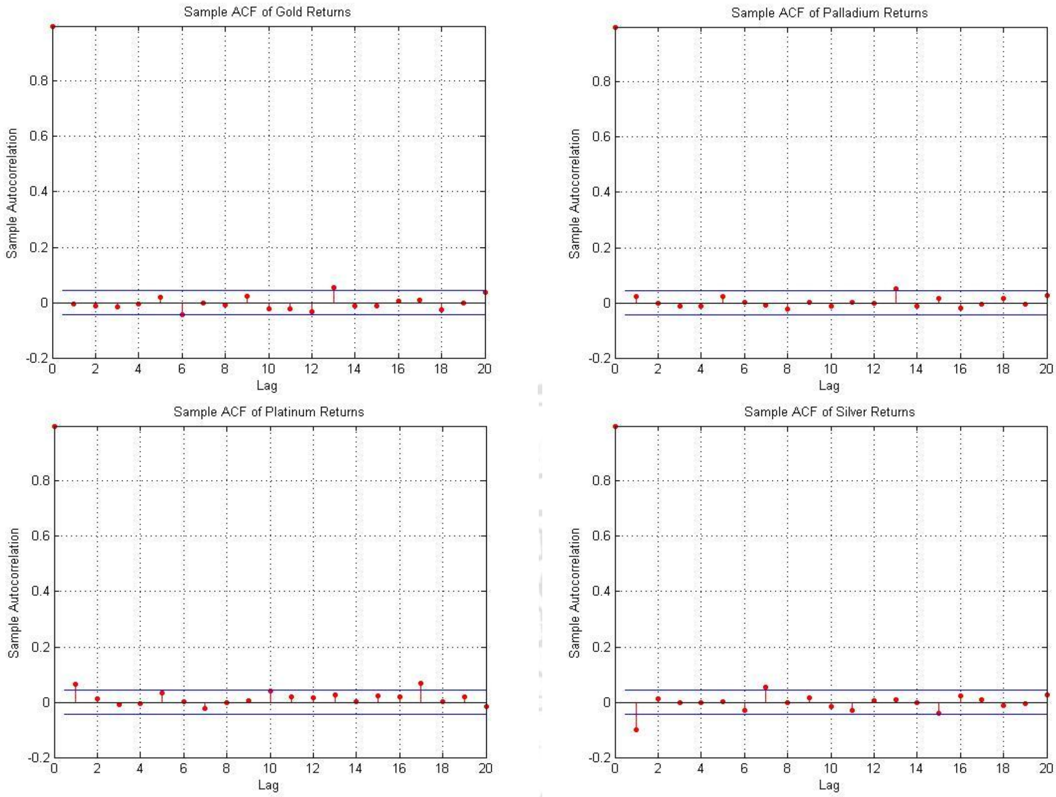

Figure 4.2 The sample autocorrelation function of precious metal returns

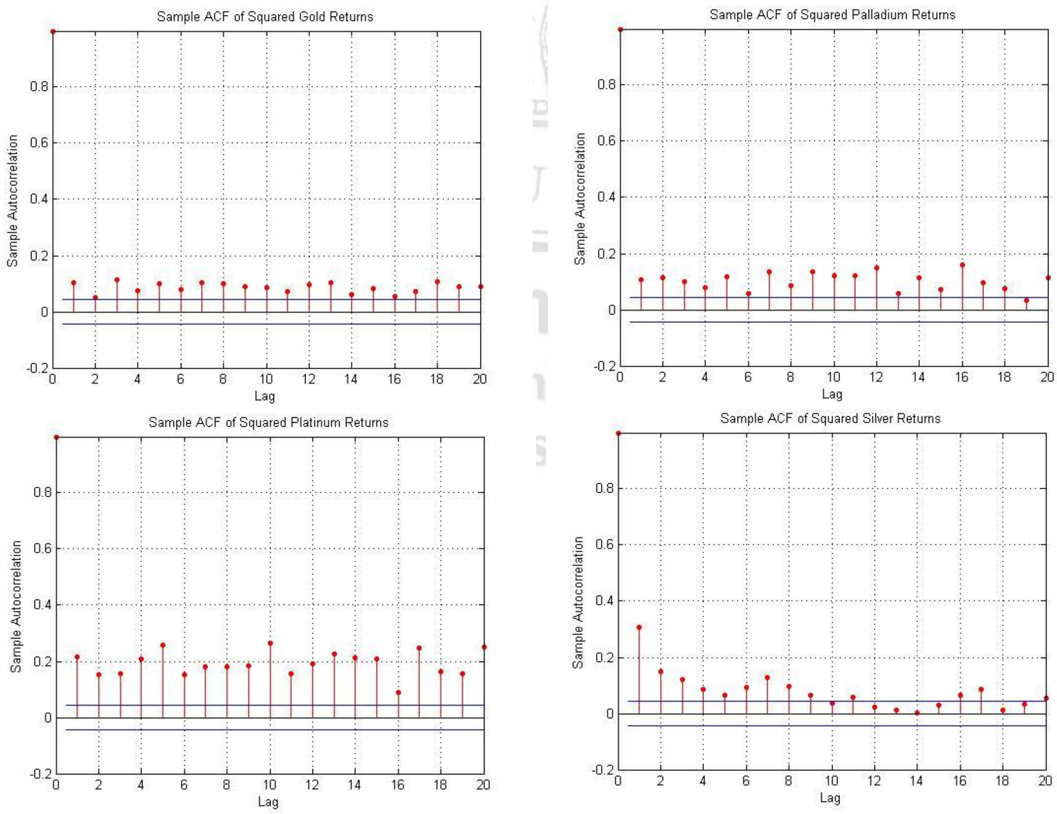

Figure 4.3 The sample autocorrelation function of squared precious metal returns

Figure 4.2 and Figure 4.3 demonstrate the degree of persistence in variance, so the asymmetric ARMA-GARCH model was used to remove the autocorrelation and to capture the conditional heteroskedasticity (Avdulaj, 2010). The ARMA-GARCH model with normal and student-*t* distribution is shown in appendix table 1 and table 4.3, respectively. The appropriated distribution was selected using Bayesian information criterion (BIC) and Akaike information criterion (AIC) which found that student-*t* distribution is appropriate for modeling because AIC and BIC in model with student-*t* distribution is less than the normal distribution. When selecting optimal lag for ARMA(p,q), we found that the return on Gold, Palladium, Platinum, and Silver satisfied ARMA(1,1), ARMA(3,3), ARMA(3,3), and ARMA(2,1) with GARCH(1,1), respectively.

Table 4.3 Estimate of ARMA (q, p) GARCH (1, 1) in student-*t* distribution result

Variable	GOLD	PALLADIUM	PLATINUM	SILVER
μ	0.00059***	0.00055**	-0.00046	-0.00138
AR(1)	0.68338**	-0.76281***	0.21323***	0.90825***
AR(2)	-	0.65244***	0.57005***	0.09116***
AR(3)	-	0.85021***	0.21249***	-
MA(1)	-0.70547***	0.77797***	-0.19034***	-0.99808***
MA(2)	-	-0.66865***	-0.57062***	-
MA(3)	-	-0.87610***	-0.23831***	-
ω	9.98E-07**	2.90E-06***	1.97E-06**	2.51E-06**
α	0.03759***	0.06524***	0.05380***	0.03051***
β	0.95863***	0.93225***	0.93695***	0.96336***
K (*t*-coefficeint)	4.02879***	4.47291***	5.914737***	5.25578***
AIC	-6.195892	-5.306617	-5.917003	-4.974453
BIC	-6.177869	-5.278274	-5.888660	-4.953848
Q (25)	29.499	26.619	15.619	22.797
Q^2 (25)	33.478	26.104	24.102	14.841
ARCH LM (30)	1.2153	1.1584	0.9524	0.5039
Jarque-Bera test	2678.741***	599.8361***	216.1603***	5055.118***
Observations	2,215	2,213	2,213	2,214

Source: Calculation

Note: *, **, and *** denote significant at 90%, 95%, and 99%, respectively

In the ARMA-GARCH model above, we obtained the ordinary residuals, then the ordinary residuals of model were divided by conditional standard deviation, $z_t = \varepsilon_t / \sigma_t$, and thus obtaining standardized residuals which are approximately identically and independently distributed. We can compare figure 4.4 and 4.5 with figure 4.2 and 4.3 which reveal that all autocorrelation is removed and a data are approximately *i.i.d.*

Moreover, the Ljung–Box (Q and Q^2) and ARCH LM statistics were tested for indication of any serial correlation and conditional heteroskedasticity, respectively. The result in table 4.3 showed that in Ljung–Box and ARCH LM test for each precious metal residuals were not at a statistical significance level of 0.01, 0.05, or 0.1. This result reveals that any serial correlation and the heteroskedasticity condition that existed in the precious metals returns series have removed and indicate no significant appearance of the ARCH effect. In terms of the Jarque-Bera test, it is the statistic to test whether the standardized residuals are normal distribution. The results showed that the standardized residuals are leptokurtic, and the Jarque-Bera statistic rejects the hypothesis of normal distribution which means that the fat-tailed asymmetric conditional distributions outperform the normal for modeling and forecasting the precious metals volatility returns.

The above testing represents that the standardized residuals of precious metals in student-*t* distribution are close to *i.i.d.* Thus, the filtering procedure has been effective in producing *i.i.d.* residuals on which EVT can be implemented (McNeil and Frey, 2000) and can suggest that the selected asymmetric ARMA-GARCH models are well specified. Therefore, now we have the data that is a required form to apply in the Extreme Value Theory and Copula model.

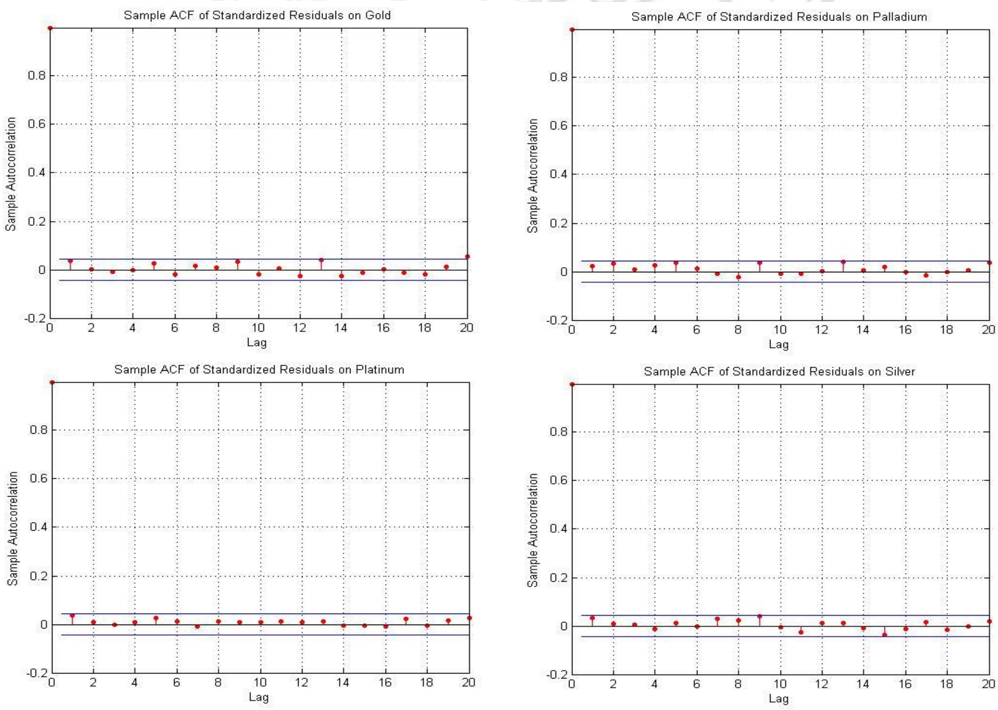

Figure 4.4 The sample autocorrelation function of precious metal residuals

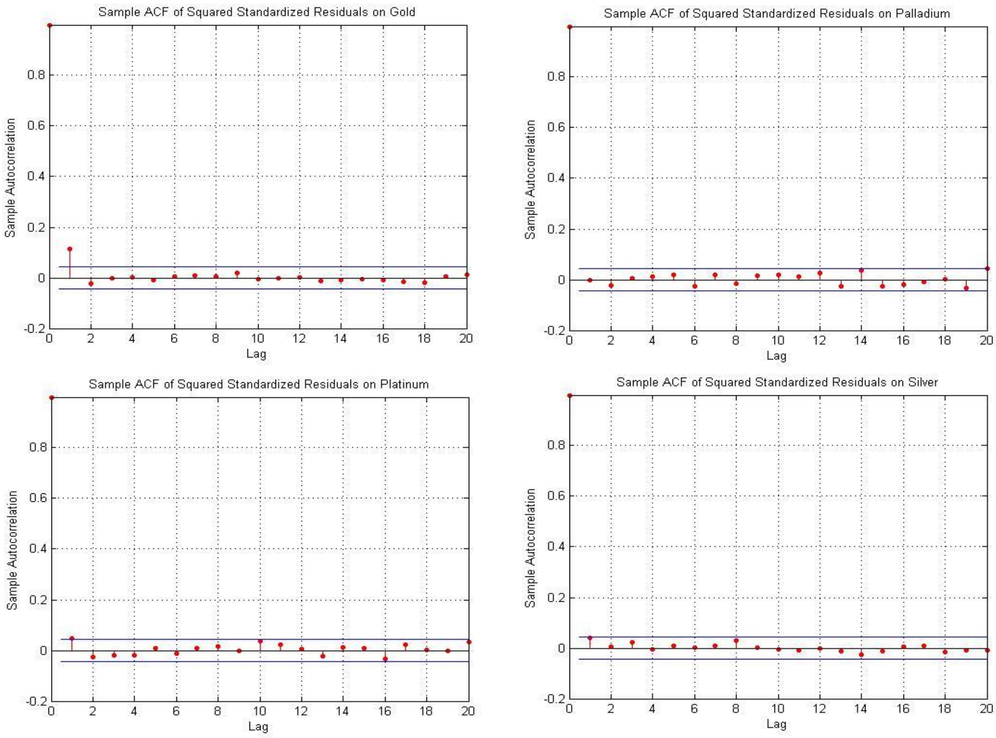

Figure 4.5 The sample autocorrelation function of squared precious metal residuals

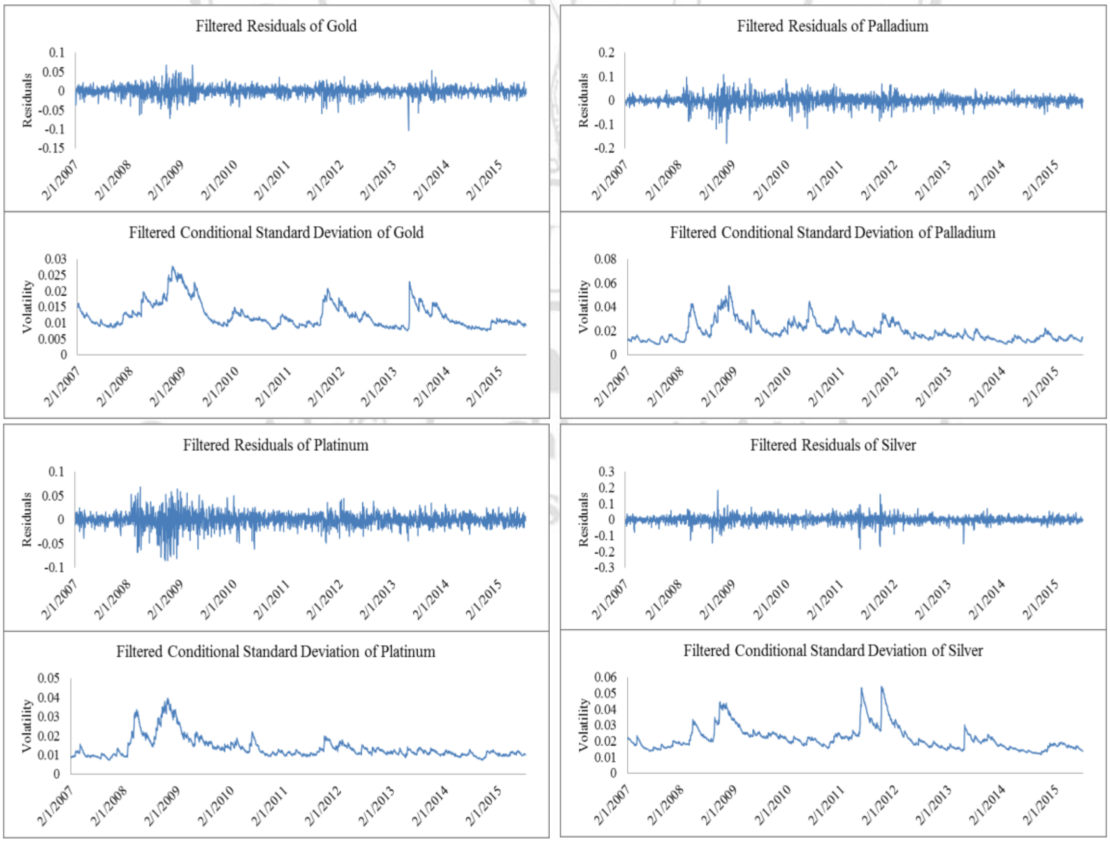

Figure 4.6 Residuals and Conditional Standard Deviation of Precious Metals

4.3.2 Extreme Value Applying

In this section, we estimate the semi-parametric cumulative distribution function for the standardized residual that obtain from ARMA-GARCH model.

EVT is then applied to those residuals which Generalized pareto distribution (GPD) specially is for tail estimation. We chose the exceedances to be the 10th percentile of the sample for the upper and lower tail of the residual distribution of the residuals (see Dumouchel, 1983) because of the appropriateness to choose the 10th percentile in the generalized Pareto model, and the same biases occurred as in the stable law analysis. We assumed that excess residuals over threshold follow the GPD and use the Gaussian kernel estimate for the remaining part.

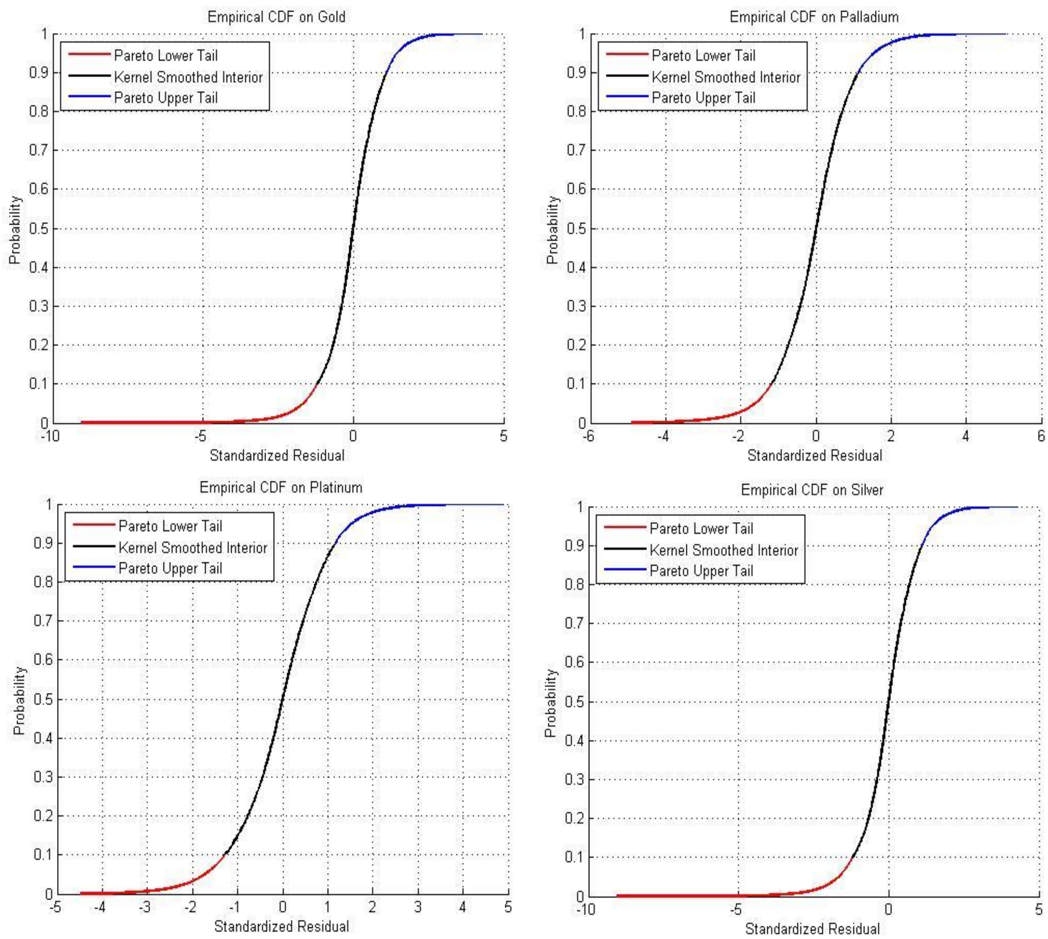

Figure 4.7 Semi-parametric cumulative distribution functions

In Figure 4.7, we plotted the semi-parametric cumulative distribution function of the 4 precious metals case which was plotted from the standard innovation from EVT. The figure represents the lower and upper tail regionns or GPD estimate, expressed by red line and blue lines, respectively and is suitable for extrapolation. Likewise, the kernel-smoothed interior or empirical distribution estimate, displayed in black line, is suitable for interpolation. Moreover, we plotted the empirical CDF of the upper tail exceedances of the residuals along with the CDF fitted by the GPD in figure 4.8 to assess the GPD fit. The diagrams indicate that the fitted distribution (blue line) close to the exceedance data (red line) thus the GPD model is a good choice to apply with each residual of precious metals.

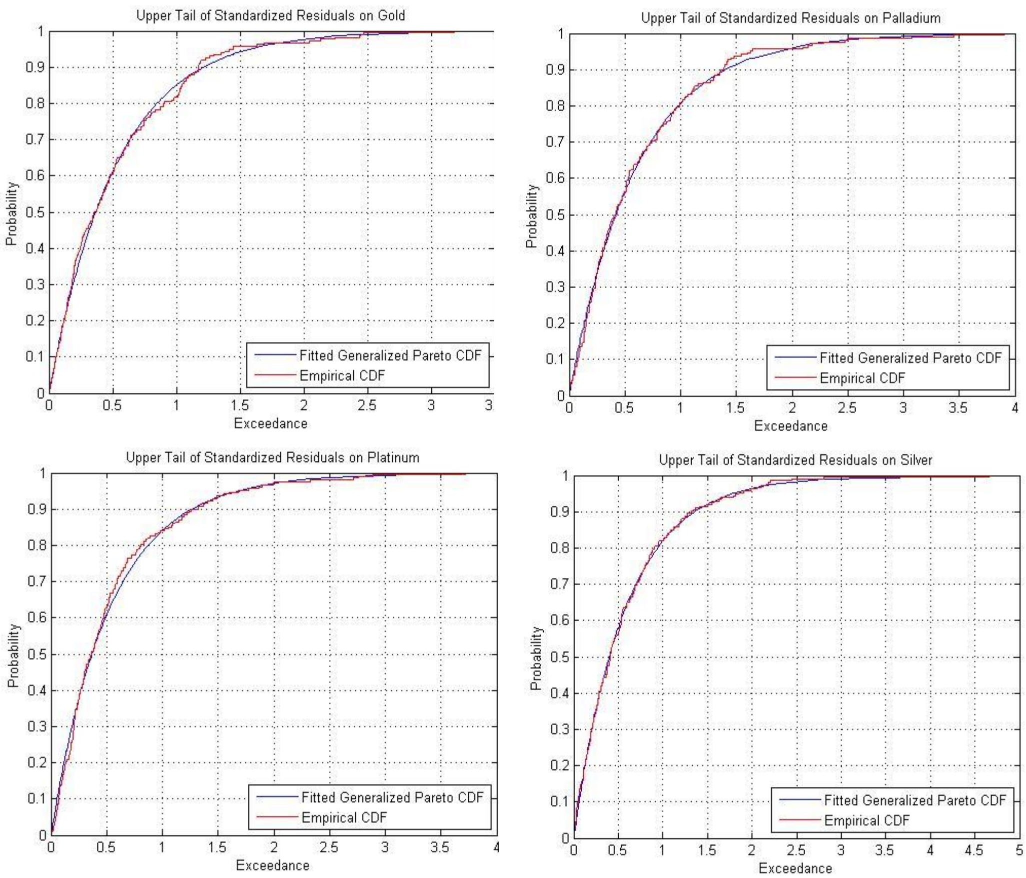

Figure 4.8 Fitted cumulative distribution functions for upper tail

We have confirmed that GPD is a good choice for each residual of precious metals. Therefore, the shape parameter ξ, scale parameter β, and threshold u for each precious metal's residuals were estimated. We chose the exceedances to be the 10th percentile of the sample and used the MEF and Hill plot of sample to define an optimal threshold, which the left tail, right tail, and threshold are demonstrated in table 4.4.

Table 4.4 Parameter estimation for each precious metal's residual

	Gold	**Palladium**	**Platinum**	**Silver**
Threshold u_L	-0.0207	-0.0313	-0.0223	-0.0346
ξ_L	0.1265	0.1291	-0.0046	0.2803
β_L	0.0093	0.0168	0.0160	0.0163
Threshold u_R	0.0187	0.0297	0.0215	0.0324
ξ_R	0.1758	-0.0586	0.0807	0.1401
β_R	0.0070	0.0170	0.0093	0.0152

Source: Calculation

4.4 Copula estimation result

After obtaining the GPD parameter and residuals $z_{it}, i = 1,2,3,4, t = 1,2,...,T$, we substituted z_{it} into equation (81) and obtained the marginal distribution $u_i = F(z_{it})$. Then the standardized residuals were transformed to uniform variates by the semi-parametric ECDF derived above (Figure 4.7) of each margin and calibrating of *t* copula to data by maximum log-likelihood method. According to the estimate of the copula parameter represented in section 3.2.3, the *t* copula can be fit by the transformed data above and then capture the dependence structure between time series. In table 4.5, we have the correlation matrix obtained from fitting the *t* copula while the degrees of freedom are 11.1430. The *t* copula was used to estimate because the Gaussian copula cannot capture the dependence of a fat tail and can show the observations in the tails more than the Gaussian (Wang et al., 2010).

Table 4.5 Empirical GARCH EVT *t* copulas parameters ($\hat{\rho}$)

$\hat{\nu} = 11.1430$	GARCH EVT *t* Copula			
	Gold	Palladium	Platinum	Silver
Gold	1.00000	0.39968	0.52048	0.46417
Palladium	0.39968	1.00000	0.72882	0.52720
Platinum	0.52048	0.72882	1.00000	0.61118
Silver	0.46417	0.52720	0.61118	1.00000

Source: Calculation

We have the empirical GARCH EVT *t* copulas parameters so the simulation algorithm described in section 3.3 was applied in this step. We are able to simulate the returns at time *t*+1 or any time to predict based on the correlation structure of *t* copula or copulas parameters in table 4.5.

4.5 Portfolio Risk Analysis

Now we can find the Value-at-Risk of portfolio. Unfortunately, Avdulaj (2010) states that there is one more thing that should be considered before calculating VaR. As we are working with log returns, we have to be careful because log returns are time additive but not portfolio additive. On the other hand, the simple returns are portfolio additive but not time additive. Thus, when we constructed the portfolio return series, we first converted the individual logarithmic returns to simple returns and multiplied each return series with its weight in the portfolio. In this way, we obtained the portfolio arithmetic return. Finally, we converted back the portfolio return to logarithmic form.

Avdulaj (2010) represented the reasoning clearly, by denoting that $r_t = logP_t / P_{t-1}$ is the log returns, $R_t = (P_t - P_{t-1}) / P_{t-1}$ is the simple returns, and *w* the weight of each index in the portfolio (*w* is column vector). Lets first convert from log return to simple return:

$$\begin{aligned} r_t = log\frac{P_t}{P_{t-1}} &\Rightarrow e^{r_t} = \frac{P_t}{P_{t-1}} \\ e^{r_t} - 1 &= \frac{P_t}{P_{t-1}} - 1 = \frac{P_t - P_{t-1}}{P_{t-1}} \\ e^{r_t} - 1 &= R_t \end{aligned} \tag{90}$$

We weighed the individual simple return for the portfolio at time *t*: $(e^{r_{i,t}} - 1) * w_i$, where $i \in \{1, ..., 4\}$ represents each index. Converting back to log returns and calculating the cumulative returns (the gain/loss during the risk horizon) will be used to construct the ECDF for the simulated returns:

$$\sum_{t=1}^{T} \sum_{j=1}^{H} log(1 + (e^{r_{j,i,t}} - 1) * w_i) \tag{91}$$

where $i \in \{1, ..., 4\}$ stands for each index. *H* is the risk horizon in days and *T* is the length of the simulated time series (also called trials, in our case we set as *T* = 100,000).

We simulated 100,000 independent random trials of dependent standardized index residuals for a risk horizon of 1, 10, and 22 trading days. Then, using the simulated standardized residuals as the *i.i.d.* input noise process, restoring it into equation (78), (79), and (80), we can obtain returns at time *t* + 1, *t* + 10, and *t* + 22.

Finally, given the each of simulated returns, we formed a 1/4 equally weighted index portfolio composed of the individual indices, and calculated the VaR at 99% confidence levels at risk horizon. The 90%, 95%, and 99% VaR estimate are shown in Table 4.6. For the results shown in table 4.6, the estimation of VaR with 90% confidence level for 1 trading day is 2.3405%. We can interpret that in 90% confidence, we expect that worst daily loss will not exceed 2.3405%. For example, if we invest \$100, worst daily loss will not exceed \$2.3405 (\$100×-2.3405%). Additionally, if we take look at a ten day and one month horizon, with 90% confidence, we expect that the worst ten days and one month loss will not exceed 3.2070% and 5.3335%, respectively. We can say that if we invest \$100, our worst ten day and monthly loss will not exceed \$3.2070 and 5.3335, respectively. For the maximum loss for a one day, ten days, and one month risk horizon, it will be 25.3954%, 51.4035%, 82.7718%, respectively. It implies that maximum loss for one day, ten days, and one month trading will be \$25.3954, \$51.4035, \$82.7718, respectively, if we invest \$100.

Table 4.6 Value at Risk based on Monte-Carlo simulation and GARCH EVT *t* Copula by using cumulative return

	1 trading day	**10 trading day**	**22 trading day**
Simulated 90% VaR:	-2.3405%	-7.3904%	-10.8867%
Simulated 95% VaR:	-3.2070%	-9.9119%	-14.4865%
Simulated 99% VaR:	-5.3335%	-15.2624%	-22.5964%
Maximum Simulated Loss:	25.3954%	51.4035%	82.7718%
Maximum Simulated Gain:	20.8516%	35.5827%	50.6751%

Source: Calculation

From the simulated returns, we can plots the profit and loss distribution of our EVT and copula model illustrating in figure 4.9 which shows the empirical CDF of the simulated portfolio returns over 1 trading day, 10 trading days, and 22 trading days or one month. VaR measures can be read from the curve such as VaR at 90% of one month is approximately -0.10, corresponding to 10% cumulative probability. We can say that, at 90% confidence, our portfolio will lose no more than 10% over the next month.

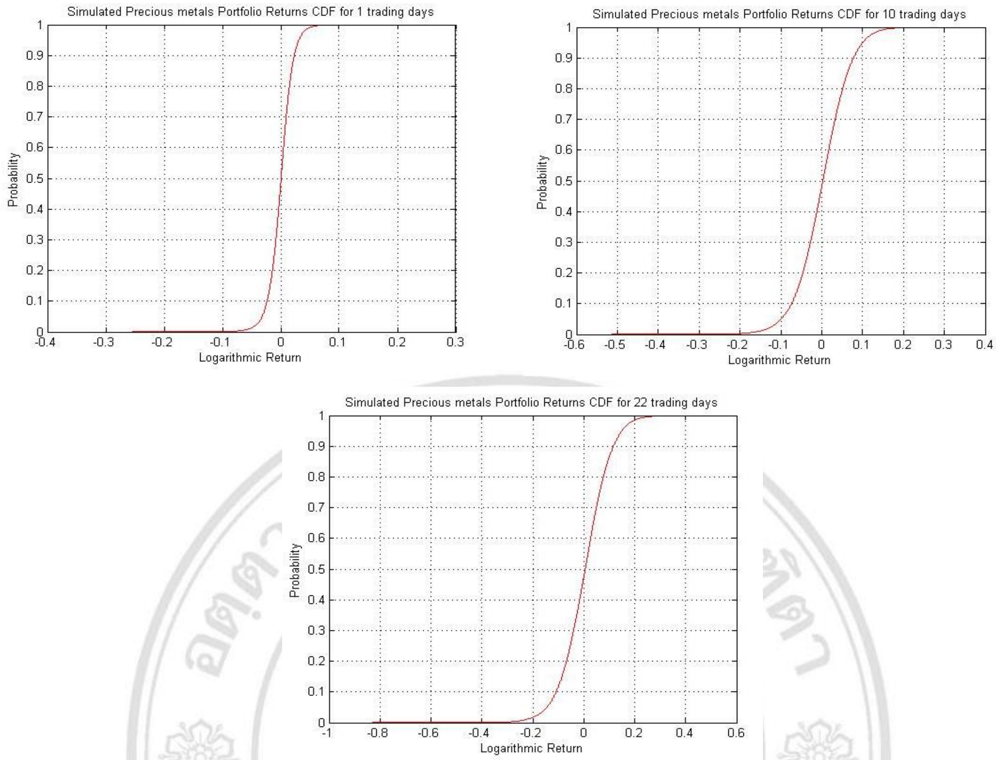

Figure 4.9 Empirical cumulative distribution function of simulated returns

The above VaR estimation is only one way to predict the risk in the future for risk management by using the cumulative return. The risk estimation can be estimated in expected shortfall value, which is easily optimized and requires a larger sample size than VaR for the same level of accuracy. Therefore, table 4.7 shows the calculation VaR and ES of the portfolio with an equally weighted portfolio of four precious metals by using simulated return. In Table 4.7, figure the estimated VaR and ES at level of 1%, 5%, and 10% under the equally weighted assumption. In period t+1, the estimation ES are higher than VaR and converges to -3.689, -4.635 and -7.146 at 10%, 5%, and 1% level, respectively.

Table 4.7 Value at risk and expected shortfall equally weighted portfolios by using simulated return

Portfolio	**Expected Value (GARCH-*t* EVT Copula)**		
	1%	**5%**	**10%**
VaR	-5.391%	-3.229%	-2.359%
ES	-7.146%	-4.635%	-3.689%

Source: Calculation

The ES shows the loss exceeds VaR, appropriating to analyze portfolio risk. So the optimal portfolio weights of the selected assets were calculated under minimized expected shortfalls with respect to maximized returns following equation (89).

Figure 4.10 shows the result of the efficient frontier of the portfolio under different expected return at a given significance level of 5%, which comes from the optimization portfolio based on mean-CVaR (ES) model. For this result, the Monte Carlo simulation was applied to simulate a set of 100,000 samples and to estimate the expected shortfall of an optimal weighted portfolio.

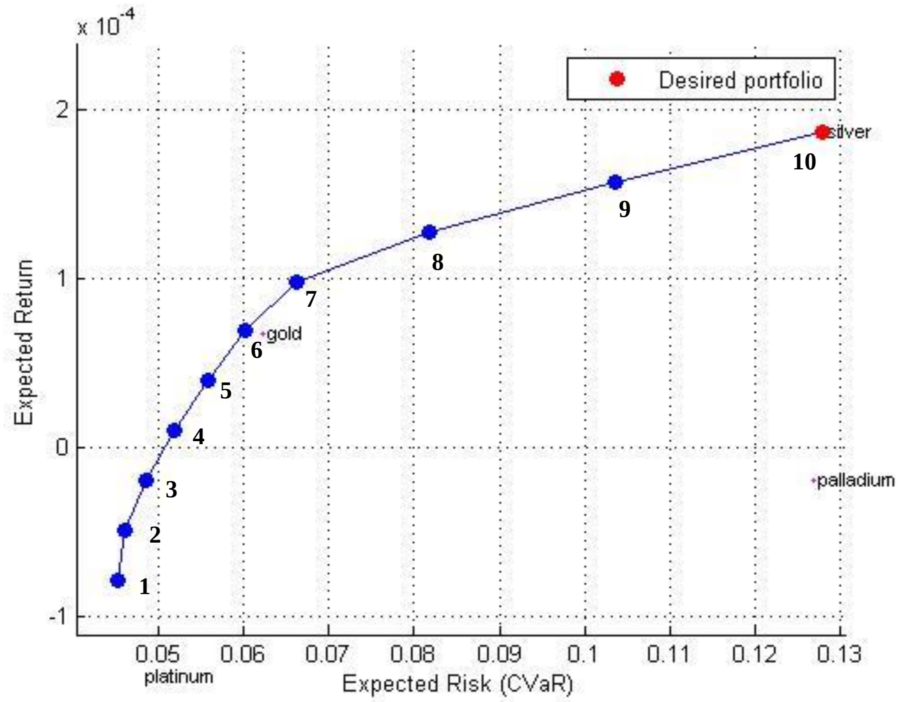

Figure 4.10 The efficient frontiers of CVaR under mean

For the discussion above, the VaR and ES of an equally weighted portfolio was focused on estimating. However, for commercial banks and individual investors, one of the major concerns is to minimize the risk of the investment portfolio. In order to address this concern, the optimal portfolio weight was calculated by minimizing the portfolio risk under minimize expected shortfall with respect to maximize returns. The result is shown in Table 4.8. This result illustrates that most of investment proportions are gold and silver, whereas palladium and platinum have little investment proportion, especially palladium, that having zero proportion in the precious metal portfolio.

Moreover, although platinum has a high investment proportion in portfolios 1 to 3, these portfolios have negative return. Therefore, most of investment proportions focus on gold and silver for portfolios 4 to 10.

Table 4.8 Optimal investment proportion of precious metal portfolio with minimum risk (ES 5%)

Portfolios	Investment proportion				Returns	Risk
	Gold	Palladium	Platinum	Silver		
1	0.280	0.000	0.720	0.000	-0.0079%	4.5220%
2	0.426	0.000	0.574	0.000	-0.0050%	4.6159%
3	0.550	0.000	0.436	0.014	-0.0020%	4.8674%
4	0.631	0.000	0.315	0.055	0.0010%	5.2007%
5	0.710	0.000	0.193	0.096	0.0039%	5.5886%
6	0.798	0.000	0.069	0.133	0.0069%	6.0205%
7	0.738	0.000	0.000	0.262	0.0099%	6.6171%
8	0.492	0.000	0.000	0.508	0.0128%	8.1744%
9	0.246	0.000	0.000	0.754	0.0158%	10.3610%
10	0.000	0.000	0.000	1.000	0.0188%	12.7779%

Source: Calculation

The result of optimum portfolio in table 6 suggests that the investment proportions in each portfolio should not have focused on palladium. The reasons are that, in 2013, the palladium price high oscillated, and in the palladium market was excess supply, so that the palladium prices declined along with gold price (Scotiabank, 2013). In addition, in 2015, the palladium price highly decreased because China's automotive market had decreasing sales volumes (Arnold, 2015).