CHAPTER 1

Introduction

There are many automated processes for which a certain exact motion of mechanical parts is required to perform a task. Since the time required to complete a motion task is often critical for performance, the race to achieve higher speed motion has led to many important developments, both in machine design and automatic control. Hi-tech products that combine high precision with high speed, such as micro-positioning equipments or hard disk drives, are now indispensable in the modern world. On the larger scale, industrial manipulation devices such as robots or gantry cranes are common examples of machines for which minimizing task completion time is important for economic reasons and more preferable for the operator/user.

This work is about how to move thing as fast as possible. The problem is considered from the stand-point of engineering design but will require application of the mathematical theory of time-optimal control. The word 'optimal' or 'optimum' mean *most desirable*, *satisfactory* or simply put, the *best*. According to the thesis title 'Simultaneous Optimization of Structure and Control', this work must not only show how to solve the time-optimal motion control problem for a given system or structure but also examine how to design the system mechanical structure to achieve the best possible time-optimal motion. The expected outcome is an optimal structure design and a corresponding time-optimal motion control strategy. This fully optimized system should require less time to complete a motion task compared with a conventional unoptimized design for the same motion task and actuation capacity.

This thesis proposes two novel mathematical methods that can be applied simultaneously for structural tuning and time-optimal control design for a class of mechanical system that is commonly seen in servo motion control applications. The first technique exploits the concept and mathematical properties of the 'reachable set' in solving the time-optimal control problem. The second technique, relies on the continuity property of the solution set and employs a first order perturbation analysis for simultaneous optimization of the system design parameters and corresponding time-optimal control input. The

(a) Hard disk drive (b) Overhead crane (c) Satellite

Figure 1.1: Machines with automatic control of motion

mathematical model considered is suitable to represent the motion of a flexible mechanical structure and includes an actuator model that accounts for limits of capacity and both linear (viscous) and non-linear (Coulomb) friction effects.

In this chapter, the motivation behind the problem will be considered in more detail. The problem statement will be given in full and this will serve as the framework for the research and hence provide the objectives and scope of the investigation. At the end of this chapter, an outline of the thesis is given.

1.1 Problem motivation

Nowadays, the world demands thing to happen quickly. For almost every task in daily life, it is preferable if it consumes less time. A typical example is the operation of the read-write head of a hard disk. This small device is required to undergo rapid motion with high precision. The time required to move between data tracks in the disk is the primary concern. The essence of the engineering design problem is to create a high speed mechanism with a compatible time-optimal control strategy that ensures minimal vibration of the components in between motions.

Instinctively, the first step in creating a high speed mechanism is to design the mechanical structure. To be able to achieve a high speed motion, the structure is designed to be light-weight but at the same time, for precise motion control, it must usually be as rigid as possible. The rigidity of the structure can help in preventing undesired vibration that might cause problems of control accuracy. However, sufficiently large magnitude control efforts or external disturbances will always excite the vibratory modes of the structures and cause vibration. If the mechanical vibration of the structure can be modelled mathematically then it should be possible to account for it in the synthesis of a control strategy.

In order to drive such a system to complete a motion within minimum time, a time-optimal control strategy that accounts for flexible structure vibration and achieves vibration cancelation is required.

In synthesizing a time-optimal control strategy, the following factors must be considered:

1. The first factor that influences the form of the time-optimal control solution is the dynamical behavior of the system, a model of which can be derived from the physical properties of the system. This dynamical model will define how the system reacts to the control input and disturbances. The correct knowledge of the mathematical model of the system can be used in creating the control input (actuation signal) which gives accurate output (motion). By considering the physical limitation of a real world system, constraints on states variable can be imposed within the system model. Moreover, non-linear dynamic effects may come to bear on motion behaviors, although these may be neglected in order to simplify the problem and to allow the creation of a useable strategy for synthesizing control solutions. It is desirable to consider the most general form of system model possible because the strategy developed based on this model can then be applied to a broader range of system types.

2. Another important factor is the desired output behavior. This may relate to the motion throughout the entire operation or only at a specific point or interval in time during a task. If only the value of the system states at the start and end time of the motion are specified, such conditions are mathematically termed the boundary conditions of the problem. These conditions are determined according to the required motion and may relate to motion specifications such as the distance traveled and the need for zero residual vibration following motion. For the time-optimal motion control problem, for a given system and same set of system parameters values, the boundary conditions will determine the total time required for a task.

3. There will always be imposed limits or other types of constraints on control input behavior. These conditions arise from physical limitations of actuation such as the safe current limit for a motor to avoid overheating or tripping a fuse. One straightforward way to improve the speed of motion is to increase the size of the actuators

to increase the limits of the control input so that a larger magnitude control effort can be applied. However, due to the first two factors, the relation between speed and actuation capacity is not always linear: doubling the size of the actuator will not always result in double the speed of motion (or in other words halve the time used). Using larger actuators will usually increase size and cost, thus, these aspects must also be considered.

From the aforementioned issues, some important questions about time-optimal motion control problems may be posed: Is there any control strategy that could achieve a time-optimal motion for a certain general class of mechanism? Does the method used to solve the problem give a solution that achieves a global optimum? What is the control input profile (signal) of the time-optimal control solution? What should be the best design of the structure for the specific task? Should the size of the actuator be increased to gain more speed and achieve a motion that can be completed in less time? These questions serve as the motivating problems that this research will aim to answer. Understanding the importance of these issues as fully as possible will be vital for achieving the main goal of developing an approach for simultaneous structure and controller design.

1.2 Research objective

The aim of this research is to create a novel strategy and mathematical methodology for simultaneous structural tuning and control synthesis for achieving time-optimal motion of a vibratory mechanical system.

1.3 Scope of investigation

In this research, the problem of simultaneous structure and control optimization will be considered for a general class of motion control problem involving a vibratory mechanical system. Among many types of general optimal control problem, only the time-optimal (i.e. minimum-time) control problem will be considered. Solutions of the problem will require calculation of the feed-forward (open-loop) control based on a mathematical model of the system. Therefore it will be assumed that the form of mathematical model of the system is known sufficiently accurately. The control input (actuation signal) is selected so that the calculated output will be as desired without the requirement for additional measurement-based feedback in the implementation.

To verify the practical suitability of the developed method and the control solutions, example cases will be simulated. Further testing on a lab-based experimental setup will be undertaken. The experimental test rig will be specially designed to realize a tunable stiffness structure for demonstration of the obtained solutions. However, this research will not focus on designing or developing a new type of physical machine that can work for real-world applications. Rather, the focus will be on the mathematical methodology and numerical tools for solving the problem. The experimental work will aim towards a working demonstration and allow more realistic assessment of feasibility and achievable performance.

1.4 Thesis outline

Following the overview of the research topic with motivation and framework as presented in this chapter, the remainder of this thesis is organized as follows:

In chapter 2, the basic approach to mathematical modelling of the dynamic behaviors of flexible structures undergoing motion will be presented. Background theory on the time-optimal control problem will be presented along with the conventional methods for solving the problem. In this chapter, the important concept of Pontryagin's Minimum Principle, used for solving optimal control problems, will be discussed. This theorem plays a fundamental role in ensuring the optimality of the control solutions considered in this work. A survey of previous research about structural tuning and structural optimization will also be presented.

Chapter 3 will describe a new approach to solve the time-optimal control problem for the defined system model. The approach is based on the concept of the reachable set and its convexity property. By using this concept, a numerical approach to find time-optimal control solutions is derived. For each step of the method, some analysis and discussion are given. Numerical examples are also presented. Various models of mechanical vibratory structures are considered and control solutions obtained. Starting from the simple linear state space equation model without friction, more complicate factors are introduced for increased generality of the model.

Following from the theoretical work in chapter 3, some more complete sets of time-optimal motion solutions for flexible structures are generated and presented in chapter 4. The results are analyzed and compared in order to better understand influencing factors and limitations for performance of time-optimal motions. This evaluation of the fundamental limits of performance will illuminate and motivate the objective of simultaneous structure and control optimization.

By exploiting the continuity property of the solution set, chapter 5 extends the scope of research by considering simultaneous structural tuning for the time-optimal control problem. A numerical iteration scheme for optimization based on the steepest descent method is presented. Numerical cases studies for a two-mode flexible structures are presented to illuminate the possible benefits of the simultaneous structure and control optimization strategy.

In order to demonstrate the effectiveness of the proposed ideas for a real system, chapter 6 presents the results of experiments on a motion control test rig. The characteristics and the components of the experimental rig will be described. The experiments verify that the theoretical methods can achieve good performance for flexible systems when performing time-optimal motions with integrated structural tuning ability.

Chapter 7 summarizes the main findings of this research with some discussion and suggestions for future work.