CHAPTER 2

Theories and Principles

2.1 Rock Physics Modeling

Seismic amplitudes are interpreted using models generated from well data and supported by a range of knowledge collectively described as rock physics. The key objectives in using of rock physics are 1) log preparation and conditioning, 2) rock characterization from logs, and 3) seismic modeling (Simm and Bacon, 2014). Many rock physics models have been published that are often presented in simple forms of relationships involving two or three parameters. Some of the most relations that are commonly used are Gardner's relations and Greenberg-Castagna.

2.1.1 Gardner's relations

Normally in rocks, compressional velocity (V_P) increases with increasing density (ρ) that means a positive relationship. Gardner et al (1974) developed a series of brine saturated lithology dependent relations of V_P and density (Figure 2-1). The equation for clastic rock is:

$$Gardner's\ equation; \quad \frac{\Delta\rho}{\rho} = \frac{1}{4}\left(\frac{\Delta V_P}{V_P}\right)$$

2.1.2 Greenberg-Castagna

The ideal shear velocity inputting to rock physic analysis is usually a strong dependent lithology but largely independent pressure, hence positive correlation between V_P and shear velocity (V_S). The equation of V_P and V_S relation in mudrocks is shown below (Castagna et al, 1985). Then, Greenberg and Castagna (1992) defined four trends for commonly occurring (brine bearing) lithology (Figure 2-2). The effect of gas sand moves the points up and lies above wet sand or Castagna sandstone trend.

$$Castagna's\ equation; \quad V_P = 1.16V_S + 1.36$$

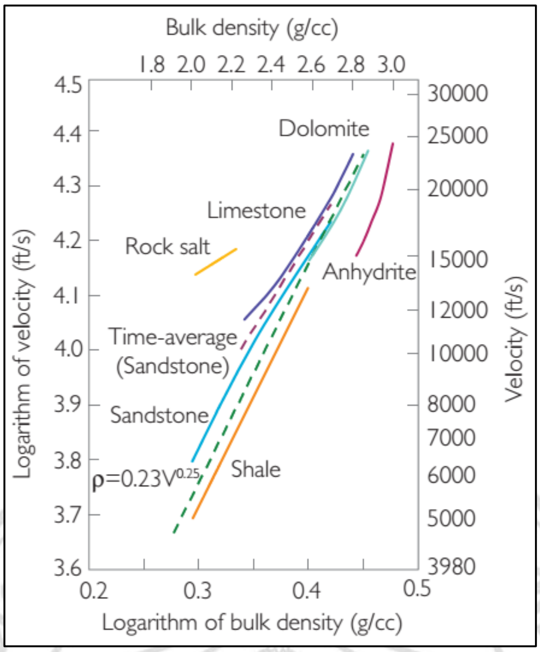

Figure 2-1. P-wave velocity and density relationships in rocks of different lithology re-drawn after Gardner et al in 1974 (Simm and Bacon, 2014)

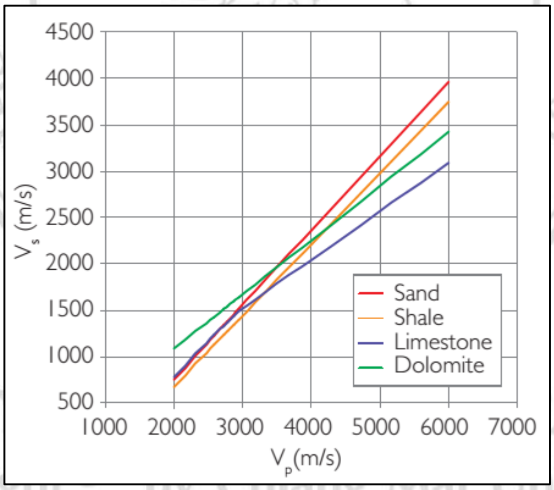

Figure 2-2. Crossplot showing V_P versus V_S of Greenberg–Castagna relations (Simm and Bacon, 2014)

2.2 AVO Response and Classification

AVO term is first proposed by Ostrander in 1982 that is the comparison of seismic amplitude changes compared to the offset between the receivers and the sources. Hence, AVO stands for Amplitude Variation with Offset or Amplitude Versus Offset. It is related to the P-wave and S-wave velocity. Without S-wave recorder, AVO can be used to infer V_S by using Aki-Richards or Zoeppritz equation if V_P and density are available.

To look at the AVO classes may identify fluid and lithology changes. This is can be used at locations far away from well control by seeing changes in AVO.

When seismic wave or P-wave encounters reflectors or layer boundaries within the Earth with velocity and density contrasts, energy of the incident wave is partitioned at each boundary. Some of the energy is mode-converted to shear wave, and then both the compressional wave (P-wave) and shear wave (S-wave) energy are partly reflected and partly transmitted at each the layer boundaries (Figure 2-3). The fraction of the incident energy that is reflected depends on the angle of incidence. Therefore, reflection amplitudes analysis of the energy should be used to detect lateral change in elastic properties of reservoir rocks, such as Poisson's ratio (Feng and Bancroft, 2006).

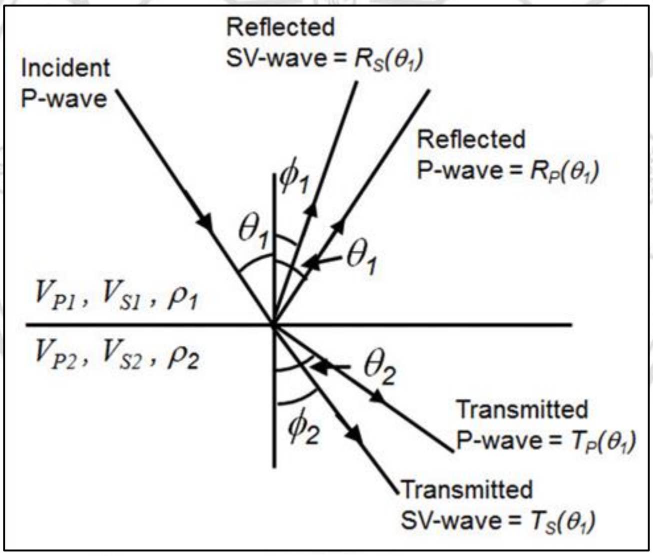

Figure 2-3. Mode conversion of an incident P-wave on the boundary between two elastic layers in the subsurface of the earth (Russell et al, 2006)

2.2.1 AVO Intercept and Gradient

The background theory for AVO is accredited to Knott (1899) and Zoeppritz (1919) who developed equations describing elastic waves as a function of reflection angle at an interface. Through the years, there have been several approaches developed to simplify these equations with a different emphasis. The most commonly used linear approximation is from Shuey (1985), who took the complicated Zoeppritz equations and produced approximations that could be measured and calculated from pre-stack data. There are two forms of the Shuey approximation.

$$\textit{Shuey three term:} \quad R(\theta) = A + B\sin^2\theta + C(\tan^2\theta - \sin^2\theta)$$

$$\textit{Shuey two term:} \quad R(\theta) = A + B\sin^2\theta$$

where θ is angle of incidence, A is the intercept and represents the reflection coefficient at normal incidence (R_0), B is the gradient (G) and denotes the slope of the reflection coefficients with $\sin^2\theta$, and C is the curvature term and describes the behavior at large angles that are close to the critical angle. The Shuey two-term approximation is typically good for angles less than 30° (Roden et al, 2014). The intercept or R_0 is controlled by the contrast in acoustic impedance, and the gradient is more complex in terms of rock properties contrasts in V_P, V_S and density.

2.2.2 AVO Classes

The classification of AVO anomalies was devised by Rutherford and Williams in 1989 and modified by Ross and Kinman in 1995 (AVO Class 2p) and Castagna and Swan in 1997 (AVO Class 4). To assume normal polarity, the five AVO classes can be sample illustrated by Veeken and Rauch-Davies (2006) as Class 1, Class 2, Class 2p, Class 3 and Class 4 (Table 2-1) (Figure 2-4).

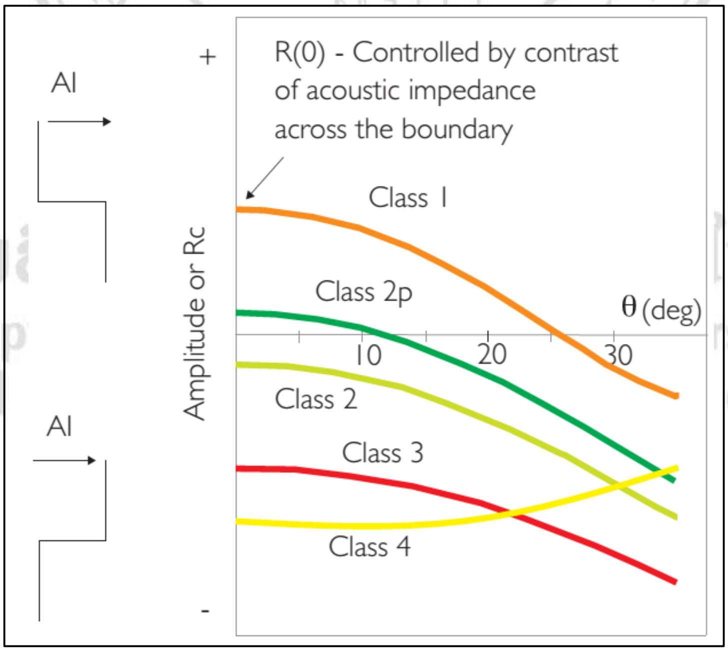

Figure 2-4. The AVO Classes modified after Rutherford and Williams in 1989, Ross and Kinman in 1995 and Castagna and Swan in 1997 (Simm and Bacon, 2014)

Table 2-1. Examples of the five AVO classes assuming normal polarity (Veeken and Rauch-Davies, 2006)

AVO	AI response at 0 angle	Response as the angle increases
Class 1	High impedance sand with decreasing AVO. The layer has higher impedance than the surrounding shale.	Rare. Low positive amplitude (peak) as the stacking changed the polarity, canceling out (dimming) the amplitude effect.
Class 2	Near-zero impedance contrast between the sand and surrounding shale.	Small positive reflectivity (peak) that changes into negative reflectivity with offset, giving a polarity flip and a dimming or brightening of the reflection on stacking.
Class 2p	Near-zero impedance contrast.	
Class 3	Low impedance sand with increasing AVO, compared to surrounding shale.	Negative reflectivity (trough) that becomes more negative, brightening the reflection on stacking. Weak amplitudes at near angle, strong amplitudes at far angle.
Class 4	Low impedance sand with decreasing AVO.	Negative amplitude becomes less negative (dims) with offset. The Shuey approximation works well for this class.

2.2.3 AVO Responses and AVO Crossplot

There are a number of terms that are generally used to describe AVO responses. The AVO plot of amplitude and angle is useful visualization of AVO responses, it is generally use in analysis of variable responses from seismic data. Plotting the intercept and gradient of each response is overcomes these limitation. Therefore, the AVO crossplot is an important to understand lithology and fluid discrimination (Simm and Bacon, 2014) (Figure 2-5).

The AVO classes describe AVO responses only for the top sand interface. In terms of sand and shale, reflections of base sand and/or hydrocarbon contact have AVO responses with positive gradients. In case, base of brine sand can have various AVO responses such as 1) Class 4, 2) phase reversal from negative to positive amplitude with increasing offset and 3) positive amplitude increasing with offset (quite low gradient). Moreover, a positive AVO with positive intercept is characteristic of hydrocarbon contacts. It is better to separate to lithology changes with positive intercept and negative gradient (Class 1) (Simm and Bacon, 2014). Figure 2-6 is illustrated some seismic responses in zero-offset of oil and brine sand at the top of a sand, and a positive impedance contrast is a positive number.

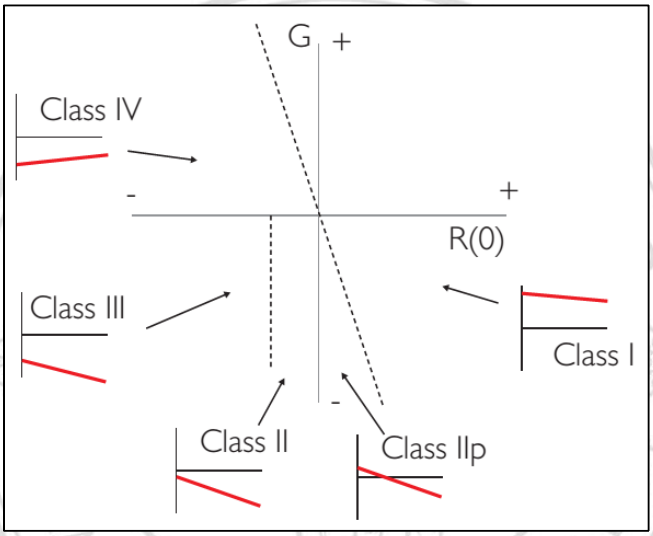

Figure 2-5. The various AVO classes are described in different area on AVO crossplot (Simm and Bacon, 2014).

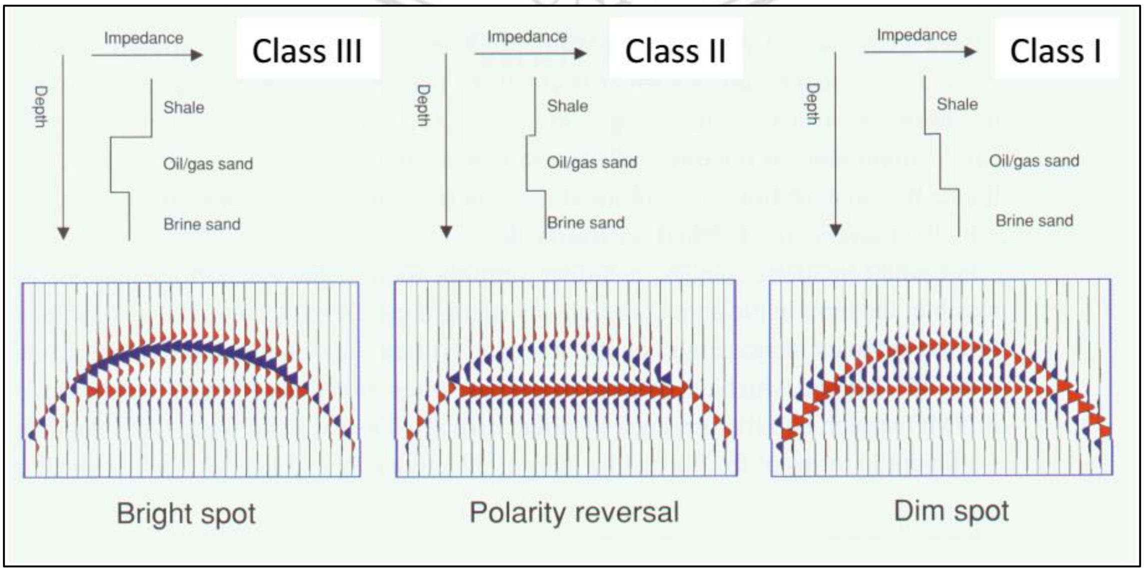

Figure 2-6. Schematic of oil and brine sand in zero-offset responses (Red is hard loop or impedance increase, and blue is soft loop or impedance decrease) (Bacon et al, 2003)

2.3 Seismic Inversion

Seismic has been used by geophysicists for almost forty years. Early inversion techniques transformed the seismic data into P-impedance (Z_P) that is the product of density and P-wave velocity (V_P) to make prediction about lithology and porosity (post-stack inversion method). However, the prediction was quite ambiguous because P-impedance is sensitive to lithology, fluid and porosity effects, and it is difficult to separate the influence of each effect. To perform a less ambiguous interpretation of inversion results, full elastic inversion must be performed by estimation of P-impedance, S-impedance (Z_S) and density when Z_S is the product of density and S-wave velocity (V_S). The reason for reflectivity that is a function of V_P, V_S and density responding to the subsurface is sufficiently different, and the changes in reflectivity as a function of angle allow seeing the difference between fluid and lithology effects. In the present, progression to the point of the inversion for P-impedance, S-impedance and density is feasible (Russell et al, 2006).

2.3.1 The General Assumptions for Seismic Inversion

1. The seismic trace can be modelled as the convolution of the earth's reflectivity and a band-limited seismic wavelet as follow.

$$S = W*R$$

where S is the seismic trace, W is the seismic wavelet, R is the reflectivity and * is represented to the convolutional operator (Figure 2-7).

2. Zero-angle (normal incidence) reflectivity is calculated by the equation at the interface of layer i and layer i+1 as

$$R_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i}$$

where Z_i is the acoustic impedance for layer i and Z_{i+1} is the acoustic impedance for layer below layer i. This equation can be used for R_P by using the corresponding impedances of Z_P, but this approach does not work well in case of the fizzy water problem. Because of low bulk modulus and low Vp of fizzy water, it is difficult to distinguish from 100% gas.

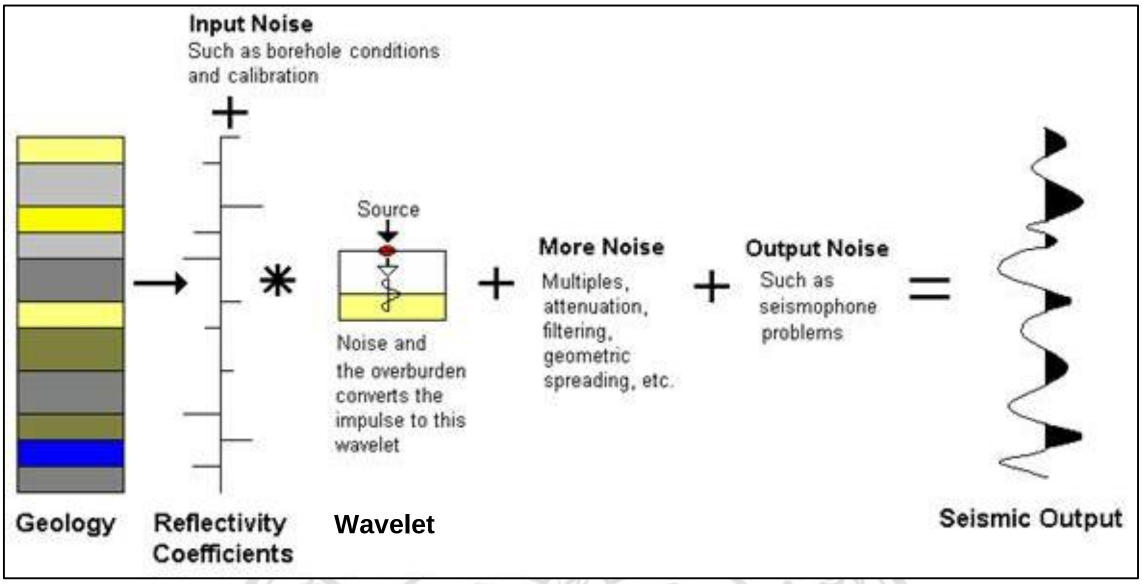

Figure 2-7. The basic convolutional model of the seismic output (or seismic trace) is Seismic Output = Wavelet convolved with Reflectivity Coefficients + Noises (after Walden and White, 1998).

3. Background trend is used as wet clastic rock. Two relationships should be hold for background trends.

3.1 Constant V_S/V_P, this ratio of the S-wave velocity and P-wave velocity should be constant within a rock layer. It is often represented by γ. Moreover, the following equation is assumed, so the impedances are related:

$$\ln(Z_S) = \ln(Z_P) + \ln(\gamma), \qquad where \;\; \gamma = \frac{V_S}{V_P}$$

3.2 Generalized Gardner, this procedure stabilizes the Aki-Richards equation by including two background regional trends that relate Z_P and density.

$$\rho = aV_P^b, \qquad and \;\; \ln(\rho) = \frac{b}{1+b}\ln(Z_P) + \frac{\ln(a)}{1+b}$$

where ρ is bulk density, Z_P is the P-wave impedance and a and b are constants.

2.3.2 Pre-stack Simultaneous Inversion

The purpose of simultaneous inversion is to invert pre-stack CDP gathers (which was applied NMO correction) to determine compression impedance (Z_P), shear impedance (Z_S) and density (ρ). In contrast, post-stack inversion ignores the fact in wet clastic rocks, Z_P and Z_S should be related. P-wave velocity (V_P) and S-wave velocity (V_S) should be linearly related when there is no complicated factor (Castagna's equation) such as the presence of hydrocarbon. Also, density should be related to V_P (Gardner's equation). Therefore, the simultaneous inversion needs to include some forms of coupling between the variables. This should be added stability to a problem of sensitive noise, and it is usually produced non-unique solutions. The actual application of pre-stack inversion uses the algorithm based on these three assumptions.

1) The linearized approximation for reflectivity holds.

2) The function of angle for PP reflectivity can be given by the Aki-Richards equations (Aki and Richards, 1980).

3) The background data is a linear relationship between P-impedance and both S-impedance and density which is expected to wet lithology.

These three assumptions can derive a final estimate of P-impedance, S-impedance and density by perturbing an initial P-impedance model (Hampson et al. 2005).

Pre-stack simultaneous inversion can be extended the theory to the pre-stack inversion case that the seismic ray strikes the boundary between two geological layers with the non-zero degrees incident angles. The results of an incident P-wave at an angle give reflected and transmitted P-waves and S-waves. The amplitudes of the reflected and transmitted waves can be computed by Zoeppritz equations (Zoeppritz, 1919) (Figure 2-3).

The problem of inversion model case, the amplitudes of velocities are found more difficult than the forward model and are the non-linear nature of Zoeppritz equations. So, pre-stack inversion had been started with the linearized Aki-Richards equation (Aki and Richards, 1980, Richards and Frasier, 1976) which was re-expressed by Fatti et al. (1994) as below equation (Hampson et al., 2005).

$$R_{PP}(\theta) = c_1 R_P + c_2 R_S + c_3 R_D$$

where $c_1 = 1 + tan^2\theta,\ c_2 = -8\gamma^2 sin^2\theta,\ c_3 = -0.5tan^2\theta + 2\gamma^2 sin^2\theta,\ \gamma = V_S/V_P$, and the linearized P-reflectivity (R_P), S-reflectivity (R_S) and density reflectivity (R_D) are

$$R_P = \frac{1}{2}\left[\frac{\Delta V_P}{V_P} + \frac{\Delta\rho}{\rho}\right] = \frac{\Delta Z_P}{2Z_P}$$

$$R_S = \frac{1}{2}\left[\frac{\Delta V_S}{V_S} + \frac{\Delta\rho}{\rho}\right] = \frac{\Delta Z_S}{2Z_S}$$

$$R_D = \frac{\Delta\rho}{\rho}$$

Then, Hampson et al (2005) used a linearized inversion approach to solve the reflectivity terms given in above equations. They extended the work of Simmons and Backus (1996) that developed a scheme to invert for P-reflectivity (R_P), S-reflectivity (R_S) and density reflectivity (R_D), and the work of Buland and Omre (2003) that called Bayesian linearized AVO inversion. The new development allows inverting directly for P-impedance, S-impedance and density. This is to extend model-based post-stack impedance inversion and combine to equation of Fatti et al. (1994) therefore this method could be seen as a generalization to pre-stack inversion for a given angle trace *S(θ)* equation.

$$S(\theta) = \left(\frac{1}{2}\right)c_1 W(\theta) D L_P + \left(\frac{1}{2}\right)c_2 W(\theta) D L_S + c_3 W(\theta) D L_D$$

where L_S is $\ln(Z_S)$, L_D is $\ln(\rho)$, W is the wavelet matrix, D is the derivative matrix, and wavelet is dependent on angle. The equation is dealing with logarithms of impedances rather than velocity.

One of the key assumptions in the simultaneous inversion is that linear relationships between the logarithms of P-impedance and S-impedance (L_P and L_S) and between the logarithms of P-impedance and the density reflectivity (L_P and L_D) can be build. That is deviations away from this linear fit given by ΔL_S and ΔL_D are observed (Figure 2-8) (Russell and Hampson, 2006).

$$\ln(Z_S) = k\ \ln(Z_P) + k_c + \Delta L_S$$

$$\ln(\rho) = m\ \ln(Z_P) + m_c + \Delta L_D$$

where ΔL_S and ΔL_D are the effect when the rock fluid is not water. Therefore, in wet rock ΔL_S and $\Delta L_D = 0$.

Combining the angle trace $S(\theta)$ equation and linear relationship equation of Z_P and both Z_S and density is gotten finally simplify the constants of simultaneous inversion equation as:

$$S(\theta) = \tilde{c}_1 W(\theta) D L_P + \tilde{c}_2 W(\theta) D \Delta L_S + c_3 W(\theta) D \Delta L_D$$

where $\tilde{c}_1 = (1/2)c_1 + (1/2)kc_2 + mc_3$ and $\tilde{c}_2 = (1/2)c_2$, and this equation can be implemented in matrix form as Figure 2-9.

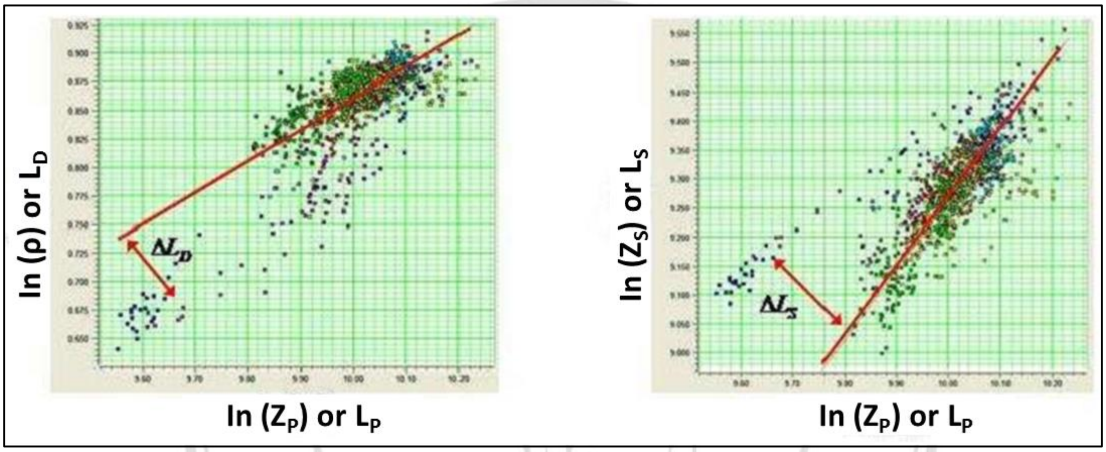

Figure 2-8. Crossplots of $\ln(Z_D)$ vs $\ln(Z_P)$ and $\ln(Z_S)$ vs $\ln(Z_P)$ where, in both cases, a best straight line fit has been added. The deviations away from this straight line, ΔL_D and ΔL_S, are the desired fluid anomalies (Russell and Hampson, 2006).

$$\begin{bmatrix} S(\theta_1) \\ S(\theta_2) \\ \vdots \\ S(\theta_N) \end{bmatrix} = \begin{bmatrix} \tilde{c}_1(\theta_1)W(\theta_1)D & \tilde{c}_2(\theta_1)W(\theta_1)D & c_3(\theta_1)W(\theta_1)D \\ \tilde{c}_1(\theta_2)W(\theta_2)D & \tilde{c}_2(\theta_2)W(\theta_2)D & c_3(\theta_2)W(\theta_2)D \\ \vdots & \vdots & \vdots \\ \tilde{c}_1(\theta_N)W(\theta_N)D & \tilde{c}_2(\theta_N)W(\theta_N)D & c_3(\theta_N)W(\theta_N)D \end{bmatrix} \begin{bmatrix} L_P \\ \Delta L_S \\ \Delta L_D \end{bmatrix}$$

Figure 2-9. Matrix form of simultaneous inversion equation

If this equation is solved by matrix inversion methods, the problem of the low frequency content will not be resolved. So, a practical approach is to initialize the solution to $[L_P, \Delta L_S, \Delta L_D]^T = [\ln(Z_{P0}), 0, 0]^T$, where Z_{P0} is the initial impedance model, and then to iterate towards a solution using the method of conjugate gradients. (Hampson et al, 2005, 2006, Russell and Hampson. 2006, Russell et al, 2006 and Hampson and Russell, 2013)