CHAPTER 1

Introduction

Let X be a nonempty set and $T(X)$ denote the set of all transformations from X into itself. Then $T(X)$ is a semigroup under the composition of maps and it is called the full transformation semigroup on X.

It is known that $T(X)$ is a regular semigroup, that is, for every $\alpha \in T(X)$, $\alpha = \alpha\beta\alpha$ for some $\beta \in T(X)$.

For a fixed nonempty subset Y of X, let

$$S(X, Y) = \{\alpha \in T(X) \mid Y\alpha \subseteq Y\}.$$

Then $S(X, Y)$ is a semigroup of total transformations of X which leave the subset Y of X invariant. This semigroup was first introduced and studied by K. D. Magill [6] in 1966. In fact, if $Y = X$, then $S(X, Y) = T(X)$. So we may regard $S(X, Y)$ as a generalization of $T(X)$.

In 2005, S. Nenthein, P. Youngkhong and Y. Kemprasit [7] characterized the regular elements of $S(X, Y)$ and gave a necessary and sufficient condition for $S(X, Y)$ to be a regular semigroup. Moreover, they counted the number of regular elements of $S(X, Y)$ when X is a finite set.

In 2011, P. Honyam and J. Sanwong [4] described Green's relations and ideals on $S(X, Y)$. Later in 2013, W. Choomanee, P. Honyam and J. Sanwong [1] characterized left regular, right regular and intra-regular elements of $S(X, Y)$ and considered the relationships between these elements. Moreover, they counted the number of left regular elements of $S(X, Y)$ when X is a finite set.

In this thesis, we study the subsemigroup of $T(X)$, namely $S(X, Y_1, Y_2)$, defined by

$$S(X, Y_1, Y_2) = \{\alpha \in T(X) \mid Y_1\alpha \subseteq Y_1, Y_2\alpha \subseteq Y_2\},$$

where $\emptyset \neq Y_1, Y_2 \subseteq X$ and $Y_1 \cap Y_2 = \emptyset$. Then $S(X, Y_1, Y_2)$ is a semigroup of total transformations on X which leave subsets Y_1, Y_2 of X invariant.

The purposes of this research are:

1. To describe Green's relations $\mathcal{L}$, $\mathcal{R}$ and group $\mathcal{H}-$classes of $S(X, Y_1, Y_2)$.

2. To characterize regular, left regular, right regular and completely regular elements of $S(X, Y_1, Y_2)$.

3. To count the number of regular elements of $S(X, Y_1, Y_2)$ when X is a finite set.

4. To consider the relationships between regular, left regular and right regular elements on $S(X, Y_1, Y_2)$.

We devide this thesis into four chapters. Chapter 1 is an introduction to the research problems. Chapter 2 deals with some preliminaries and some useful results those will be used in later chapters. Chapter 3 is the main results of this research work consisting of the following three sections:

1. Transformation Semigroups with Two Invariant Subsets.

2. Green's Relations on $S(X, Y_1, Y_2)$.

3. Regularity of $S(X, Y_1, Y_2)$.

Finally, chapter 4 is for conclusion.