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# LIST OF ABBREVIATIONS AND SYMBOLS

$\mathcal{A}$	attainable set
$\mathcal{J}$	cost function
${\cal H}$	Hamiltonian
$\mathcal{P}$	set of tunable system parameters
$\mathcal{R}$	reachable set
$\mathcal{T}_{c}$	set of switching times
$\mathcal{U}$	set of admissible control input
$C_U$	relative actuation capacity
с	viscous friction coefficient
d	required direction for state transfer
$F_c$	Coulomb friction coefficient
Ι	overall mass moment inertia of the system
J	Jacobian matrix due of changing of $y$ due to change in tunable
	parameters
$K_T$	torque constant of the motor
k	number of tunable parameters
m	number of input component
N	number of the flexible mode presenting in model
n $a$ $a$	order of the system
$n_k$	outward normal direction for the hyperplane at $k$ th iteration
p	tunable system parameters
$S^{-}$	overall speed of motion
$T_F$	Coulomb friction coefficient arises collocate at motor
$T_M$	torque produced by motor
t	time variable
$\underline{U},\overline{U}$	lower and upper bound of control input
u	input vector
W	sensitivity matrix of $y$ due to change in tunable parameters
w	point within reachable set

X(t)	inverse of state transition matrix
x	state variables vector
$x_0$	initial state
$x_f$	final state
$Y_k$	set of linearly independent vectors of $y$ at $k$ th iteration
y	state variable in y-space
Ζ	sensitivity matrix due to change in tunable parameters
$\alpha, \beta$	linear combination of $y$ forming $d$ and $w_k$ repectively
δ	step size
$\epsilon$	arbitrarily small number used as stopping criterion
$\gamma$	distance to be traveled
λ	co-state variables vector
$\lambda_0$	vector of initial co-state variable
$\eta$	an outward normal direction for boundary of reachable set
ν	initial velocity
θ	angular displacement
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# ข้อความแห่งการริเริ่ม

- วิทยานิพนธ์นี้ได้นำเสนอวิธีการแก้ปัญหาการหาค่าเหมาะสมที่สุดเชิงเวลา โดยมีพื้นฐานอยู่บน ความคอนเวกซ์ของเซตที่เข้าถึงได้
- แนวกิดการหาก่าเหมาะสมที่สุดของโกรงสร้างและการกวบกุมกวบกู่กันได้ถูกนำเสนอขึ้น และ สามารถที่จะลดเวลาที่น้อยที่สุดในการเกลื่อนที่ของระบบสั่นเชิงกลได้



# STATEMENT OF ORIGINALITY

- 1) This thesis propose the method for solving time-optimal control problem based on the convexity of the reachable set.
- The simultaneous optimization of structure and time-optimal control has been developed which provides further reduction in minimum-time of motion for vibratory mechanical system.

