CHAPTER 5

Optimizing Stock Returns Portfolio Using the Dependence Structure Between Capital Asset Pricing Models : A Vine Copula-based approach

This chapter is developed from the original paper namely, "Optimizing Stock Returns Portfolio Using the Dependence Structure Between Capital Asset Pricing Models: A Vine Copula-based approach". The contents are extracted from the original paper that was published in "Causal Inference in Econometrics", Studies in Computational Intelligence Volume 622, pp 319 – 331. This paper can be found in the appendix C. The methodology of this study was described in Chapter 2.

5.1 Introduction

An important task of financial institutions is evaluating the exposure to market and credit risks. Market risks arise from variations in prices of equities, commodities, exchange rates, and interest rates. Credit risks refer to potential losses that might occur because of a change in the counterparty's credit quality such as a rating migration or a default. The dependence on market or credit risks can be measured by changes in the portfolio value, or gains and losses.

The classical portfolio theory was originally conceived by Markowitz in 1952, the idea that explained the return of the portfolio by mean and variance. Since econometrics concerns quantitative relations in modern economic life, its analysis consists mainly of determining the impact of a set of variables on some other variable of interest. For example, we wish to determine how return on market X affects return on asset Y in a stock exchange. Now this problem is a regression problem, namely, capital asset pricing model (CAPM). We regress the values of the variable of interest Y, usually called the dependent variable in the explanatory variable X, often called the independent variable. This regression problem is formulated by Sharpe (1964) and Lintner (1965).

Many pieces of research on the CAPM model is used to explain the diversification of the risk parameter and the performance of portfolios. The investigated issue from Zabarankin et al. (2014) purposed drawdown parameter in CAPM model to provide tools for hedging against market drawdowns. Fabozzi and Francis (1977), Levy (1974) used CAPM measure risk parameter for a various period. The contributions to the CAMP are the papers of Papavassiliou (2013), Chochola et al. (2014), Da et al. (2012).

A typical risk assessment situation is this. Consider a portfolio consisting of n assets whose possible losses are random variables $X_1, X_2, \ldots, X_n$. We are interested in the overall risks of the portfolio at some given time, i.e., the total loss $Y = X_1 + X_2 + \cdots + X_n$. The value-at-risk (VaR) is a commonly used methodology for estimatingof risks. The essence of the VaR computations is the estimation of high quantiles (see, Autchariyapanitkul et al. (2014)) in the portfolio return distributions. Usually, these computations are based on the assumption of normality of the financial return distribution. However, financial data often reveals that the underlying distribution is not normal. The standard value-at-risk is $F_Y^{-1}(\alpha)$, the maximum possible total loss at level $\alpha \in [0,1]$, i.e.,

$$P(Y \geq F_Y^{-1}(\alpha)) \leq 1 - \alpha \qquad (5.1)$$

In order to obtain the distribution F_Y of Y, we need the joint distribution of the regressors $(X_1, X_2, \ldots, X_n)$, since, clearly, we cannot “assume” that the $X_i' s$ are mutually independent. A multivariate normal distribution will not work, in the view of empirical work of Mandelbrot (1963) and Fama (1965) financial variables are rather heavy-tailed. Not only do we need copulas to come up with a realistic multivariate model (i.e., a joint distribution for $(X_1, X_2, \ldots, X_n)$, but we also need copulas to describe quantitativelythe dependence among assets.

Vine copulas started with Harry Joe in 1996. He gave a construction of multivariate copulas in terms of bivariate copulas, expressed in terms of distribution functions. Thus, it suffices, besides estimating the marginals, to come up with a high dimensional copula to arrive at a joint distribution for the marginal. In one hand, while lots of parametric bivariate copulas models exist in the literature, there seems not to be the case for higher dimensional copulas. On the contrary, we want a high dimensional copula to capture, say, pairwise dependencies between capital asset pricing models. First, we modeled pairwise dependencies by bivariate copulas and then glue them together to obtain the global high dimensional copula. Zhang et al. (2014) used vine copula methods estimate CVAR of the portfolio based on VaR measurement, and showed that D-vine copula model is superior to C-vine and R-vine copulas. Also, to study construct dependence structure, So and Yeung (2014) used the time varying vine copulas based GARCH model to show that Kendall 's tau and linear correlation of the stock return change over time. Moreover, an enormous number of papers about vine copulas that we can found in a study of Aas et al. (2009), Guegan and Maugis (2011), Roboredo and Ugolini (2015).

In this paper, we intend to use C-vine and D-vine copulas to examine the dependence structure between CAPM models. Then, use the joint distribution that minimize expected shortfall with respect to the expected returns to show the optimal weight of stocks in portfolios. Similarly to the work of Autchariyapanitkul (2014) introduced multivariate t-copula to optimize stock returns in portfolio analysis.

5.2 Data

We used the stock returns in SET50 index. The data consist of the returns from the 8 big capitalization companies, high return. There are Banpu Public Company Limited (BANPU), Bank of Ayudhya Public Company Limited (BAY), Bangkok Bank Public Company Limited(BBL), Central Pattana Public Company Limited (CPN), Land and Houses Public Company Limited (LH), Pruksa Real Estate Public Company Limited (PS), Thanachart Capital Public Company Limited (TCAP) and Thai Oil Public Company Limited (TOP). All the weekly data are extracted from DataStream from

March 2009 until Jan 2014 with a total of 260 observations for each selected companies. The stock return prices are calculated by $r_t = \ln(P_t) - lnP_{t-1}$. Table 5.1 presents a summary of the variables.

Table 5.1 Summary statistics

Statistics	SET50	BANPU	BAY	BBL	CPN	LH	PS	TCAP	TOP
Mean	0.0025	-0.0036	0.0028	0.0020	0.0055	0.0015	0.0020	0.0016	-0.0002
Median	0.0041	-0.0051	0.0000	0.0025	0.0048	0.0000	0.0000	0.0000	0.0000
Maximum	0.0706	0.1802	0.1341	0.1000	0.1268	0.1638	0.1650	0.1475	0.1377
Minimum	-0.0706	-0.1324	-0.1658	-0.1039	-0.1406	-0.1947	-0.1926	-0.1581	-0.2173
Std. Dev.	0.0253	0.0422	0.0425	0.0344	0.0426	0.0503	0.0567	0.03825	0.0413
Skewness	-0.3412	0.1823	-0.1674	0.1789	-0.1494	0.1803	-0.3417	-0.2854	-0.3675
Kurtosis	3.8347	4.6789	4.2171	3.2542	3.6822	4.2344	3.8459	4.6276	6.2557
Jarque-Bera	12.5927	31.9757	17.2625	2.0867	6.0089	17.9156	12.8106	32.2261	120.6802
PROB	0.0080	0.0010	0.0034	0.3097	0.0454	0.0030	0.0077	0.0010	0.0010

Note: All values are the log return.

Table 5.1 summarizes the statistics, including the mean, standard deviation, maximum , minimum returns , skewness, kurtosis , Jarque-Bera of the 8 stocks and SET50. We can see that the returns are mostly positive, except in the case of BANPU and TOP. The returns mostly present a negative skewness, except in the case of BANPU, BBL and LH. In addition, all returns have a high kurtosis above 3. Regarding to the value of Jarque-Bera, non-normality distribution is presented in all returns. These means that the marginal distributionof these returns are not a normal distribution and there have a heavy tail to the left and high kurtosis.

5.3 Empirical results

This study aims to apply the copulas approach to estimate the optimal weight in portfolio analysis. Given CAPM equation of each stock returns, we can calculate the joint dependency structure via C-vine and D-vine to carry out the optimization process. The standardized residuals of from each equations are, then, transform to be the uniform [0,1] using the empirical cumulative distribution function. The estimated CAPM models are shown Table 5.2

Table 5.2 Parameters estimation from CAPM models

Para-meters	BANPU	BAY	BBL	CPN	LH	PS	TCAP	TOP
$\boldsymbol{\beta_0}$	-0.0060 (0.0021)	0.0003 (0.0021)	-0.0005 (0.0014)	0.0030 (0.0022)	-0.0014 (0.0024)	-0.0009 (0.0029)	-0.0008 (0.0018)	-0.0028 (0.0020)
$\boldsymbol{\beta_1}$	0.9653 (0.0846)	1.0098 (0.0835)	1.0248 (0.0554)	0.9695 (0.0855)	1.2592 (0.0956)	1.2617 (0.1153)	0.9556 (0.7280)	1.0543 (0.0773)
$\boldsymbol{\sigma^2}$	0.0012	0.0012	0.0005	0.0012	0.0015	0.0022	0.0009	0.0010
$\boldsymbol{R^2}$	0.3350	0.3620	0.5700	0.3320	0.4020	0.3170	0.4000	0.4190
KS test	0.0811	0.7856	0.4211	0.8055	0.4854	0.6835	0.4326	0.0678

According to table 5.2. The result shows the positive beta value (β_1) for all stock returns these mean that the market returns have a positive effect to stock returns. We observe that the PS return presents the highest rate of return and highest beta value (β_1).There are five stocks, consisting BAY, BBL, LS, PS and TOP, present the value of beta coefficients (β) which are above 1 (β > 1) while the rest of 3 stocks are BANPU, CPN and TCAP offered less expected return rate less than stock market does. Their beta coefficients (β) are below 1 (β < 1).This indicates that BAY, BBL, LS, PS and TOP return have the high expected risk and their price will be volatile than the SET market while BANPU, CPN and TCAP return has the low expected risk and their price will be less volatile than the SET market.

In addition, the Kolmogorov–Smirnov (KS) test is used as the uniform test for the transformed marginal distribution functions of these CAPM residuals. The result shows that none of the KS test accepts the null hypothesis. Therefore, it is evident that all the marginal distributions are uniform on $[0,1]$.

Table 5.3 Estimated Results of C-vine copula

Pairs	Families	Parameter 1	Parameter 2	AIC	Upper-Lower tail dependence	Kendall's tau
1,2	Frank	-1.1185 (0.4133)		-5.2231	(0,0)	-0.1227
1,3	Gumbel	1.0630 (0.0372)		-9.6632	(0,0.804)	0.0566
1,4	Frank	-0.8447 (0.4256)		-1.9134	(0,0)	-0.0931
1,5	Frank	-1.2610 (0.4217)		-6.7918	(0,0)	-0.1379
1,6	Frank	-0.8455 (0.4355)		-1.2042	(0,0)	-0.0932
1,7	rotated BB8 Copula(90)	-1.2335 (0.1805)	-0.9483 (0.0876)	-0.2098	(0,0)	-0.0868
1,8	Clayton	0.0837 (0.0626)		-3.7487	(0.0003,0)	0.0398
2,3|1	rotated Gumbel Copula(180)	-1.0667 (0.0370)		-10.4321	(0,0)	-0.0625
2,4|1	Clayton	0.2005 (0.0691)		-8.1257	(0.0312,0)	0.0909
2,5|1	Clayton	0.2150 (0.0784)		-1.0440	(0.0397,0)	0.0950
2,6|1	rotated Gumbel Copula(180)	1.0453 (0.0338)		1.4950	(0.0591,0)	0.0433
2,7|1	Frank	-0.2854 (0.4018)		-6.7201	(0,0)	-0.0316
2,8|1	rotated Clayton Copula(180)	-0.0551 (0.0596)		0.9857	(0,0)	-0.0268
3,4|1,2	rotated BB8 Copula(270)	-1.3483 (0.3361)	-0.8992 (0.1688)	-3.0042	(0,0)	-0.1037

Table 5.3 (Continued)

Pairs	Families	Parameter 1	Parameter 2	AIC	Upper-Lower tail dependence	Kendall's tau
3,5|1,2	Gaussian	-0.1832 (0.0596)		-6.6535	(0,0)	-0.1172
3,6|1,2	Student-t	-0.0835 (0.0725)		-3.1033	(0.0041,0.0041)	-0.0532
3,7|1,2	Gaussian	-0.1824 (0.0596)	9.8817 (4.8793)	-6.5825	(0,0)	-0.1167
4,5|1,2,3	rotated Gumbel Copula(180)	1.1679 (0.0527)		-19.5084	(0.1896,0)	0.1437
4,6|1,2,3	Clayton	0.1389 (0.0678)		-3.5559	(0.0068,0)	0.0649
4,7|1,2,3	Frank	0.8175 (0.4013)		-2.1411	(0,0)	0.0902
4,8|1,2,3	Clayton	0.0698 (0.0551)		0.0668	(0.00004,0)	0.0337
5,6|1,2,3,4	Gaussian	0.0817 (0.0620)		0.2960	(0,0)	0.0520
5,7|1,2,3,4	Rotate Joe copula(180 degree)	1.0722 (0.0522)		-1.4888	(0.0912,0)	0.0399
5,8|1,2,3,4	Gaussian	-0.0643 (0.0619)		0.9325	(0,0)	-0.0409
6,7|1,2,3,4,5	Clayton	0.1207 (0.0658)		-2.5254	(0.0042,0)	0.0569
6,8|1,2,3,4,5	Frank	0.5211 (0.4039)		0.3410	(0,0)	0.0609
7,8|1,2,3,4,5,6	Frank	0.8498 (0.3956)		-2.6209	(0,0)	0.0937

( ) standard error is in parenthesis, 5% level of significant. *1=BANPU, 2=CPN, 3=TOP, 4=PS, 5=LH, 6=TCAP, 7=BBL, 8=BAY.

Table 5.4 Estimated Results of D-vine copula

Pairs	Families	Parameter 1	Parameter 2	AIC	Upper-Lower tail dependence	Kendall's tau
1,2	Gaussian	-0.198 (0.0586)		-57.8570	(0,0)	-0.1235
2,3	Frank	0.8544 (0.4129)		-2.2602	(0,0)	0.0942
3,4	Frank	-0.8447 (0.4256)		-1.9134	(0,0)	-0.0935
4,5	Frank	-1.1185 (0.4133)		-5.2231	(0,0)	-0.1298
5,6	rotated Gumbel Copula (180 degrees)	1.0518 (0.0366)		-1.4600	(0.0671,0)	0.0492
6,7	Clayton	0.1536 (0.0737)		-3.9940	(0.0109,0)	0.0713
7,8	rotated Clayton Copula (90 degrees)	-0.0406 (0.0499)		1.2452	(0,0)	-0.0198
1,3|2	rotated BB8 Copula (270 degrees)	-1.3060 (0.2259)	-0.9483 (0.0876)	-5.9783	(0,0)	-0.1113
2,4|3	rotated BB8 Copula (270 degrees)	-1.2359 (0.1959)	-0.9325 (0.1116)	-0.4941	(0,0)	-0.819
3,5|4	Clayton	0.2005 (0.0691)		-10.4321	(0.0315,0)	0.0911
4,6|5	Frank	-0.7854 (0.4253)		-1.3920	(0,0)	-0.0867
5,7|6	Gumbel	1.4941 (0.2797)	0.9002 (0.1083)	-9.4433	(0,0)	0.1443
6,8|7	Frank	0.5461 (0.4103)		0.2335	(0,0)	0.0604
1,4|2,3	Gumbel	1.0622 (0.0369)		-9.1137	(0,0.0795)	0.0585
2,5|3,4	Frank	-0.4316 (0.3998)		0.8355	(0,0)	-0.0478
3,6|4,5	Clayton	0.1500 (0.0668)		-5.0929	(0.0098,0)	0.0697

Table 5.4 (Continued)

Pairs	Families	Parameter 1	Parameter 2	AIC	Upper-Lower tail dependence	Kendall's Tau
4,7|5,6	Frank	-0.9638 (0.4094)		-3.5019	(0,0)	-0.1061
5,8|6,7	rotated Clayton Copula (90 degrees)	-0.0776 (0.0607)		-0.0134	(0,0)	-0.0373
1,5|2,3,4	Gaussian	-0.0976 (0.0618)		-0.4291	(0,0)	-0.0622
2,6|3,4,5	Clayton	0.1386 (0.0652)		-4.2823	(0.0068,0)	0.0649
3,7|4,5,6	rotated Gumbel Copula (180 degrees)	1.1893 (0.0539)		-21.7787	(0.2089,0)	0.1591
4,8|5,6,7	Gaussian	0.0560 (0.0597)		1.1273	(0,0)	0.0343
1,6|2,3,4,5	Student-t	-0.0585 (0.0718)	10.8479 (6.0147)	-0.9194	(0.0343,0.0343)	-0.0372
2,7|3,4,5,6	rotated Joe Copula (180 degrees)	1.0944 (0.0556)		-4.3223	(0.1161,0)	0.0515
3,8|4,5,6,7	Clayton	0.0702 (0.0576)		0.1823	(0.00005,0)	0.0339
1,7|2,3,4,5,6	Frank	-1.0796 (0.4115)		-4.8556	(0,0)	-0.1185
2,8|3,4,5,6,7	Gaussian	0.1336 (0.0604)		-2.6796	(0,0)	0.0853
1,8|2,3,4,5,6,7	rotated Clayton Copula (90 degrees)	-0.0926 (0.0611)		-0.8799	(0,0)	-0.0442

( ) standard error is in parenthesis, 1=BANPU, 2=CPN, 3=TOP, 4=PS,5=LH, 6=TCAP, 7=BBL, 8=BAY.

Table 5.3 and Table 5.4 show the estimated results for C-vine and D-vine copulas, respectively. According these estimation results, there is evident that the D-vine structure for these eight returns is more appropriate than the C-Vine one because the sum values of AIC and BIC are the smallest for D-vine. Thus, in this study, we choose

D-vine to analyze the co-movement and dependency between these eight returns. Table 5.4 present the estimated result from D-vine copula and the result show that most of the estimated parameters are significant at 5% level. We, then consider in the Kendall 's tau values and found that the Rotate Gumbel copula (180 degrees) of (3,7|4,5,6) pair has the highest correlation, followed by Rotate copula BB8 (180 degrees) of (5,7|6) pair and Frank copula 4,5 pair, respectively. The relations of each pairs mostly jointed by Frank family, consisting (2,3) ,(3,4) , (4,5) ,(4,6|5) ,(6,8|7) , (2,5|3,4) and (1,7|2,3,4,5,6) pairs. Consider the tail dependence, it is evident that there is significant co-movement and tail dependence for (5,6), (6,7), (3,5|4), (1,4|2,3), (3,6|4,5), (2,6|3,4,5), (3,7|4,5,6), (1,6|2,3,4,5), (2,7|3,4,5,6) and (3,8|4,5,6,7) pairs.

Next, we used values of the D-vine copula to estimate the CVAR and efficient portfolio with the maximum expected return for a minimum loss.

We applied the Monte Carlo simulation to generate a set of 1,000,000 simulated returns in order to compute the Value at risk (VaR) and Expected Shortfall (CVaR) of this portfolio.

Table 5.5 Expected shortfall of equally weights portfolios

	Expected Returns	VaR	CVaR
10%	0.9405	-0.7537	-1.4289
5 %	0.9405	-1.2657	-1.8687
1 %	0.9405	-2.2458	-2.7599

Table 5.5 shows VaR and CVaR at levels of 1%, 5% and 10% with equally weighted. We notice that the estimated CVaR converges to -1.4289, -1.8687 and -2.7599 at 10%, 5% and 1% levels in period $t+1$, respectively. In the case of VaR, we can indicates that it might be 1%, 5%, and 10% sure that this portfolios will fall more than 7.537%, 1.2657%, and 2.2452%. If we take ES into account, it might be 1%, 5%, and 10% sure that this portfolios will fall more than 1.4289%, 1.8687%, and 2.7599%.

Furthermore, given significant level of 5%, we optimize the portfolio by using the mean-CVaR model and obtained the efficient frontier of the portfolio under various expected returns. Figure 5.1 illustrates the efficiency frontier which are represented by 10 portfolios in the table 5.6. In this section, we also provide the optimal weight investment for these eight stock returns in SET market. The results seem to have a financial interpretation. For example, in portfolios 1, the investors who are risk adverse and want to minimize their risk of portfolio, they can allocate there investment in BANPU 10.86%, BAY 10.24%, BBL 27.25% CPN 11.79% LH 7.59% PS 1.54% TCAP 12.29% and TOP 18.45%. In contrast the investors who are risk lover and want to maximize the return, they can invest only in CPN.

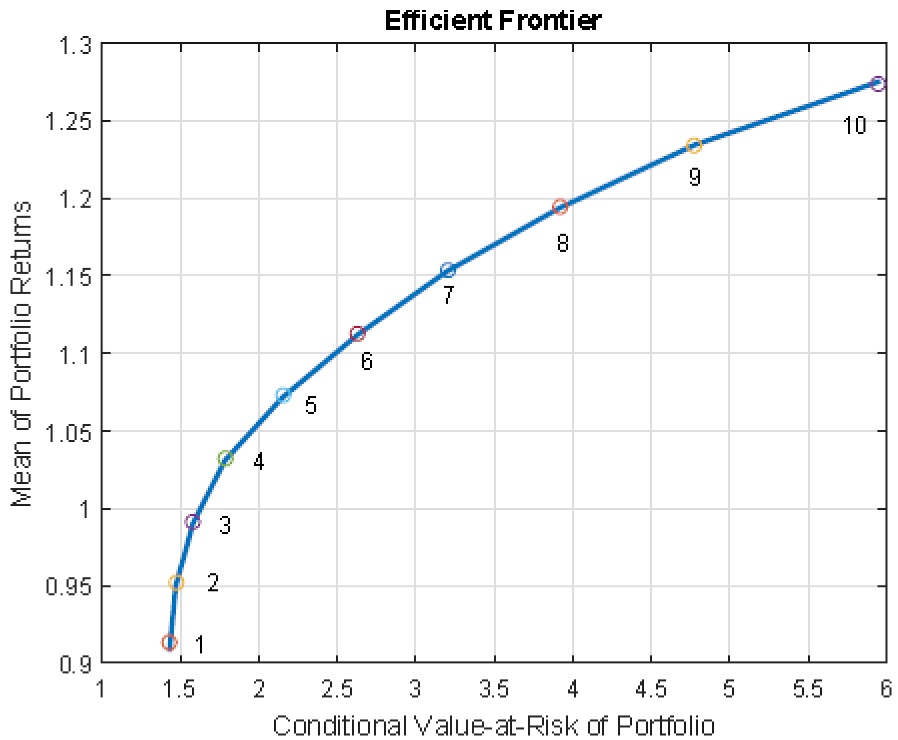

Figure 5.1 The efficient frontiers of CVaR under mean

Table 5.6 Optimal weighted portfolios for CVAR at 5 %

Portfolios	$BANPU_{w_1}$	BAY_{w_2}	BBL_{w_3}	CPN_{w_4}	LH_{w_5}	PS_{w_6}	$TCAP_{w_7}$	TOP_{w_8}	Returns
1	0.1086	0.1024	0.2725	0.1179	0.0759	0.0154	0.1229	0.1845	0.9129
2	0.0604	0.1152	0.2866	0.1415	0.0715	0.0246	0.1187	0.1814	0.9522
3	0.0130	0.1259	0.2979	0.1662	0.0695	0.0339	0.1153	0.1782	0.9916
4	0.0000	0.1449	0.2857	0.2270	0.0695	0.0467	0.0873	0.1391	1.0320
5	0.0000	0.1676	0.2652	0.3020	0.0667	0.0618	0.0500	0.0867	1.0727
6	0.0000	0.1872	0.2512	0.3798	0.0626	0.0729	0.0143	0.0320	1.1133
7	0.0000	0.1985	0.1801	0.4810	0.0530	0.0874	0.0000	0.0000	1.1539
8	0.0000	0.1961	0.0530	0.6114	0.0329	0.1065	0.0000	0.0000	1.1942
9	0.0000	0.1164	0.0000	0.7714	0.0000	0.1123	0.0000	0.0000	1.2340
10	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	1.2736

5.4 Conclusions

In this paper, we have determined the risk in portfolio management by employingVaR and used the mean-CVaR model to optimize portfolios. We used the C-vine and D-vine copula to measured dependence structure between capital asset pricing model (CAPM) affects the returns of portfolios. We carried our analysis in two steps. First, we examined the dependence structure of stock returns obtained from CAPM equations. Second, we investigated how the dependence structure of the asset pricing model influences portfolio optimization. We used an optimization procedure to allocate risk in the portfolios. It is feasible to reason that vine copulas can be explained dependency structure of the asset in the portfolio management.