APPENDIX A

Flow Chart for Location Problems

Three main methods are proposed in this thesis: 1) to reduce the size of the location problem, 2) to solve the facility location problem and 3) to solve the $p-$center problem. Flow charts for three methods and all submethod are given in this appendix. These following notations are introduced for the flow charts.

Let $I = \{1, 2, 3, \ldots, n\}$ be a set of clients or customers,

$J = \{1, 2, 3, \ldots, m\}$ be a set of potential facility sites,

f_j be facility setup cost for facility $j \in J$,

$c_{i,j}$ be the transportation cost from client $i \in I$ to facility $j \in J$.

D be the sorted distinct entries of the transportation cost $c_{i,j}$

h_i be supply of client $i \in I$,

s_j be capacity of facility $j \in J$,

β be the first feasible or a minimum number of opened facilities,

γ be a maximum number of opened facilities,

C^*_λ be an optimal cost with λ opened facilities,

$J0$ be a set of opened facilities,

$J1$ be a set of unopened facilities,

$\text{argmin}\{f(x) : x \in X\}$ be a value of x for which $f(x)$ is minimum,

$\text{argmax}\{f(x) : x \in X\}$ be a value of x for which $f(x)$ is maximum.

The number of clients and candidate places in the facility location problem are generally large in the real world problems. If the method to solve the facility location problem cannot solve the facility location problem in an appropriate time, the problem can be relaxed by reducing its size. The flow chart for problem size reduction method is in Figure A.1. Inputs are set of clients or customers (I), set of potential facility sites (J), supplies of client (h_i), and transportation cost from client $i \in I$ to facility $j \in J$ ($c_{i,j}$). There are 2 majors steps in the method. First, the set of nodes having a degree fewer than or equal to 2 are constructed. In the second step, for every node in the set that is constructed in the first step, find the nearest nodes with degree more than two. Next, combine the node found in the second step with its corresponding node in the first step. Redo this process until the set constructed in the first step is empty. Return the set of nodes for relaxed problem ($\bar{I}$).

Figure A.2 shows the flow chart of facility location method. Inputs of this method are the set of clients or customers (I), set of potential facility sites (J), facility setup costs (f_j), supplies of client (h_i), facility capacities (s_j), transportation cost from client $i \in I$ to facility $j \in J$ (c_{ij}), and minimum and maximum number of opened facilities (β and γ, respectively). First, predict the number for opened facilities (λ) using bisection method between the minimum and maximum number of opened facilities. Next, use Method A or B or C to find the optimal total cost with the number of opened facility being the prediction found in the first step (C^*_λ) and the optimal total cost with $\lambda + 1$ opened facilities ($C^*_{\lambda+1}$). Consider the total cost obtained in the second step. With the predicted number of facilities, if the total cost at optimum is less than that with $\lambda+1$ ($C^*_\lambda < C^*_{\lambda+1}$), update the maximum number of opened facilities to the prediction found in the first step. Otherwise, the minimum number of opened facilities is updated to that number. Redo the processes in the first step until the maximum number of opened facilities equals to minimum number of opened facilities plus 1. Return optimal total cost with λ opened facilities (C^*_λ).

The flow chart of the method to choose facility whose priority is given to the setup cost, or Method A, is shown in Figure A.3. Inputs are set of clients or customers (I), set of potential facility sites (J), facility setup costs (f_j), supplies of client (h_i), facility capacities (s_j), transportation costs from client $i \in I$ to facility $j \in J$ (c_{ij}), and the prediction number for opened facilities (λ). First, let the opened facility set ($J0$) be an empty set. Next, find an unopened facility with the minimum setup cost and add that facility to the set of opened facilities. After that, solve knapsack problem (KP) to assign the client to the opened facility. All processes are repeated until the number of opened facilities equals to the prediction number for opened facilities. After that, return the set of opened facilities and calculate the optimal total cost with λ opened facilities (C^*_λ).

Figure A.4 shows the flow chart of the method to choose facility whose priority given to the transportation cost or Method B. Inputs are set of clients or customers (I), set of potential facility sites (J), facility setup costs (f_j), supplies of client (h_i), facility capacities (s_j), transportation costs from client $i \in I$ to facility $j \in J$ (c_{ij}), and the prediction number of opened facilities (λ). First, let the opened facility set ($J0$) be an empty set. Next, find an unopened facility having the minimum transportation cost and add that facility to the set of opened facilities. After that, solve knapsack problem (KP) to assign the client to the opened facilities. All processes are repeated until the number of opened facilities equals to the prediction number for the opened facilities. Calculate the optimal total cost with λ opened facilities (C^*_λ) and return the set of opened facilities.

Figure A.5 shows the flow chart of the Method C for solving $p-$center problem. In this method, the facility is chosen by giving priority to the distribution of facility location. Inputs of this method are the set of clients or customers (I), set of potential facility sites (J), facility setup costs (f_j), supplies of client (h_i), facility capacities (s_j), transportation costs from client $i \in I$ to facility $j \in J$ (c_{ij}), the set of sorted distinct entries of the transportation costs (D), and the prediction of the opened facility numbers (p). Note that the lower and the upper bounds of the $p-$center problem are the smallest and the largest elements in D, respectively. First, set the index of the lower bound to 1 and that of the upper bound to the number of the elements in D. Next, calculate the index of coverage radius (ε) to be the round up of the median of lower and upper bound indices, and set the coverage radius (δ) to be the element in D with that index (D_ε). After that, solve the maximal client coverage problem (MCP) using the coverage radius found in the second step. If the solution of the maximal client cannot cover all clients, set the index of the lower bound to the index of the coverage radius found in the second step. Otherwise, set the index of the upper bound to the index of that radius. If the index of the upper bound is next to the index of the lower bound, the optimum coverage radius is the upper bound. Otherwise, go to the second step.

The flow chart of maximal client coverage method is shown in Figure A.6. This method used to find the maximal clients can cover with given radius and number of the opened facility. Inputs are the set of clients or customers (I), the set of potential facility sites (J), the facility setup costs (f_j), supplies of clients (h_i), facilities capacities (s_j), transportation costs from client $i \in I$ to facility $j \in J$ (c_{ij}), coverage radius (δ), and the prediction of the open facility numbers (p). The first step, construct the set of clients in the coverage radius for each facility (J^j_δ), and the set of facilities in the coverage radius for each client (I^i_δ). If there are some facilities uniquely covered some clients in the specific coverage radius, open those facilities. Otherwise, open the facility that has a maximum number of clients in the coverage radius. Next, assign clients to the opened facility by solving the knapsack problem and delete all clients who have been assigned to facility from the unassigned clients. Reprocess the first step until the number of opened facilities is equal to the prediction of the opened facility numbers (p). Return the set of covered clients (I^*).

Figure A.7 shows the flow chart of the knapsack problem. The opened facility (j^*) with facility capacities (s_{j^*}), set of clients (I), and supplies of client (h_i) are inputs to the method. The client candidate set is constructed in the first step. Next step, assign client in the candidate set who have maximum supply to the facility. Update the facility capacities and the candidate client set. If the candidate client set is empty, stop and return the set of assigned clients. Else, go back to the second step. Return the set of assigned clients ($I0$).

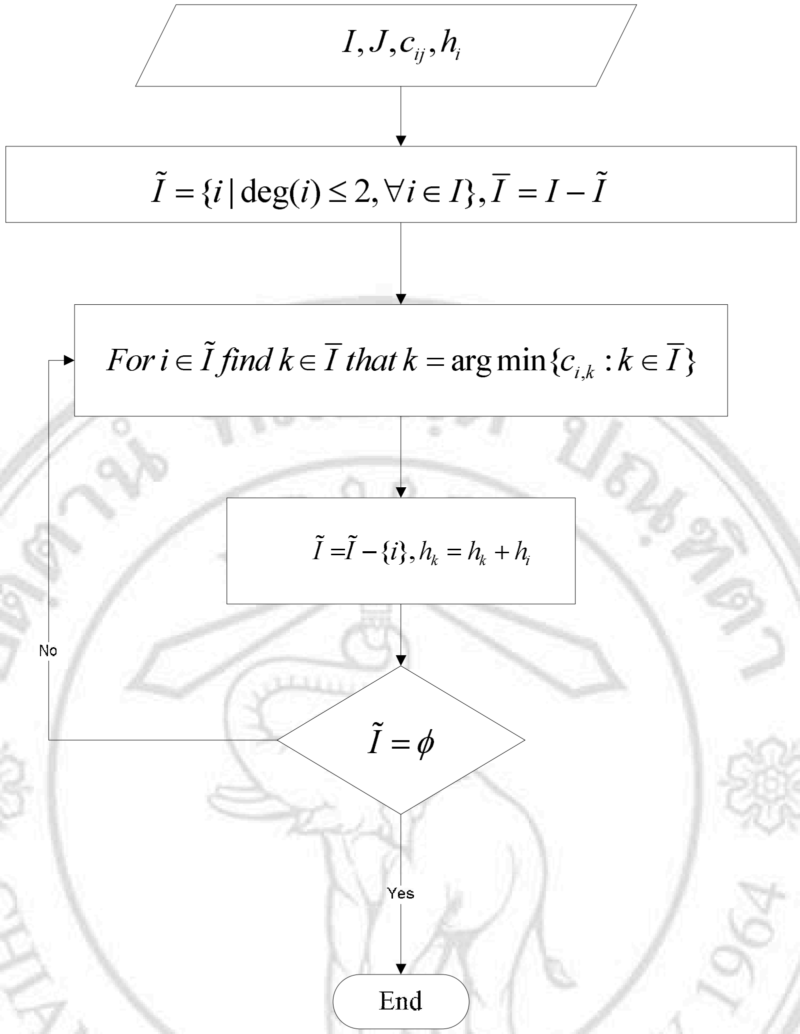

Figure A.1: Flow chart of the size reduction method.

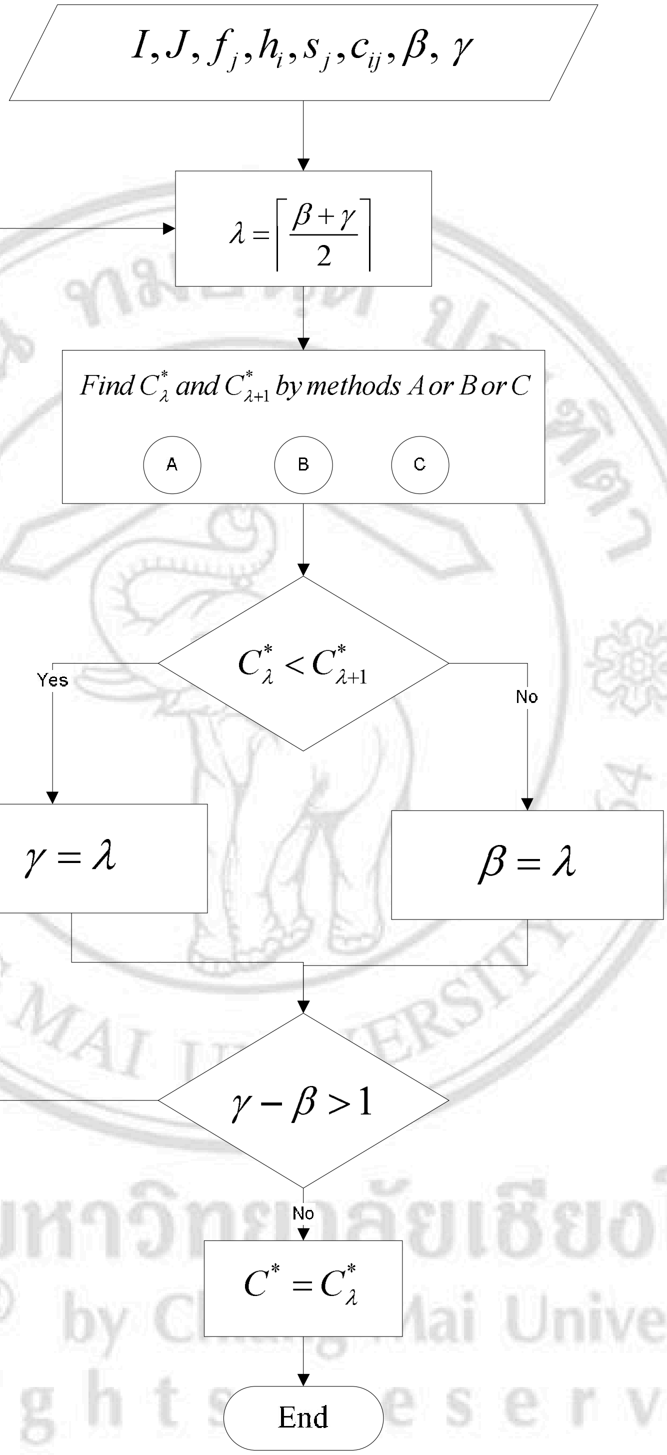

Figure A.2: Flow chart of facility location method.

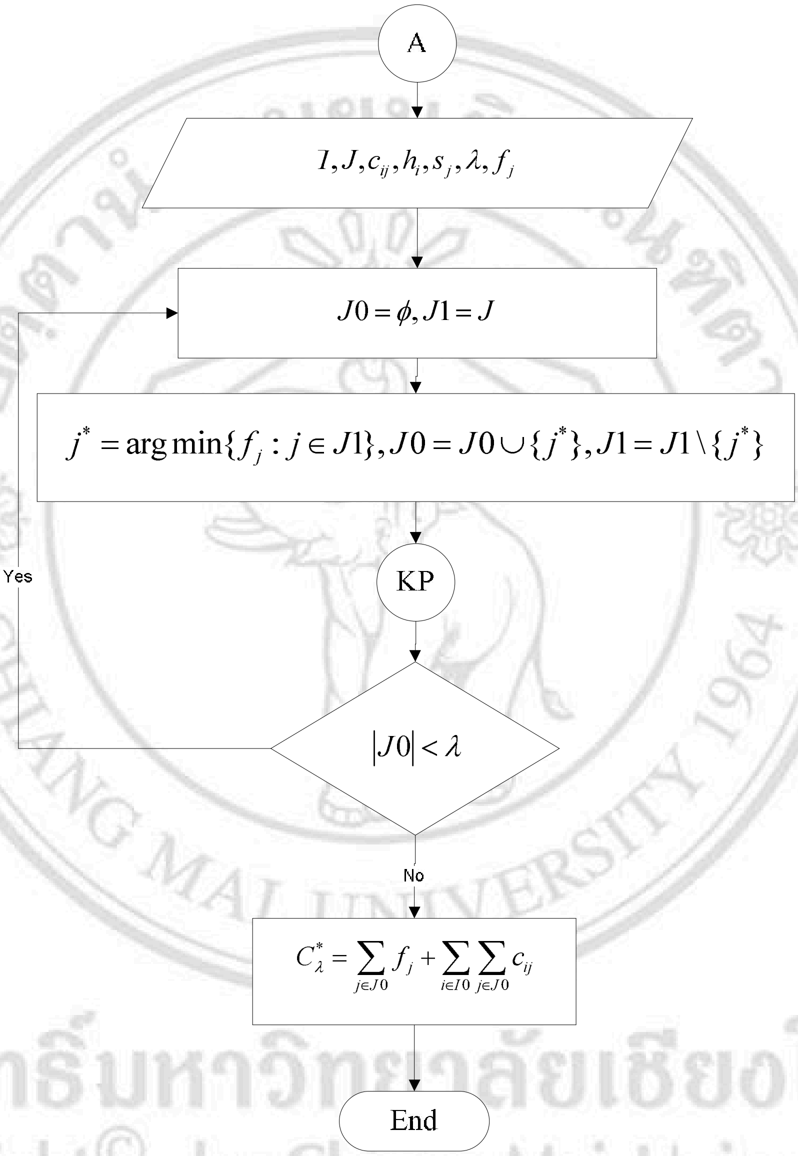

Figure A.3: Flow chart of Method A.

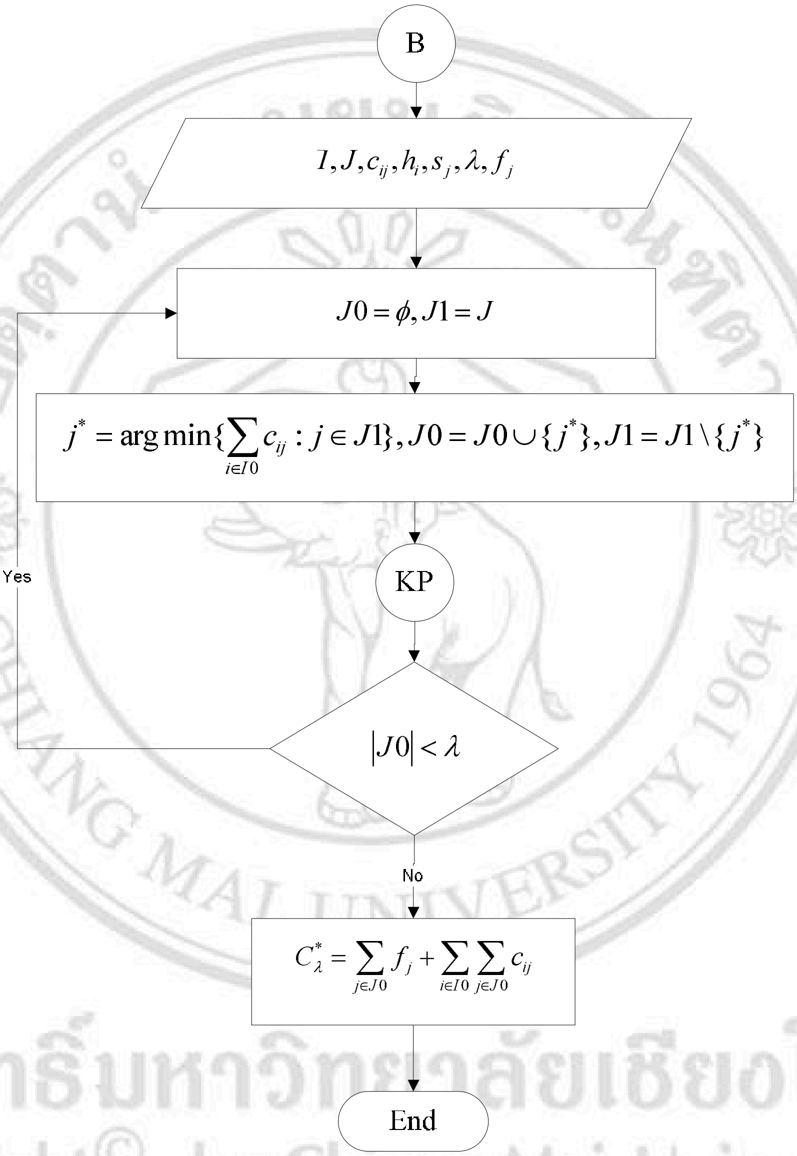

Figure A.4: Flow chart of Method B (p-median).

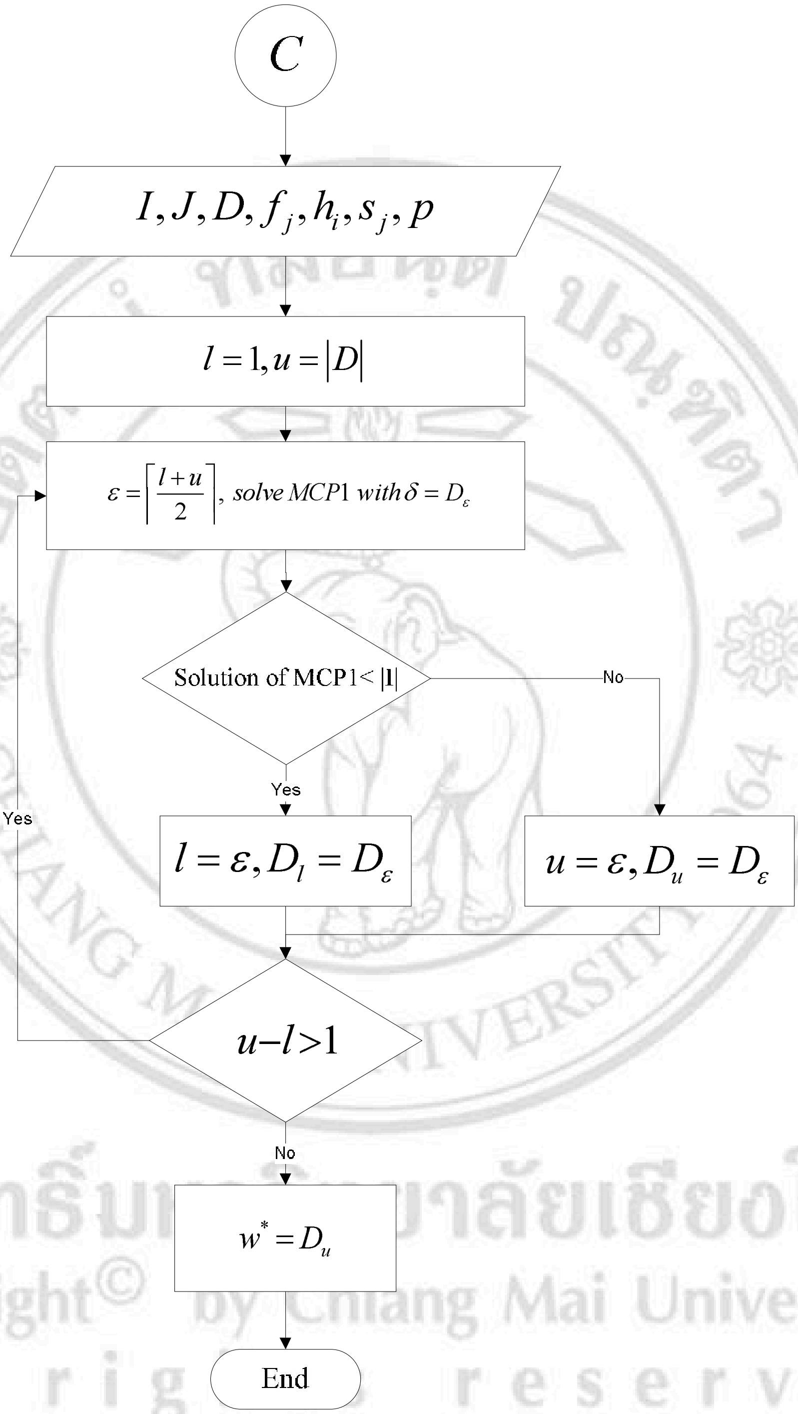

Figure A.5: Flow chart of Method C (p-center).

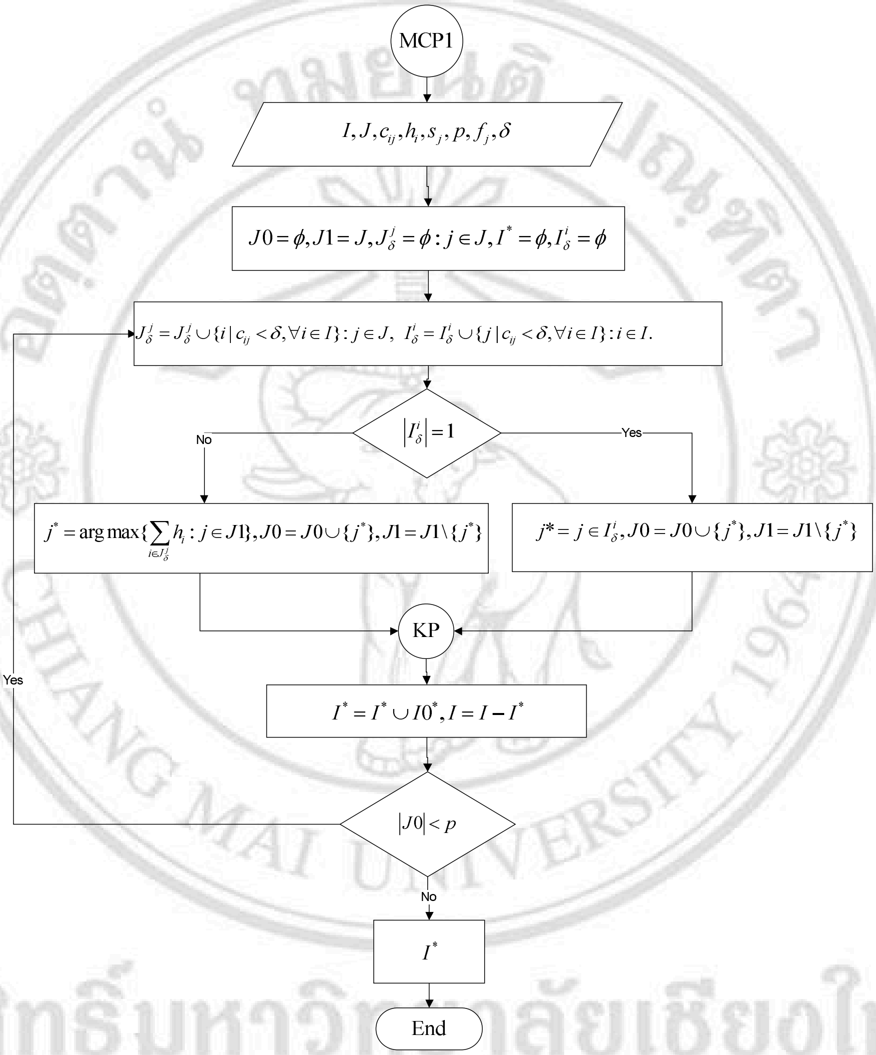

Figure A.6: Flow chart of maximal client coverage method.

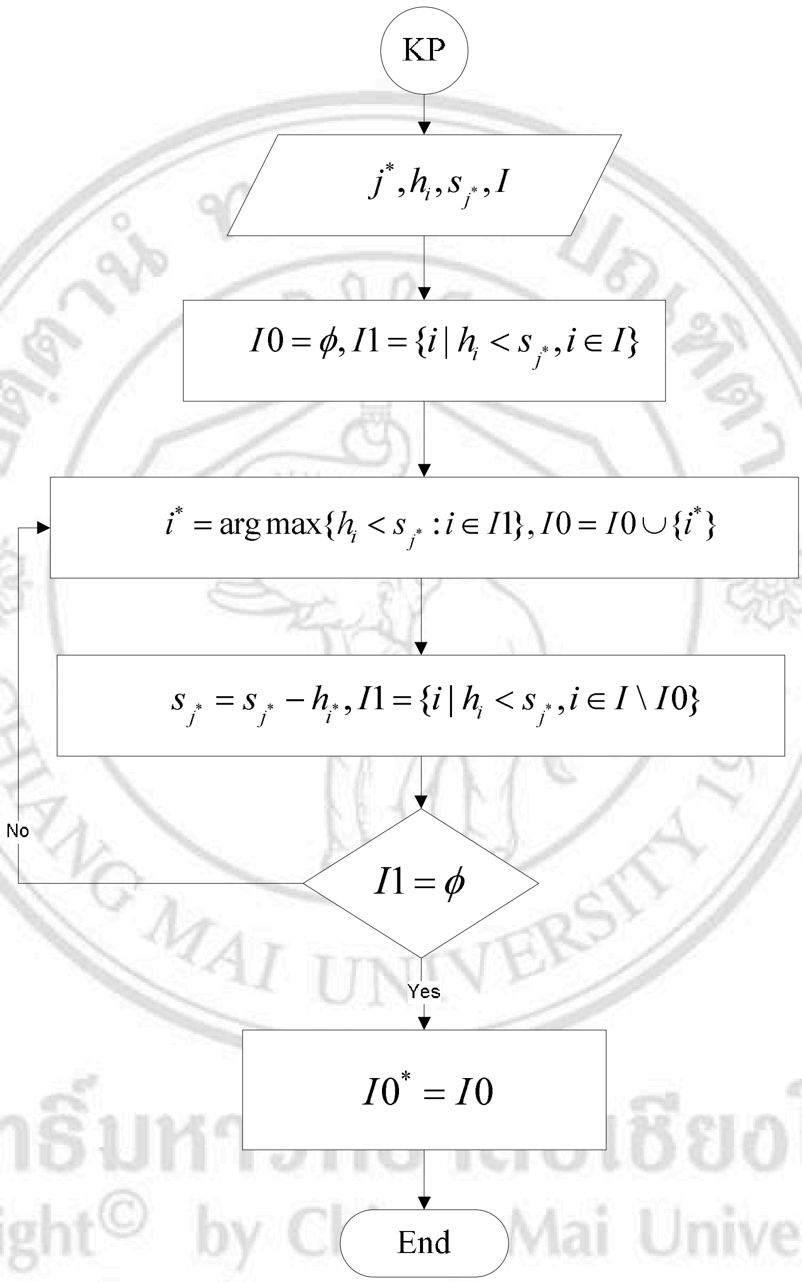

Figure A.7: Flow chart of method for solving knapsack problem.

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Publication(s)

1) S. Dantrakul and C. Likasiri. A maximal client coverage algo--rithm for the p-center problem. *Thai Journal of Mathematics,* 10:423-432, 2012.

2) S. Dantrakul and C. Likasiri. An Algorithm and its error bound of reduced facility location problem with case study of Chiang Mai waste management system. *Joutnal of Industrial Technology Ubon Ratchatani Rajabhat University,* 3:41-51, 2013.

3) S. Dantrakul, C. Likasiri and R. Pongvuthithum. Applied p-me--dian and p-center algorithms for facility location problems. *Expert Systems with Applications,* 41:3596-3604, 2014.