# CHAPTER 4

## Capital Asset Pricing Model with Interval Data

This chapter is developed from the original paper "Capital Asset Pricing Model with Interval Data". The contents are extracted from the original paper that was published in "Integrated Uncertainty in Knowledge Modelling and Decision Making" LNAI 9376, pp 163 – 170. This paper can be found in the appendix C. The methodology of this study was described in Chapter 2.

### 4.1 Introduction

Capital asset pricing model provides a piece of information of asset return related to the market return via its systematic risk. In general, asset returns of any interested asset and market returns are calculated from a single-valued data. Most of the papers in financial econometrics use only closed price taking into account for calculation but in the real world stock price is moving up and down within the range of highest price and lowest price. So, in this paper we intend to use all the points in the range of high and low to improve the results in our calculations. We also put an assumption of a normal distribution on these interval-valued data.



An enormous number of research on CAPM model with single-valued data could be found in much financial research topic, the reader is referred to, e.g., William F. Sharpe (1964) and John Lintner (1965) only a single-valued of interest was considered. Many various technics were applied to the original CAPM model that we can found in the work from Autchariyapanitkul et al. (2015), the authors used quantile regression under asymmetric Laplace distribution (ALD) to quantify the beta of the asset returns in CAPM model. The results showed that this method can capture the stylized facts in financial data to explain the return of stocks under quantile, especially under the middle quantile levels. In Barnes and Hughes (2002), the beta risk is significant in both tails of the conditional distribution of returns. In Chen et al. (2012), the authors used a couple of methods to obtain the time-varying market betas in CAPM to analyze stock in the Dow Jones Industrial for several quantiles. The results indicated that smooth transition quantile method performed better than others methods.

Interval-valued data has become popular in many research fields especially in the context of financial portfolio analysis. Most of the financial data are usually affected by imprecision, uncertainty, inaccuracy and incompleteness, etc. The uncertainty in the data may be captured with interval-valued data. There are several existing research in the literature for investigating this issue. see Billard (2007), Carvalho et al. (2012), Cattaneo and Wiencierz (2002), Diamond (1990), Gil et al. (2002), Kőrner and Näther (1998), Manski (2002), Neto and Carvalho (2008). However, in these research papers are lacking in a foundation and theoretical background to support this idea.

The connection between the classical linear regression and the interval-valued data that share the important properties could be found for the work by Sun and Li (2015). In their paper, they provided a theoretical support framework between the classical one and the interval-valued linear regression such as least squares estimation, asymptotic properties, variances estimation, etc. However, in their paper only one of an explanatory variable can use to describe the responding variable. In this paper, we intend to apply the concept of the interval-valued data to the CAPM model. We replace a single value of market returns and asset returns with the range of high and low historical data into the model.



**4.2 Data**

The data contains 259 weekly interval-valued returns in total during 2010- 2015 are obtained from Yahoo. We compute the log returns on the following stock, namely, Chesapeake Energy Corporation (CHK) and Microsoft Corporation (MSFT). Due to significant capitalization and high turnover volume.

In this study, we use Treasury bills as a proxy. From Autchariyapanitkul et al. (2015) and Mukherji (2011) suggested that Treasury bills are better proxies for the risk-free rate, only related to the U.S. market.

### 4.3 Empirical Results

We consider the following financial model that is so called Capital Asset Pricing Model (CAPM). Only two sets of weekly interval-valued data of Chesapeake Energy Corporation (CHK) and Microsoft Corporation (MSFT) are used to explain the relationship of the assets and market returns. The fitted model is based on the least square estimation. The parameters estimation by the likelihood for an interval values as follows:

$$\max_{a,b,k_0} \ln L(a,b,k_0 \mid ([\underline{x_i}, \overline{x_i}], [\underline{y_i}, \overline{y_i}]), i = 1, \ldots, n) = \max_{a,b,k_0} \{ \sum_{i=1}^{n} ln\emptyset(\frac{\tilde{y}_i - a\tilde{x}_i - b}{k_0\sqrt{a^2 \Delta x_i^2 + \Delta y_i^2}})\} \qquad (4.1)$$

where $\emptyset(.) \sim N(0,1)$.

Table 4.1 and Table 4.2 report the estimated results for CHK and MSFT from equation (4.1).

**Table 4.1** Estimated parameter results for CHK

| | Interval-valued data | | Point-valued data | |
|---|---|---|---|---|
| Parameters | Values | Std. Dev. | Values | Std. Dev. |
| $\beta_0$ | -0.0021 | 0.0233 | -0.0191 | 0.0055 |
| $\beta_1$ | 0.9873 | 0.0914 | 0.7226 | 0.0713 |
| k | 0.44472 | 0.0845 | - | - |
| MSE | - | - | 0.0360 | - |
| LL | 525.7021 | - | 361.1400 | - |
| $\sigma\chi^2$ | 259.00 | - | - | - |
| AIC | -1045.04 | - | -716.28 | - |

For example, the simple linear regression model for the asset returns (Y) and the market returns (X) for interval valued data for CHK is written to be

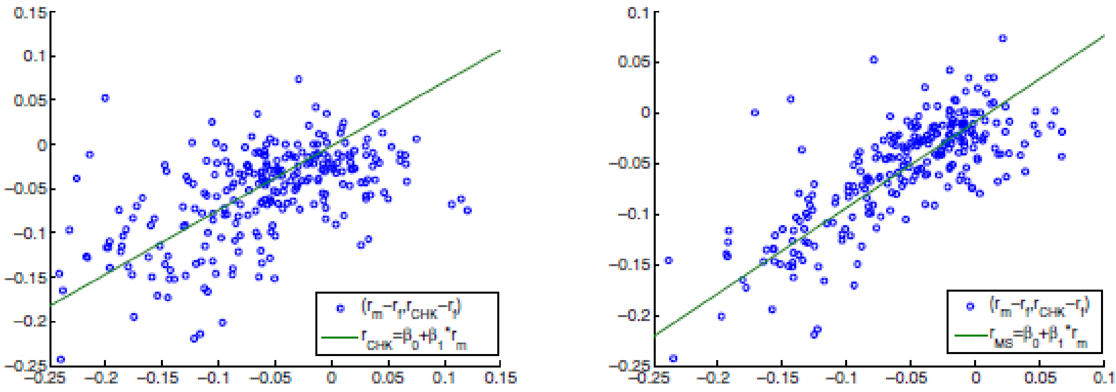
$$R_A = -0.0021 + 0.9873 R_M. \quad (4.2)$$

**Table 4.2** Estimated parameter results for MSFT

| | Interval-valued data | | Point-valued data | |
|---|---|---|---|---|
| Parameters | Values | Std. Dev. | Values | Std. Dev. |
| $\beta_0$ | -0.0004 | 0.0015 | -.0.0088 | 0.0035 |
| $\beta_1$ | 1.0086 | 0.0220 | 0.8489 | 0.0005 |
| k | 0.4017 | 0.0170 | - | - |
| MSE | - | - | 0.0025 | - |
| LL | 692.3808 | - | 478.9365 | - |
| $\sigma\chi^2$ | 259.00 | - | - | - |
| AIC | -1378.76 | - | -951.87 | - |

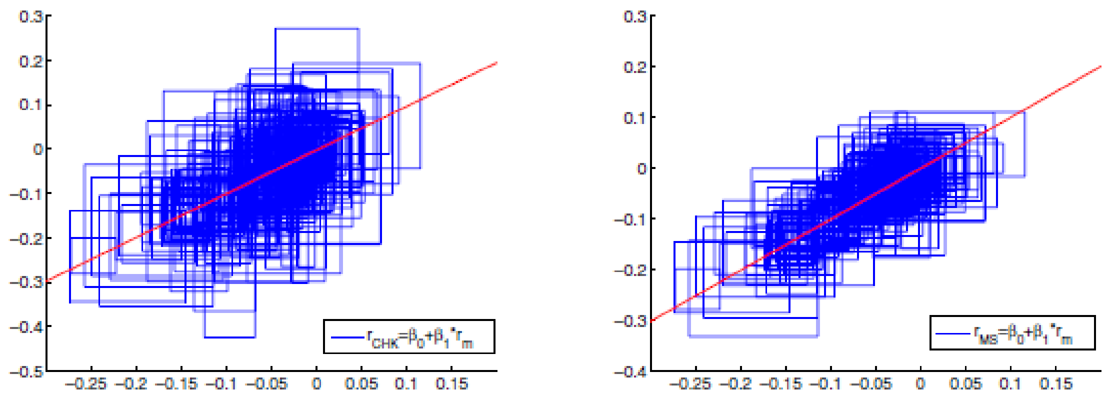
From the linear equation, the return of a stock is likely to increase less than the return from the market. A non-parametric chi-square test is used to validate the method of interval-valued data. The theoretical $\chi^2_{n-2}$gives the value of CHK, $\chi^2_{n-2} = 303.2984$ compare with the empirical value $\chi^2_{emp} = 259.000$ confirm that the market returns can be used to explain the asset returns. The model selection criteria Akaike information criterion (AIC) was employed to compare these two techniques. The AIC of interval-valued data gives a value of $-1051.4402$ is smaller than the AIC of pointed-valued data, which indicate that the results from the interval-valued method is more preferable than the deterministic one.

The relationship between market return and asset return are plotted in Figure 4.1 and Figure 4.2 for pointed-valued data and interval-valued data, respectively. The rectangular are the high and low interval-valued data, and the straight line is the securities characteristic line, the slope of this straight line represent the systematic risk beta. All investments and portfolio of investments must lie along a straight line in the return beta space.




(a) CHK (b) MSFT

Fig. 4.1 The securities characteristic line for point valued data.




(a) CHK (b) MSFT

Fig. 4.2 The securities characteristic line for interval- valued data.

## 4.4 Conclusions

The systematic risk has played as the critical role of financial measurement in capital asset pricing model. Academic and practitioners attempt to estimate its underlying value accurately. Fortunately, there have been the novel approaches to evaluating the beta with interval-valued data. We used every price range of real world data to obtained the single value of the systematic risk same as the results from the conventional CAMP model.

In this paper, we use our approach to an interval-valued data in CAPM for only one stock in $S\&P500$ for a demonstration. With this, a method can be used to investigate the linear relationship between the expected asset returns and its asymmetric market risk by including all of the levels of prices in the range of an interval-valued data. The results clearly show that the beta can measure the responsiveness to the asset returns and market returns. However, only a systematic risk is calculated through the model, and we neglect the unsystematic risk under CAPM assumption. CAPM concludes that the expected return of a security or a portfolio equals the rate on a risk-free security plus a risk premium.

By AIC criterion, it should be noticed that the estimation by using interval-valued data more reasonable than just used the single valued in the calculations. Not only one explanatory variable can be used to explain the outcome variable but with this method also allowed us to use more than one covariate in the model.



For future research, we are interested to use this method to the time series models such as ARMA, GARCH model. Additionally, we can use this method to the model with more than one explanatory variables such as Fama and French (1993). A three-factor model can be extended the CAPM by putting size and value factors in the classical one.