CHAPTER 3

System Design and Methodology

This chapter presents the overall idea of the proposed Fuzzy Co-occurrence Matrix (FCOM). To create FCOM, we utilize the fuzzy clustering into Gray Level Co-occurrence Matrix (GLCM) to create the member ship values and the quantization image. Then, FCOMs are created in each of member ship planes. The FCOM texture descriptor and algorithm are introduced in section 3.1. Then, section 3.2 explains the properties of the FCOM. Finally, the textural features that can be extracted by FCOM are presented.

3.1 Fuzzy co-occurrence matrix texture descriptor

To create the FCOM, we incorporate the fuzzy clustering algorithms with the GLCM for the texture feature extraction. In particular, we utilize the fuzzy clustering in the quantization task. Then, we create the fuzzy gray level co-occurrence matrix in each of the given orientation and distance. Finally, the texture features are calculated from each plane. Figure 3.1 shows the fuzzy co-occurrence matrix texture feature extraction diagram.

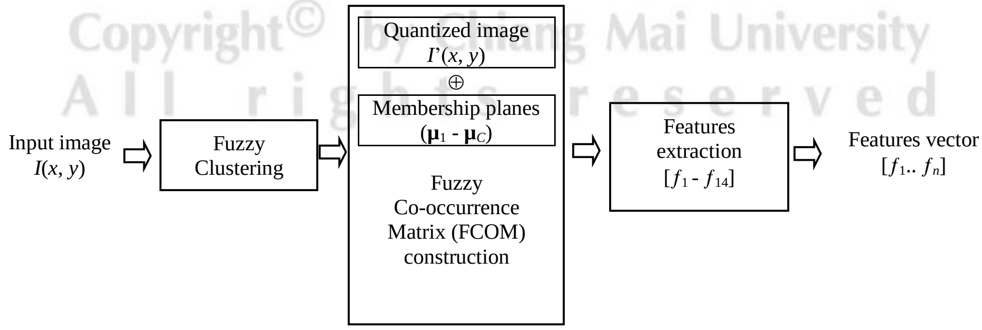

Figure 3.1 The fuzzy co-occurrence matrix texture feature extraction workflow.

In order to create the FCOM, we first implement the fuzzy clustering on an original gray scale image *I* with the fuzzifier *m* and cluster *C*. In each direction θ and distance *d*, we create the FCOM planes for each cluster. Finally, we obtain *C* planes of FCOM with the size of $C \times C$. The FCOM algorithm is as follows:

```
For each direction (θ)
    For each pixel (p)
        Find a pixel (q) that is d apart from p
        Set all FCOMs to zero
        For each cluster (C)
            FCOM(i, j) = FCOM(i, j) + (upi + uqi)
            (where upi and uqi are the membership
            values of pixels p and q in cluster i
            and j respectively)
        End For C
    End For p
End For θ.
```

From the FCM algorithm as mentioned in section 2.2 and the FCOM algorithm above, the analysis of complexity is as follows:

Computational time = T(FCM) + T(FCOM)

$= \Theta(C \times n) + \Theta(n \times n)$

$\in \Theta(n^2)$.

For example, Fuzzy *C*-Means (FCM) clustering with the number of cluster $C = 3$ is applied to image **I**(x, y) where *C* is similar to gray quantization levels. This will create one quantized image **I**$'(x, y)$ (each pixel is assigned to a cluster with the highest membership value), and three membership planes ($\boldsymbol{\mu}_{C1}$, $\boldsymbol{\mu}_{C2}$, and $\boldsymbol{\mu}_{C3}$). Then, the fuzzy co-occurrence matrices are constructed by applying the scalar cardinality in each membership plane. To create FCOM with orientation $\theta = 0°$ and distant $d = 1$, this will generate three FCOM planes. One membership plane will generate one FCOM plane as one direction and one distance. If we set θ = 0°, 45°, 95°, and 135° with distant $d = 1$, this will generate twelve FCOM planes. Then, the FCOM texture features are computed from each FCOM plane. The FCOM planes and feature dimensional can be computed by the following equations:

$$\text{FCOM planes} = C \times \text{Number of } \theta \times \text{Number of } d, \tag{3.1}$$

$$\text{Feature dimensions} = \text{Number of FCOM planes} \times \text{Number of features}. \tag{3.2}$$

For example, the 4 features, i.e., contrast, correlation, energy, and homogeneity are computed from each FCOM plane. This will generate 12 (3 FCOM planes × 4 features × 1 orientation × 1 distance) feature dimensions. We will have 42 feature dimensions when 14 texture features are computed from each FCOM plane. The expected result of the FCOM for this example is shown in figure 3.2.

μ_{C1}	1	2	…	*m*
1	0.1	0.5	…	0.3
2	0.5	0.6	…	0.1
:	:	:	:	:
n	0.2	0.1	…	0.3

FCOM (1, *i*, *j* | θ = 0°, *d* = 1)

	1	2	3
1	1.1	1.5	0.0
2	0.6	0.0	0.3
3	0.0	0.4	0.0

μ_{C2}	1	2	…	*m*
1	0.7	0.4	…	0.4
2	0.2	0.2	…	0.9
:	:	:	:	:
n	0.8	0.0	…	0.4

I'*(x, y)*

Scalar Card.	1	2	…	*m*
1	2	1	…	2
2	1	1	…	2
:	:	:	:	:
n	2	3	…	2

FCOM (2, *i*, *j* | θ = 0°, *d* = 1)

	1	2	3
1	0.4	1.9	0.0
2	1.1	0.0	0.8
3	0.0	0.4	0.0

μ_{C3}	1	2	…	*m*
1	0.2	0.1	…	0.3
2	0.3	0.2	…	0.0
:	:	:	:	:
n	0.0	0.9	…	0.3

FCOM (3, *i*, *j* | θ = 0°, *d* = 1)

	1	2	3
1	0.5	0.6	0.0
2	0.3	0.0	0.9
3	0.0	1.2	0.0

Figure 3.2 An example of the fuzzy co-occurrence matrix construction.

3.2 Fuzzy co-occurrence matrix properties

The properties of FCOM are extended from the GLCM. There are generated by the fuzzy clustering characteristics. FCOM properties are as follows:

1. The cluster centers are rearranged in the ascending order.

2. FCOM(d, θ, C) is a square matrix of the size of $C \times C$, where C is the number of clusters.

3. The summation of FCOM(d, θ, C) is in [0, 2 × Number of pixels pair] since the membership values of each pixel in each cluster are in [0, 1], where the number of a pixel pair in each direction can be calculated from the following equations:

$$(N_x - d) \times N_y \text{ where } \theta = 0°, \tag{3.3}$$

$$(N_x - d) \times (N_y - d) \text{ where } \theta = 45°, 135°, \tag{3.4}$$

$$N_x \times (N_y - d) \text{ where } \theta = 90°. \tag{3.5}$$

N_x and N_y denote rows and columns of the image, respectively. The summation of all pixel pairs, FCOM(d, θ), is equal to 2 × the number of pixel pairs. When the fuzzifier, m, of the clustering is 1, then FCOM becomes GLCM since the fuzzy clustering has become a k-means clustering.

4. FCOM(d, θ, C) of a test image is the same if image **I** and image **J** and their clustering results are the same. The FCOM(d, θ, C) of image **I** is equal to **J** when a gray level of pixel in **I** is equal to **J.** The image **I** and **J** are equal when

4.1. **I** and **J** have the same number of rows, N_y, and columns, N_x,

4.2. $\mathbf{I}(x, y) = \mathbf{J}(x, y)\ \forall x \in N_x, y \in N_y$.

5. Computational complexity of FCOM $\in \Theta(n^2)$

3.3 Fuzzy co-occurrence matrix texture features

Similar to the textural features suggested by Haralick et al. [4], the set of 14 texture features can be computed from the following equations.

Angular Second Moment$_k$ or Energy$_k$:

$$f_1 = \sum_{i,j} \{FCOM(k,i,j)\}^2, 1 \le k \le C \tag{3.6}$$

Contrast$_k$:

$$f_2 = \sum_{i,j} |i - j|^2 FCOM(k,i,j), 1 \le \mathrm{k} \le \mathrm{C} \tag{3.7}$$

Correlation$_k$:

$$f_3 = \sum_{i,j} \frac{(i - u_i^k)(j - u_j^k) FCOM(k,i,j)}{\sigma_i^k \sigma_j^k}, 1 \le k \le C \tag{3.8}$$

where μ_x, μ_y, σ_x, and σ_y are the mean and standard deviation of p_x and p_y, respectively. The p_x, p_y, μ_x, μ_y, σ_x, and σ_y are computed from the following equations:

$$p_x = \sum_j^{N_g} FCOM(k,i,j), 1 \le k \le C, \tag{3.9}$$

$$p_y = \sum_i^{N_g} FCOM(k,i,j), 1 \le k \le C, \tag{3.10}$$

$$\mu_x = \sum_i i p_x(i), \tag{3.11}$$

$$\mu_y = \sum_j j p_y(j), \tag{3.12}$$

$$\sigma_x^2 = \sum_i (p_x(i) - \mu_x(i))^2, \tag{3.13}$$

$$\sigma_y^2 = \sum_j (p_y(j) - \mu_y(j))^2. \tag{3.14}$$

Sum of Squares$_k$ or Variance$_k$:

$$f_4 = \sum_{i,j} (i - \mu_i^k)(j - \mu_j^k) FCOM(k,i,j), \text{for } 1 \le k \le C. \tag{3.15}$$

Inverse Difference Moment$_k$ or Homogeneity$_k$:

$$f_5 = \sum_{i,j} \frac{FCOM(k,i,j)}{1 + (i + j)^2}, \text{for } 1 \le k \le C \tag{3.16}$$

Sum Average$_k$:

$$f_6 = \sum_{i,j} (ij) FCOM(k,i,j), \text{for } 1 \le k \le C. \tag{3.17}$$

Sum Variance$_k$:

$$f_7 = \sum_{i,j} (ij - f_6)^2 FCOM(k,i,j), \text{ for } 1 \le k \le C. \tag{3.18}$$

Sum Entropy$_k$:

$$f_8 = -\sum_{i,j} FCOM_{x+y}(k,i,j)\log(FCOM_{x+y}(k,i,j)), \text{ for } 1 \le k \le C, \tag{3.19}$$

$$\text{where } FCOM_{x+y} = \sum_{\substack{i,j \\ i+j=l}} FCOM(k,i,j), \text{ for } l = 2, 3, \ldots, 2N_g, 1 \le k \le C.$$

Entropy$_k$:

$$f_9 = -\sum_{i,j} FCOM(k,i,j)\log(FCOM(k,i,j)), \text{ for } 1 \le k \le C \tag{3.20}$$

Difference Variance$_k$:

$$f_{10} = \text{variance of } p_{x-y} \tag{3.21}$$

Difference Entropy$_k$:

$$f_{11} = -\sum_{i,j} FCOM_{x-y}(k,i,j)\log(FCOM_{x-y}(k,i,j)), \text{ for } 1 \le k \le C, \tag{3.22}$$

$$\text{where } FCOM_{x-y} = \sum_{\substack{i,j \\ |i-j|=l}} FCOM(k,i,j), \text{ for } l = 0, 1, \ldots, N_g - 1, 1 \le k \le C.$$

Information Measure of Correlation$_k$:

$$f_{12} = \frac{HXY - HXY1}{\max(HX, HY)}, \tag{3.23}$$

$$f_{13} = (1 - \exp[-2.0(HXY2 - HXY)])^{\frac{1}{2}}, \tag{3.24}$$

where

HX and HY are entropies of p_x and p_y, and

$$HXY = -\sum_{i,j} FCOM(k,i,j)\log(FCOM(k,i,j)), \text{ for } 1 \le k \le C, \tag{3.25}$$

$$HXY1 = -\sum_{i,j} FCOM(k,i,j)\log(FCOM_x(i) FCOM_y(j)), \text{ for } 1 \le k \le C, \tag{3.26}$$

$$HXY2 = -\sum_{i,j} FCOM_x(i) FCOM_y(j) \log\left(p_x(i) p_y(j)\right). \quad (3.27)$$

Maximal Correlation Coefficient$_k$:

$$f_{14} = \left(\text{Second largest eigenvalue of } Q\right)^{\frac{1}{2}}, \quad (3.28)$$

where

$$Q(i, j) = \sum_l \frac{FCOM(k,i,l) FCOM(k,j,l)}{p_x(i) p_y(l)}. \quad (3.29)$$