CHAPTER 3

Forecasting risk and returns: CAPM model with belief functions

This chapter is developed from the original paper namely, “Forecasting risk and returns : CAPM model with belief functions”. The contents are extracted from the original paper that was published in “Econometrics of Risk”, Studies in Computational Intelligence Volume 583, pp 259 – 271. This paper can be found in the appendix A. The methodology of this study was described in Chapter 2.

3.1 Introduction

Most investors focus on the stock market return forecasting. The aim is to gain high profit by using the best trading strategies. The more successful in stock return prediction, the more profitable it becomes in stock market investment. The uncertainty and volatility of stock prices have an effect on the investor’s decision. The knowledge on the dependence pattern between stock and market returns can help portfolio investors to diversify their assets better as well as reducing their risk at the suitable moments. The Capital Asset Pricing Model (CAPM) is a foundation and widely used model for evaluating the risk of a portfolio of assets with respect to the market risk which was introduced by Sharpe (1964). The CAPM is a linear model that estimates asset prices using the information on the risk free rate and the market returns. The CAPM takes into account the non- diversifiable risk, which is captured by the parameter β. The CAPM non-diversifiable risk depends on the correlation between particular stock and overall stock market. Essentially, the standard CAPM model depends on the assumptions of normality of returns and quadratic utility functions of investors.

However, the numerous empirical evidences that have been carried out to analyze the applicability of CAPM in different stock markets have failed to maintain this relationship due to the inadequacy of the market beta alone in explaining the variations in stock returns and the assumptions of CAPM model. For example, Isa et al. (2008) applied CAPM in the Malaysian stock market by using the linear regression method, which was carried out on four models. The result indicated that both of the standard CAPM models with constant beta and time varying beta are statistically insignificant. On the other hand, the CAPM models conditional on segregating positive and negative market risk premiums are statistically significant. Nikolaos (2009) evaluated of CAPM's validity in the British Stock Exchange. The result showed that under the two steps procedure, the CAPM does not have a statistical significance in portfolio selection. Choudhary and Choudhary (2010) applied the CAPM model for the Indian stock. There is a lack of substantiating the theory's basic result illustrating that there is higher risk (beta) is associated with higher levels of return. Masood et al. (2012) examined the validity of the CAPM in the capital markets of the Pakistan. The least squares method (OLS) is used to find the beta of the stocks in the first step and then searches for the regression equations in second step. The result showed that there is no support with the CAPM. The intercept term is equal to zero. Also, there is a positive relation between the risk and return. In addition, the market risk premium is a significant explanatory variable for the determining to see if the stock's risk premium are rejected. Zhang and Meng (2013) analyzed the CAPM model in the Chinese stock market. The main problem of their studies was found that the effective test method did not exist.

From the above literature reviewed, CAPM is a useful tool to estimate the stock market return in different stock index. It can be concluded that there is no one model that can claim to have the absolute ability to predict the expected stock return by using the standard CAPM model. Then, there is a need of accurate forecast model that consistently predict uncertainty and volatility of the stock market prices. The stock market investor would be able to make decisions on the investment that is more informed and accurate. Therefore, various techniques are used for handling the uncertainty data. One such method applied is the Dempster-Shafer belief function theory, which is a useful tool for forecasting. Many studies have applied the belief function model to predict the uncertainty data. For instance, Nampak et al. (2014) used the belief function model in order to forecast groundwater of specific area in Malaysia.

Abdallah et al. (2014) cooperated the statistical judgements with expert evidence by using belief function for prediction the future centennial sea level which climate change is considered. Kanjanatarakul et al. (2014) used the Bass model for innovation diffusion together with past sales data and the formalism of belief functions to quantify the uncertainty on future sales. In their studies, a piece of evidence as a belief function was considered which can be viewed as the distribution of a random set. Furthermore, two main reasons for using the belief function formalism in this paper are the following:

1) The belief function approach does not require the statistician to arbitrarily provide a prior probability distribution when prior knowledge is not available.
2) We wish to measure the weight of statistical evidence that pertains to some specific questions, whereas confidence and prediction intervals are related to sequences of trials.

For more discussion on the comparison bet the belief function approach and classical methods of inference, the reader can find more information with the regards to the work done by Kanjanatarakul et al. (2014). In this contributions, we propose and alternative method for drawing inference via a likelihood based on a belief function approach for estimation of linear regression of CAPM. The objectives of this study are to (1) analyze the dependence pattern between the CHK stock and market returns and to (2) forecast the CHK stock returns using belief functions.

3.2 Data

The data contain of 209 weekly returns during 2010-2013 are obtained from Yahoo to compute the log returns on the following securities. Integrated oil and Gas Company - A company that participates in every aspect of the oil or gas business, which includes the discovering, obtaining, producing, refining, and distributing oil and gas. The log returns prices by using the formula:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \tag{3.1}$$

where P_t and P_{t-1} are the weekly closing prices at time t and $t-1$ respectively. Mukherji (2011) indicated that the treasury bills are better proxies for the risk-free rate than longer–term treasury securities regardless of the investment horizon, which is only related to the U.S. market. In this paper, the treasury bills stand for the risk free rate. The daily returns of the treasury bills are adjusted to the weekly returns and can be used in this manner by using the compound interest that take form:

$$I_{wj} = \{\prod_{i=1}^{N}(1 + I_{di})\} - 1, \qquad (3.2)$$

where $I_{wj}, j = 1, \ldots, N$ is the weekly interest rate and $I_{dj}, j = 1, \ldots, N$ is the daily interest rate.

3.3 Empirical Results

This study aim to analyze the dependence pattern between the CHK stock and market returns and to (2) forecast the CHK stock returns using belief functions. First, we calculate the beta coefficients in CAPM model by using Maximum likelihood method. The parameters $\beta_0, \beta_1 and\ \sigma^2$ are shown in Table 3.1

Table 3.1 Parameter estimation results

Stock name	Parameters	
CHK	β_0	$-0.001(0.0031)$
	β_1	$1.436(0.1417)$
	σ^2	$0.0020(1.917e^{-4})$

Table 3.1 presents the results of the parameters estimation for the CAPM. The result shows the positive beta value (β_1) for CHK stock returns these mean that the market returns have a positive effect to stock returns. This indicates that CHK return has the high expected risk and their price will be volatile than the S&P500 market.

Figure 3.1 displays two-dimensional marginal contour functions, with one of the three parameters fixed to its MLE.

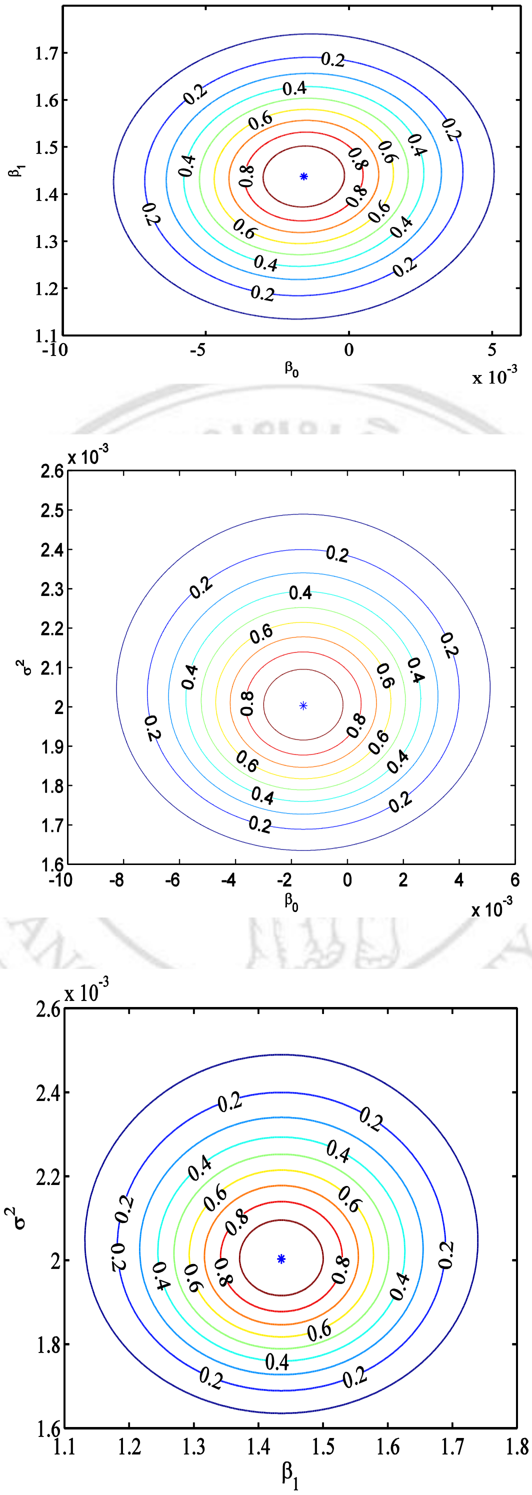

Fig.3.1 Displays two –dimensional marginal contour functions

According to figure 3.1 shows the marginal contour functions for parameters $\beta_0, \beta_1, \sigma^2$. These three plausibilities will be used to perform plausibility intervals for each of the three parameters.

To predict the expected return of the asset $y_{i,n+1}$ for a new market portfolio return $X_{i,n+1}$ we compute the minimum and maximum of $y_{i,n+1}$ given $X_{i,n+1}$ by

$$y_{i,n+1} = \beta_0 + \beta_1 X_{i,n+1} + \sigma F^{-1}(u_s), \tag{3.3}$$

under the constraint $pl(\theta) \geq \omega_s$, where $F^{-1}(u_s)$ is the inverse cumulative distribution function (cdf) of the normal distribution and u, ω are independent random variables with the same uniform distribution $U([0,1])$.Given equation (3.3), we randomize independently N pairs of the random number, $(u_s, \omega_s); S = 1,2, \dots, N$ resulting in N intervals $[y_{i,s}^L(u_s, \omega_s), y_{i,s}^U(u_s, \omega_s)]$ For any $A \subset R$, the stock returns $Bel_{yi}(A)$and $Pl_{yi}(A)$ can be estimated by equation (3.4).

$$y_i = \alpha + \beta x + \varepsilon_i \tag{3.4}$$

The estimated lower and upper expectations of $r_{i,n+1}$ are then:

$$\bar{y}_{i,s}^L = \sum_{s=1}^{N} \frac{y_s^L(u_s, \omega_s)}{N}, \tag{3.5}$$

$$\bar{y}_{i,s}^U = \sum_{s=1}^{N} \frac{y_s^U(u_s, \omega_s)}{N}, \tag{3.6}$$

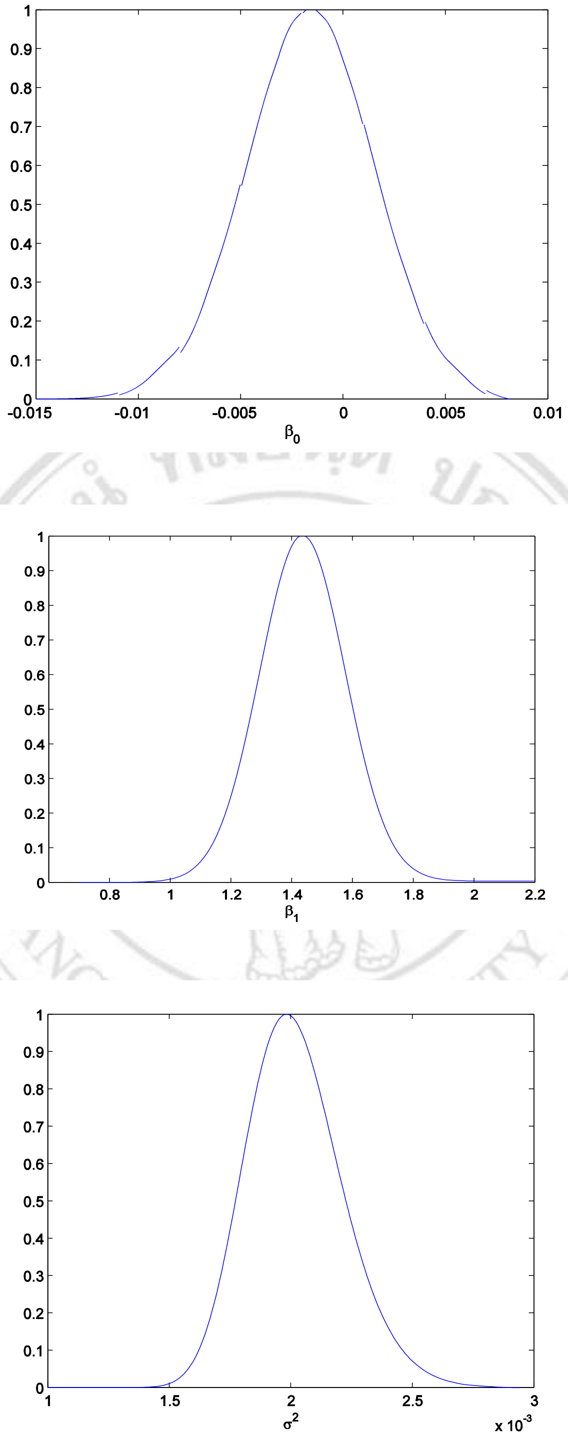

Figure 3.2 parameters $\beta_0, \beta_1, \sigma^2$.

Figures 3.3 displays the lower and upper cdfs $Bel_{yi}([-\infty, y_i)$ and $Pl_{yi}([-\infty, y_i)$ .This function give us the summary of the predictive belief function Bel_{yi} .

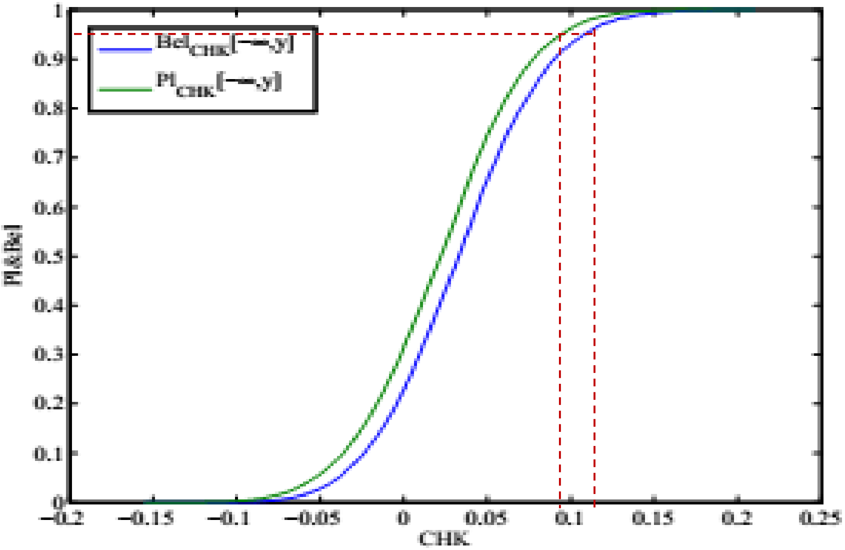

Fig.3.3 Lower and Upper cumulative distribution function

Figure (3.3) shows the upper and lower bound of stock return via CAPM using belief function. In other words, it exhibits the forecasting lower and upper cdfs $Bel_{yi}([-\infty, y_i)$ and $Pl_{yi}([-\infty, y_i)$. For example, the stock return will have the range of the plausibility and belief equal to (0.09-0.11) at $\alpha = 0.95$.

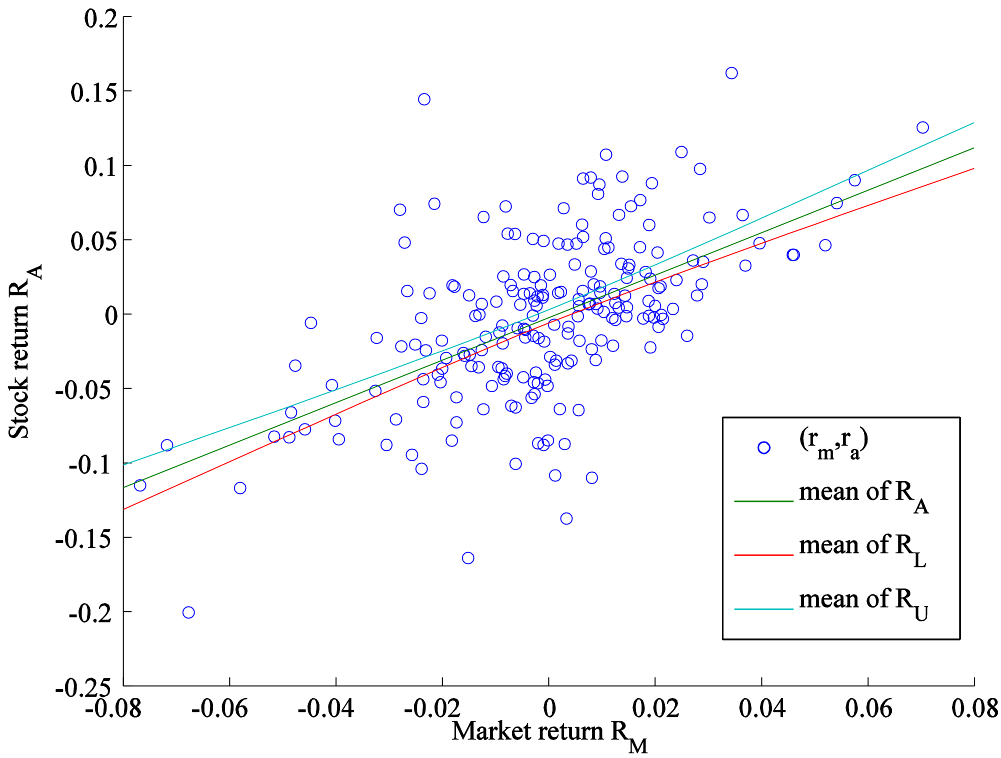

Fig.3.4 Lower and Upper interval of stock return via CAPM using belief function

The another representation of uncertainty prediction can be defined as the lower–upper expectations of stock returns, the uncertainty and randomness estimation are considered. From the empirical result, the gap between the lower and upper cdfs is quite narrow, which shows that estimation uncertainty is small as compared to random uncertainty. Therefore, the investor can use these results to increase the gain of portfolio investment (Autchariyapanikul et al., 2014).

3.4 Conclusions

In this paper, we presented the method of standard CAPM with normal distribution for CHK stock in S&P500 in the belief function framework. The Dempster-Shafer belief function theory was used in order to identify the uncertainty. The statistical prediction based on historical data and a financial model. This method consists of two steps. First, a belief function is defined from the normalized likelihood function given the past data which is referred to the uncertainty on the parameter vector θ .Second, the return of stock y_i is illustrated as $\varphi(\theta, u)$, where u is a stochastic variable with known distribution. Then, belief on θ and u are transferred through φ , resulting in a belief function on y_i. This approach has been adapted to the prediction of the stock returns. A possible extension of this work is to consider uncertainty on the independent variable r_m, which can also be expressed as a belief function and combined with other uncertainties to compute a belief function on y_i.