CHAPTER 5

Conclusion and Discussion

In this thesis, we defined two measures of complete dependence for random vectors which are tools to quantify the amount of information of a random vector contained in another one. One is the measure of complete dependence φ based on conditional distribution functions and another one is ζ_p based on linkages. Normally, the value of any measure of complete dependence ψ of a random vector Y given a random vector X is always between zero and $\psi(Y|Y)$. Moreover, $\psi(Y|X)$ is equal to zero, the minimum value of the measure, if and only if these two random vectors X and Y are independent, that is, X does not contain any information about Y. In addition, $\psi(Y|X)$ is equal to $\psi(Y|Y)$ which is its maximum value if and only if Y is a function of X meaning that X contains all information of Y.

Concerning the defined measure of complete dependence φ based on conditional distribution functions, both random vectors Y and (Y, Y) contain the same information. Thus, the measure of (Y, Z) depending on X and the measure of (Y, Y, Z) depending on X have the same value. Also, for a measurable function f, since $f(X)$ is a function of X, it contains less information of Y than X does, so the measure of Y depending on $f(X)$ will have no more value than the measure of Y depending on X.

There are some common properties between the measure ζ_p and the measure φ such as the value of the measure of Y depending on $f(X)$ is always less than the value of the measure of Y depeding on X. Furthermore, we have proved that the measure ζ_p of Y depending on (X, Z) has more value than the measure ζ_p of Y depending on X since there are more information of Y depending on (X, Z) than on X.