CHAPTER 4

MULTI-OBJECTIVE FUNCTION SOLVING

4.1 MULTI-OBJECTIVE OPTIMIZATION

Multi-objective (MO) optimization, also known as multi-criteria or multi-attribute optimization is the process of simultaneously optimizing two or more conflicting objectives subject to certain constraints [40]. Multi-objective optimization problems can be found in various fields such as product and process design, finance, aircraft design, the oil and gas industry, automobile design, or wherever optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. Maximizing profit and minimizing the cost of a product, maximizing performance and minimizing fuel consumption of a vehicle, and minimizing weight while maximizing the strength of a particular component are examples of multi-objective optimization problems.

For nontrivial multi-objective problems, one cannot identify a single solution that simultaneously optimizes every objective. While searching for solutions, one reaches points such that, when attempting to improve an objective further, other objectives suffer as a result. A tentative solution is called non-dominated, pareto optimal, or pareto efficient if it cannot be eliminated from consideration by replacing it with another solution which improves an objective without worsening another one. Finding such non-dominated solutions, and quantifying the trade-offs in satisfying the different objectives, is the goal when setting up and solving a multi-objective optimization problem.

In mathematical terms, the multi-objective problem can be written as:

$$\min_{x} \quad [\, f_1(x), f_2(x), \dots, f_k(x)] \tag{4.1}$$

where

x is the input variable, and

f_k is the *k*th objective function.

The multi-objective optimization is applied to solve the power system problems. For examples, ref [41, 42], presented the summary of pareto multi-objective optimization. The content of these papers are to give fundamental knowledge on solving MO optimization problems. The focus is on the intelligent meta-heuristic approaches, techniques for efficient generation of the pareto frontier. The general formulation of MO optimization, pareto optimally concepts, and solution approaches with examples of MO problems in the power systems fields are given.

4.2 Fuzzy C-means clustering

Fuzzy C-means (FCM) clustering is one of widely wellknown soft clustering algorithm and it originally introduced by J. Bezdek in 1981 [43]. The method of FCM can group member of data set which are on multidimensional space into different clusters. The used data set is allowed to belong to different clusters by certain degrees called membership grade. The main advantage of FCM is that it allows member of data set to clusters as degrees in [0,1]. This gives the flexibility to express that member of data set can belong to more than one cluster. Details of FCM are included as follow.

1) Defining group of data clustering which aim to set as a condition for providing the information. These include stopping criteria, defining fuzzy parameters and define centroids of information.

2) Evaluating membership value of data to clustering.

3) Evaluating new centroids of information and checking for new membership value, after that clear old membership value.

4) If the optimal centroids are set, evaluate new membership function and objective function. Else evaluate membership value from latest centroids.

The objective function can be evaluate from

$$J = \sum_{i=1}^{c} \sum_{j=1}^{n} \left(\mu_{ij} \right)^{m} d^{2} \left(X_{j}, Z_{i} \right) \tag{4.2}$$

where

J	Objective function value of fuzzy C-means,
$X = \left\{ X_{1,} X_{2}, ..., X_{n} \right\}$	Data set,
n	number of data,
c	number of clusters,
m	fuzzy parameter which must be more than 1,
μ_{ij}	membership value of X_j in the *i*th cluster, and
$d^{2}\left(X_{j}, Z_{i} \right)$	distance squared between X_j and the *i*th cluster centroid.

where

$$Z_{i} = \frac{\sum_{j=1}^{n} \left(\mu_{ij} \right)^{m} X_{j}}{\sum_{j=1}^{n} \left(\mu_{ij} \right)^{m}} \tag{4.3}$$

$$\mu_{ij} = \frac{\left[\dfrac{1}{d^{2}\left(X_{j} - Z_{i} \right)} \right]^{\frac{1}{(m-1)}}}{\sum_{i=1}^{c} \left[\dfrac{1}{d^{2}\left(X_{j} - X_{i} \right)} \right]^{\frac{1}{(m-1)}}} \tag{4.4}$$

Flowchart of FCM is shown as below.

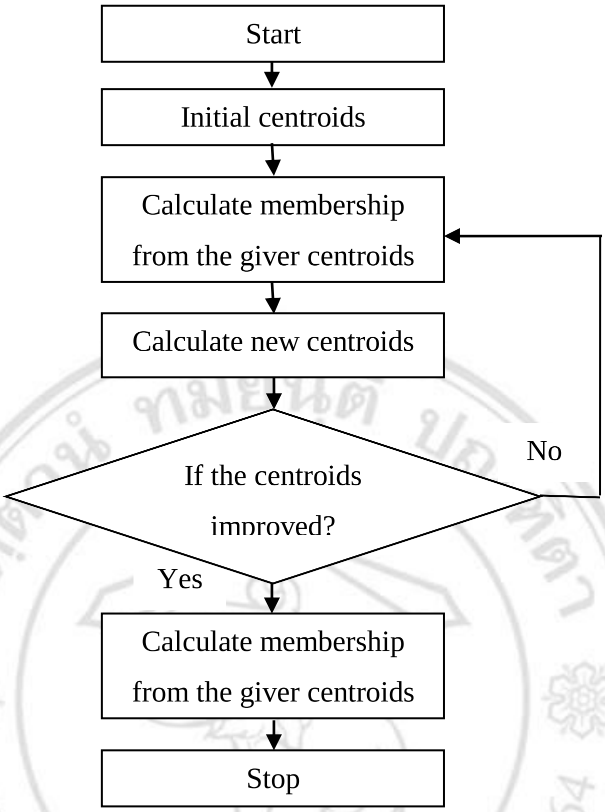

Figure 4.1 Flowchart of FCM.

In this thesis, FCM is used to cluster particles.

4.3 Tournament scheme selection

The tournament scheme selection [44] is used to determine and select the best cluster and the best compromise particle which handle the best score of multi-objective function value. The selection method utilized is a tournament scheme selection, which can be computed from equation 4.5 and equation 4.6.

$$w_t = \begin{cases} 1 & if \quad f_k > f_r \\ 0 & otherwise \end{cases} \tag{4.5}$$

$$s_k = \sum_{t=1}^{Nt} w_t \tag{4.6}$$

where

w_t weight value of each opponent,

f_k objective value of the *k*th particle,

f_r fitness of the *r*th opponents which is all particles except itself,

$\lfloor x \rfloor$ the greatest integer less than or equal *x*,

u uniform random number in the interval [0,1],

P population size,

s_k total score of each *k*th particle, and

Nt number of the opponents.

4.4 FCM with tournament scheme selection

In this thesis, FCM and tournament scheme selection which is named FCM/S are proposed. Details of FCM/S with tournament scheme selection are described as follow.

Step 1: Fuzzy C-means is used to cluster the data.

Step 2: The center of each clusters is applied with tournament scheme selection. The cluster which has the most score of tournament scheme selection is the first priority and the others are sort up to score of tournament scheme selection.

Step 3: Best cluster and runner up are determined.

Step 4: All clusters are sorted from best to last by scoring.

Step 5: Pareto-optimal set is created by sorted cluster by sorting first to last of particle group size.

Step 6: Apply tournament scheme selection to particle group size for best compromise particle. Flowchart of FCM/S with selection techniques is shown as Figure 4.2.

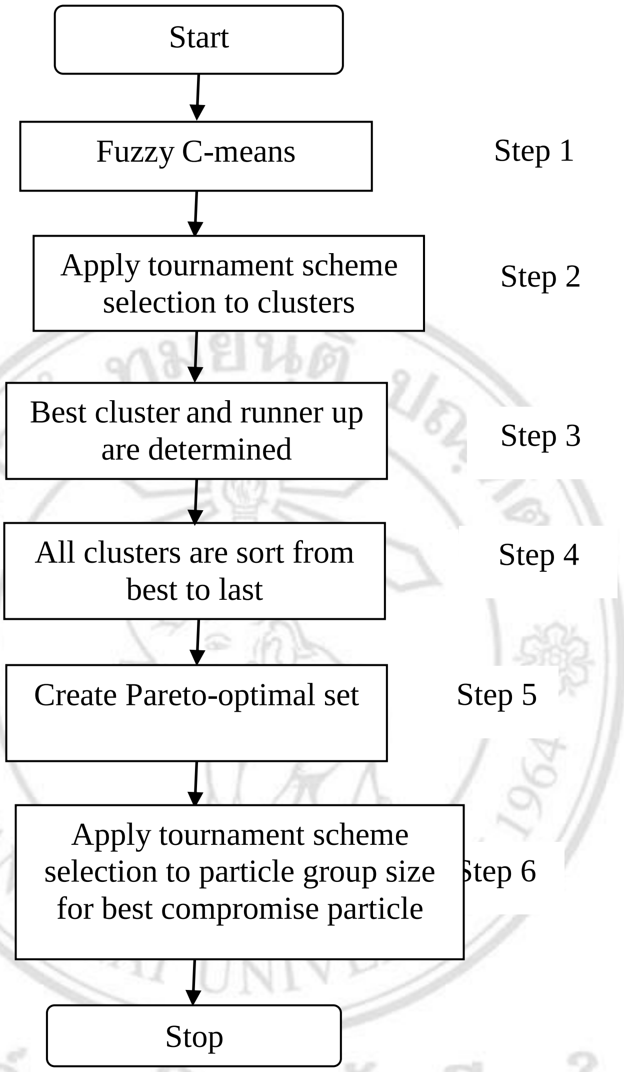

Figure 4.2 Flowchart of FCM/S.

Special features and merits of FCM/S can be described as follows:

1. FCM/S uses fuzzy C-mean which can cluster the data to many clusters up to defining.
2. Tournament scheme selection can select the member of first priority clusters into pareto frontier.

4.5 Hybrid PSO with FCM/S

The FCM/S is integrated to hybrid PSO by replacing Performing Competition and Selection. Flowchart of hybrid PSO with FCM/S is shown as Figure 4.3 which can be described as follows.

Step 1: Generation of initial condition of each particle. Initial searching points and velocities of each particle are usually random within the allowable range.

Step 2: Update particle velocity and position. Searching points of each particle are modified.

Step 3: Evaluation of searching point of each particle. The objective function value is calculated for each particle.

Step 4: Performing FCM/S

Step 5: Search and update tabu list.

Step 6: If the stop criteria is reach, the iteration will terminate. Else go to step2.

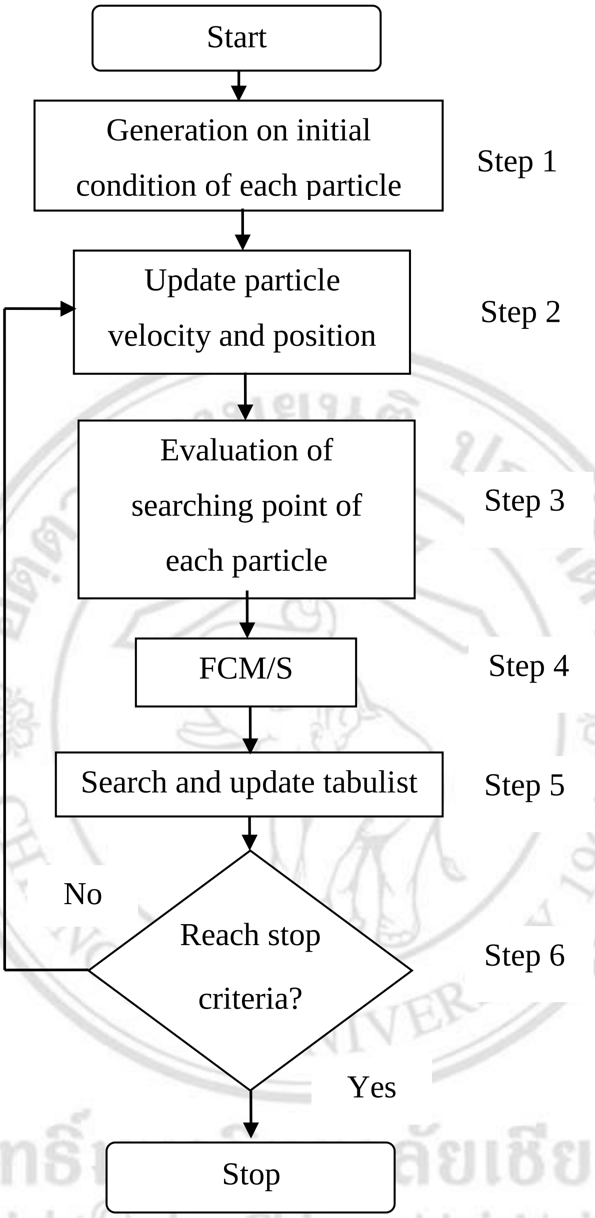

Figure 4.3 Flowchart of hybrid PSO with FCM/S

Moreover FCM/S is integrated to EP, TS, and PSO to compare with hybrid PSO with FCM/S. The flowchart of EP with FCM/S is shown in Figure 4.4 which can be described as follows.

Step 1: The initial population is initialized randomly using sets of uniform random number distribution ranging over the limitation of each control variable

Step 2: Evaluation of searching point of each individual. The objective function value is calculated for each individual.

Step 3: A new population is generated by using Gaussian mutation operator.

Step 4: Evaluation of searching point of each individual. The objective function value is calculated for each individual.

Step 5: Performing FCM/S

Step 6: If the stop criteria is reach, the iteration will terminate. Else go to step2.

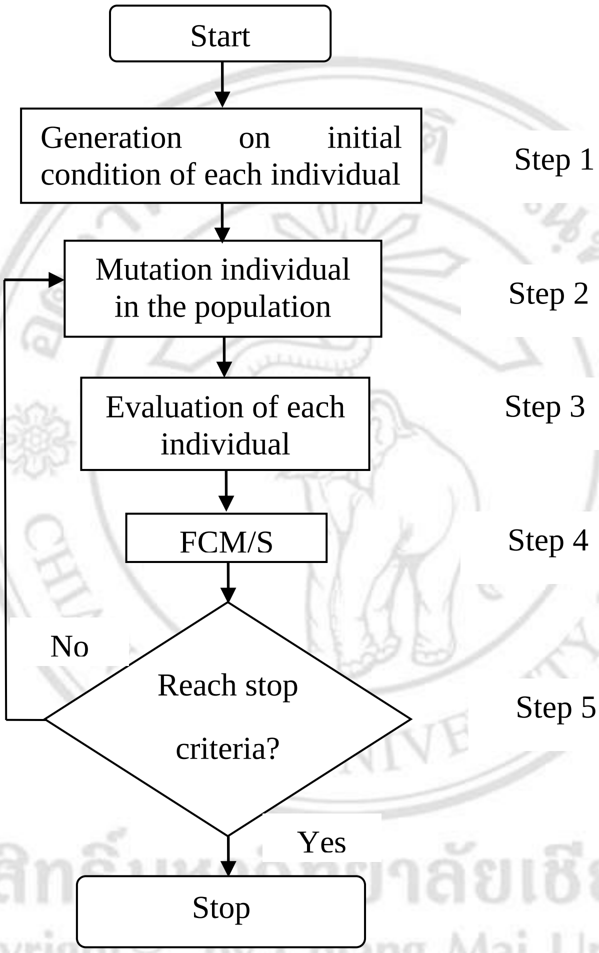

Figure 4.4 Flowchart of EP with FCM/S.

The flowchart of TS with FCM/S is shown as Figure 4.5 which can be described as follows.

Step 1: The initial population is initialized randomly using sets of uniform random number distribution ranging over the limitation of each control variable

Step 2: A new population is generated by using perturbation [17].

Step 3: Evaluation of searching point of each individual. The objective function value is calculated for each individual.

Step 4: Performing FCM/S

Step 5: Search and update tabulist, The best neighbor which is not tabu is selected as new individual.

Step 6: If the stop criteria is reach, the iteration will terminate. Else go to step2.

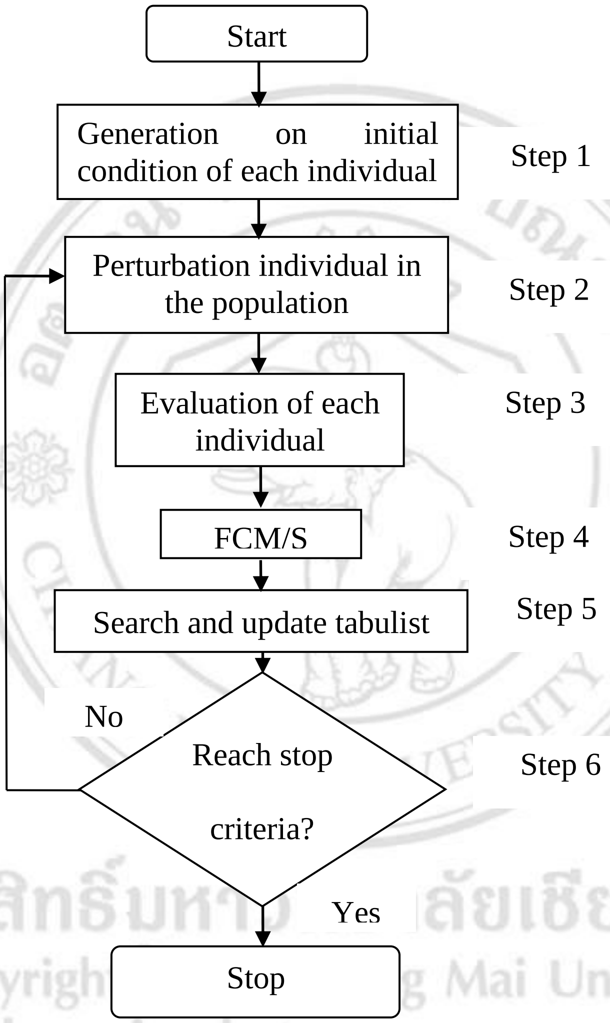

Figure 4.5 Flowchart of TS with FCM/S.

The flowchart of PSO with FCM/S is shown as Figure 4.6 which can be described as follows.

Step 1: Generation of initial condition of each particle. Initial searching points and velocities of each particle are usually random within the allowable range.

Step 2: Update particle velocity and position. Searching point of each particle are modified.

Step 3: Evaluation of searching point of each particle. The objective function value is calculated for each particle.

Step 4: Performing FCM/S

Step 5: If the stop criteria is reach, the iteration will stop. Else go to step2.

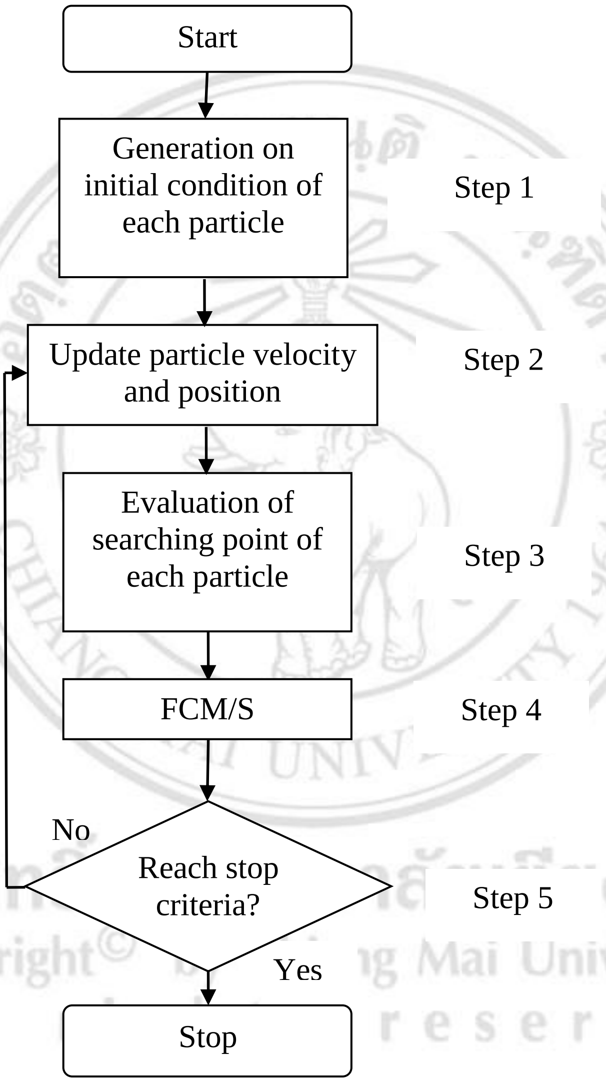

Figure 4.6 Flowchart of PSO with FCM/S.