CHAPTER 7

Conclusions

7.1 Conclusion

This thesis has presented a class of time-optimal control problem involving motion of a flexible structure. The possibility to simultaneously optimize structural parameters of the system together with the control input has been investigated. To solve the problem, two novel mathematical methodologies have been proposed, as follows:

1. A method for solving the time-optimal control problem based on the convexity of the reachable set has been proposed. The concepts of state transfer and the reachable set allow the possible boundary conditions for motion to form a convex set within the state space with optimum points on the boundary. These boundary points can be reached within the same minimum time by using the corresponding outward normal direction as an initial value for the co-state variable, and using this to generate the optimium control input. If the direction of the final point is given (accoring to specified boundary conditions) then iteration over the outward normal direction will converge the solution to the desired final point. In this way the time-optimal control solution is acquired.

2. A method for simultaneous optimization of structure and time-optimal control has been reported. By exploiting the convexity and continuity of the solution set, a first order perturbation of system parameters and a gradient-based searching method can be used as an efficient and reliable method for further optimization of the system design/configuration.

The proposed methods were applied to time-optimal motion control problems involving flexible structures undergoing rest-to-rest and motion-to-rest maneuvers with zero-residual-vibration condition. Numerical and experimental cases have been selected to demonstrate the potential of the methods. The main contributions that can be highlighted are:

- By generating solutions directly from the initial co-state values, the true optimality of the time-optimal control solution can be guaranteed by the Pontryagin minimum principle. There is no need to verify the optimality of the solution by calculating the co-state afterward. Moreover, the assumption about the number of the switching times can be avoided. Therefore, more complicated solutions for multi-mode flexible structure models can be easily acquired, as demonstrated by numerical examples. However, the higher number of flexible modes means that more time is taken to complete the calculation.

- Due to the transformation of the state from $x(t)$ into $y(t)$ in accordance with the definition of the reachable set, the convex optimization cannot only be applied to a linear system but also to a simple non-linear system with additional constant input. This type of situation arises if there is Coulomb friction present in a motion system.

- The sets of solutions give a clearer view of the influence of the system parameter values on the performance of the controlled system which is indicated by the overall speed of motion. It can be seen that the time-optimal speed achievable in a flexible structure case can be close to the upper limit (corresponding to the rigid-body case) when the travel time is certain multiples of the natural period. The results can also be interpreted as showing the effect of change in action capacity on the overall speed, allowing improved selection of the size/capacity of the actuator.

- The simultaneous optimization procedure can significantly reduce the time-of-motion compared to the tradition input-only optimization scheme by matching the tunable system parameters with the boundary conditions, i.e. matching the damped natural frequency with the distance traveled. Achievable benefits depend on the boundary conditions but are most significant when the time-of-motion is of order of the natural period of vibration or less.

- In the experiments, feedback control was needed to ensure that the actual movement matched the projected trajectory for the time-optimal open loop system. In the case study, the effect of the feedback signal that modified the control input is small and has a small effect on optimality if the allowed limits on control effect are made slightly large than the limits used for the optimal control calculation.

7.2 Comments and future work

Theoretical challenges. To model the effect of the Coulomb friction an additional term with signum function is needed within the state-space model equation. However, the signum function is then assumed to be constant to simplify the calculation. For a more general model, other non-linear terms might be added to the system model yet the concept of reachable set is still applicable. Although the reachable set no longer has the global convexity property, the convex optimization algorithm may still be applied with some restrictions or modification. This should be investigated further.

Not only changing the form of the model but also the cost function could be considered for more general optimal control problem. Although the Pontryagin's principle can still be applied, the concept and the properties of the reachable set need to be studied carefully before geometric solution methods can be employed.

Robustness of the time-optimal control solution and implementation is an issue that is important for practical users. Therefore, it is helpful to investigate the robustness properties of the time-optimal control solution further. By allowing a near-minimum-time control input, an addition constraint could be introduced so that the control solution becomes more robust to error in the system model. Another possible approach is to incorporate robustness measures that can be evaluated and allow further consideration of the convex optimization to be made

Programming and software. When performing the convex optimization process, the time-interval must be discretized in order to locate the switching times where the sign change occurs in the switching function. A small interval is required to ensure detecting all switching times and this causes the long time to finish the calculation. It should be possible to predict the approximate location of switches so that a fine discretization does not need to be applied over the whole time interval of motion. More efficient code could also reduce the computation time as well as reduce numerical errors.

Hardware and real-world applications. The range of cases that could be tested on the experimental system was limited by the possible travel distance i.e the length of the ball screw. In fact, it is the ratio of move distance and the natural period that has a key role in the pattern of motion. One possibility to extend the motion range is to use the rotating motion instead. The corresponding mathematical model might change as well but the model should still be equivalent to the standard (modal) form. A pendulum with variable rope length instead of a continuous beam is one of the example where the control methods could be implemented and tested.

For the current design of the experimental system, the stiffness of the structure was tuned by manually adjusting a screw. This is simple but makes it hard to accurately achieve the desired natural frequency. The development of a scheme for automated tuning/smart structure for the flexible structure is an encouraging possibility. It should be noted that some standard continuously variable actuation approaches could be a better alternative, e.g. dual actuation approach. However, an additional actuator that can control the vibration is not strictly necessary since the vibration suppression can be performed using the existing control input.

In terms of application, since the convex optimization relies only on the knowledge of the state-space model of the system, then it is possible to apply the algorithm to other time-optimal control applications beside motion control cases, such as the time-optimal control with heat equations [45], [88].