CHAPTER 2

Theory

2.1 Porosity

Porosity is the fraction of the void space over the total volume in a material. The range of the porosity is 0 to 1 (0 to 100 in terms of percentage). In rocks, the pore space can be interconnected or not connected. The mathematical expression for porosity determination in the rock is stated as:

$$\phi = \frac{V_b - V_{gr}}{V_b} = \frac{V_p}{V_b} \quad \text{(2-1)}$$

Where,

ϕ = Porosity

V_b = Bulk volume of the rock

V_{gr} = Grain volume

V_p = Pore volume

The amount of porosity is largely affected by the roundness, sorting, size, packing, shape, and orientation. Well-sorted materials have higher porosity than the poor-sorted materials. Again, smaller grains can significantly decrease the total porosity effectively by filling the pores (Schlumberger, 1991). The presence of shale or clay in rock decreases the effective pore space. So, the shale or clay porosity has to be subtracted from the total porosity to obtain the effective porosity (Sheriff, 2002).

2.1.1 Volume of Shale Calculation

The gamma ray log is measured in API (American Petroleum Institute) unit. In rock formations the radioactive elements (K, Th, and U) radiate the concentration of gamma rays. Usually shale contains more radioactive minerals than sand or carbonate.

As radioactive elements tend to be concentrated in clay and shale, the volume of shale or clay can be measured using the gamma ray log. The gamma ray log can thus be used to identify lithologies and correlate zones (Asquith and Krygowski, 2004).

The shale volume (V_{sh}) is calculated in decimal fraction or percentage and it ranges from 0 to 1 (0 to 100 in terms of percentage). The mathematical expression for volume of shale determination in the rock is stated as (Asquith and Krygowski, 2004):

$$V_{sh} = \frac{GR_{log} - GR_{min}}{GR_{max} - GR_{min}} \quad \text{(2-2)}$$

Where,

V_{sh} = Shale volume

GR_{log} = Gamma ray reading

GR_{min} = Minimum gamma ray log (clean sandstone)

GR_{max} = Maximum gamma ray log (Shale, Clay, etc.)

2.1.2 Porosity from Density Log

The density log actually measures the electron density in the formation. It measures the bulk amount of electrons of a formation (Asquith and Gibson, 1982). The density log gives the bulk density of the formation including matrix and fluid density. Fluids usually have a lower density than the rock matrix. So the bulk density decreases if the fluid contains in much amount in the formation. On the other hand, fluid stays in the pore or void space. So the pore space controls the amount of density. The mathematical expression for porosity calculation from the density log in the rock is stated as (Asquith and Gibson, 1982):

$$\phi_D = \frac{\rho_{ms} - \rho_b}{\rho_{ms} - \rho_f} \quad \text{(2.3)}$$

Where,

ϕ_D = Porosity from density log

ρ_{ms} = Matrix density

ρ_f = Bulk density reading from density log

ρ_b = Fluid density

Some common matrix and fluid density values are shown in the Table 2-1:

Table 2-1: Matrix and fluid density (Asquith and Gibson, 1982)

Matrix	**Density (g/cc)**	**Fluid**	**Density (g/cc)**
Quartz	2.65	Fresh water	1.00
Calcite	2.71	Salt water	1.15
Dolomite	2.87	Oil	0.85
Anhydrite	2.96		

2.1.3 Porosity from Sonic Log

Sonic log measures the interval transit time (Δt) of the signal in the formation. The higher the porosity contains, the higher the travel time is found. The porosity can be determined from the sonic log. The mathematical expression for porosity calculation from the sonic log in the rock is stated as (Wyllie et al., 1958):

$$\phi_s = \frac{t-t_o}{t_f-t_o} \qquad (2\text{-}4)$$

Where,

ϕ_S = Porosity from sonic log

t = Travel time interval in the formation

t_o = Travel time interval in the matrix

t_f = Travel time interval in the fluid

2.1.4 Root Mean Square (RMS) Porosity

Porosity can be determined using various tools and can also be extracted from density and sonic logs. The readings of different porosity logs can widely vary. For example, the neutron porosity sees hydrogen atoms in the formation. So usually it gives higher porosity in shale and clay because of containing bound water. On the other hand, density and sonic log are not affected too much by the bound water in shale and clay. So porosity is found less in this case. It is a debatable issue which one is the correct porosity log. In this situation the root mean square porosity calculation can be a useful tool. It takes values from two or more porosity logs and might give a reasonable porosity measurement. The mathematical expression for the RMS porosity calculation from logs in the rock is stated as:

$$\phi_{RMS} = \sqrt{\frac{x_1^2+x_2^2+x_3^2+\cdots+x_n^2}{n}} \quad \text{……………………………} \quad (2\text{-}4)$$

Where,

ϕ_{RMS} = Root Mean Square Porosity

x = Porosity values from different logs

n = Number of logs

2.2 Seismic Attributes

Nowadays seismic attributes are widely used to extract the quantity of the seismic data in order to enhance the information in the reflection seismology. It helps to obtain a better understanding of geophysical and geological interpretation model. It extracts more subtle details than a traditional seismic image (Barnes, 2001). Seismic attributes are an aid for the geoscientist in reservoir characterization and also a tool for quality control. One of the common uses of seismic attributes is seismic amplitude that reports the maximum and minimum amplitude reflected to the acoustic impedance contrast. In many cases, the porosity of the formation is corresponded by the amplitude of reflection (Taner, 2001).

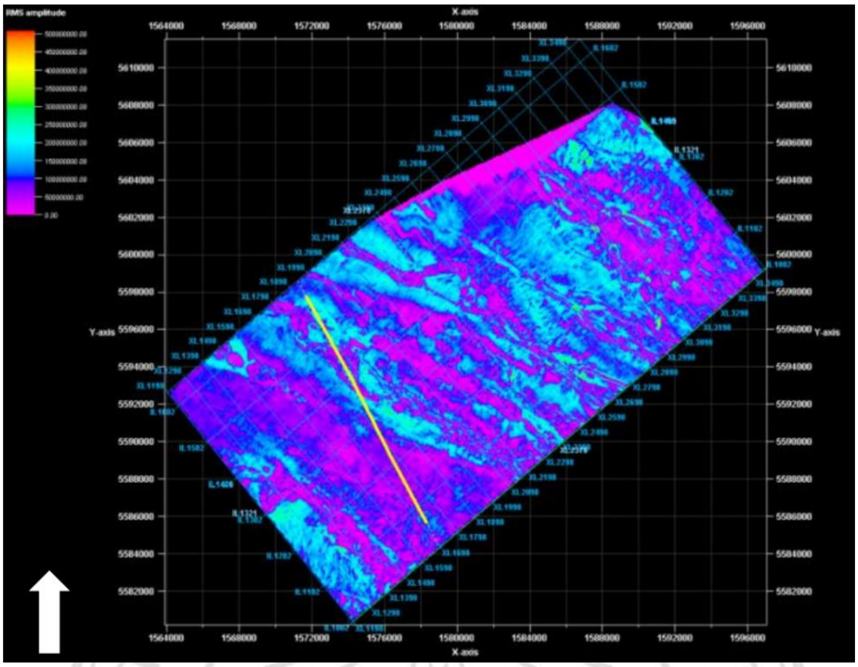

Figure 2-1: RMS amplitude attribute shows the high value in blue color that represented high contrast of acoustic impedance in Hector field, New Zealand (modified from ARCO Petroleum, 1992)

2.3 Seismic Inversion

Seismic data is used in exploration geophysics to investigate the properties of the subsurface structures of the earth. Seismic traces are the result of the convolution of seismic reflectivity and wavelet. Moreover, the reflectivity of the seismic data is expressed in terms of acoustic impedances of two adjacent layers. Seismic inversion aims to remove the wavelet from the seismic traces to get the reflectivity series back and to obtain the acoustic impedance (AI) from the seismic reflectivity.

To acquire seismic data controlled seismic energy sources, such as vibroseis, dynamite, etc. are needed. These sources produce the seismic waves and it is possible to know the approximate depth of the reflectors by knowing the travel time of a reflected wave to reach at a receiver (Geotrade, 2009). The recorded reflection data are processed and the seismic section is displayed as the final output. A seismic section consists of a number of seismic traces which is the convolution of the seismic reflectivity of the earth and a band-limited seismic wavelet. It can be expressed mathematically as follow:

$$S_t = W_t * r_t + n \qquad (2.5)$$

Where,

S_t = Seismic trace

W_t = Seismic wavelet

r_t = Reflectivity

n = Noise component

And the convolution operator is denoted by * in Equation 2-5.

Acoustic impedance (AI) is the product of density and velocity of the subsurface rock. It can be expressed mathematically as follow:

$$AI_i = V_i * \rho_i \quad (2\text{-}6)$$

Where,

AI_i = Acoustic Impedance of the i^{th} layer

V_i = Acoustic velocity of the i^{th} layer

ρ_i = Density of the i^{th} layer

The seismic energy reflects and refracts at the interfaces between two adjacent layers when the seismic wave passes through the earth subsurface layers. The reflectivity depends on the acoustic impedances of two interfacing layers. Mathematically it can be expressed as follow:

$$r_i = \frac{AI_{i+1} - AI_i}{AI_{i+1} + AI_i} \quad (2\text{-}7)$$

Where,

r_i = The reflectivity at the interface of i^{th} and $(i+1)^{th}$ interfaces

AI_i = Acoustic impedance of the i^{th} layer

If one seismic trace is recorded by geophone or hydrophone by following the Equation 2-5, then the acoustic impedance can be calculated by inverting the Equation 2-7 (Lindseth, 1979). Equation 2-8 can be applied to series of reflectivity derived from a seismic trace.

$$AI_{i+1} = AI_i \left[\frac{1+r_i}{1-r_i}\right] \quad (2.8)$$

Where,

r_i = The reflectivity at the interface of i^{th} and $(i+1)^{th}$ interfaces

AI_i = Acoustic impedance of the i^{th} layer

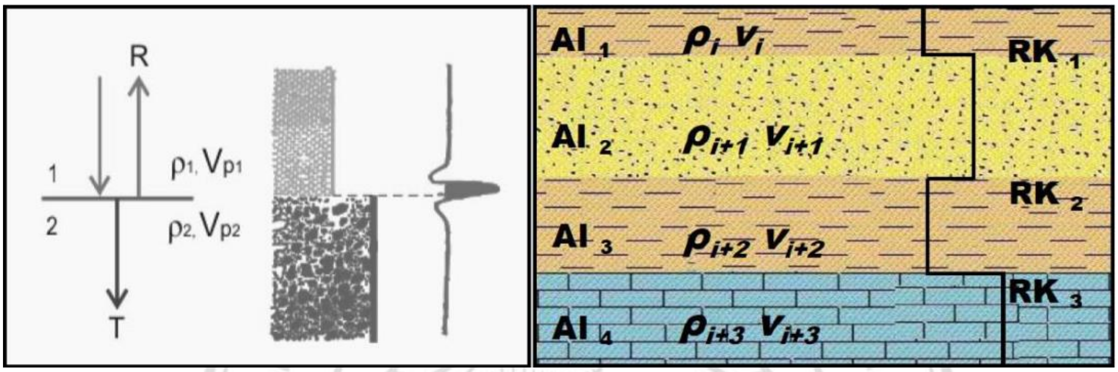

Figure 2-2: The illustration of the relationship between acoustic impedance and reflection coefficient (Bhatia, 1986)

Seismic inversion can be of two types: Pre-stack and Post-stack. Seismic inversion can be applied to pre-stack and post-stack data and it translates seismic reflection data into quantitative reservoir rock properties (Francis, 2005). With the seismic data it may include other reservoir measurements such as core data and well log data.

Based on the extracting methodss seismic inversion can be deterministic inversion and stochastic inversion. Deterministic inversion extracts an absolute solution and assumes that the inputs are known and not subjected to error. The stochastic inversion, in turns extracts a non-unique solution and assumes that the sub-surface properties are fixed and unknown (Francis, 2005). The detailed information of the sub-surface cannot be extracted from the conventional seismic interpretation but the seismic inversion provides detailed geological information to facilitate the interpretation by increasing the resolution of the seismic data and improving the estimation of rock properties. It replaces the seismic signature by blocky responses which correspond to acoustic impedance layers (Pendrel, 2006). The velocity and the density of the rock can be obtained directly from well logs or estimated mathematically from the acoustic impedance data. Porosity can be calculated from the empirical relationship between the acoustic impedance and porosity in an inverse problem (Dolberg et al., 2000), and the inverse problem can also be solved by minimizing the difference between the seismic and synthetic data.

2.4 Model-based Deterministic Post-stack Seismic Inversion

Model-based inversion technique was introduced by Russell and Hampson in 1991. In this technique a generalized linear inversion algorithm is used and it is assumed that the seismic trace and the wavelet are known. This inversion method tries to modify an initial model until the resulting synthetic matches the seismic trace (Cooke and Schneider, 1983). This is an effective inversion method if the proper geological knowledge is available and a reliable model can be created. The basic approach is to minimize this function:

$$J = [weight_1\ x\ (S - W * R)] + [wieght_2\ x\ (M - H * R)] \quad \text{.................. (2-9)}$$

Where,

S = Seismic trace

W = Wavelet

R = Final reflectivity

M = Initial guess model

H = Integration operator

And the convolution operator is denoted by * in Equation 2-9.

The first part of the Equation 2-9 models the seismic trace, whereas the second part models the initial guess impedance. The Hampson-Russell software implements a “Constrained Model Inversion”. $Weight_2$ is 0, where the final impedance values are set within upper and lower values (using a percentage of the average impedance for the log). A "soft" constraint, meaning that the initial guess impedance is considered as a separate component which is added to the seismic trace, can be used to extract additional information but the “hard” constraint is recommended because it prevents small amounts of noise in the data or modeling errors (STRATA user guide, 2009).

The summary of the model-based deterministic post-stack inversion can be described as follows (Gavotti, 2014):

- As it is assumed that the wavelet is known, its effect can be removed from the seismic data. For example it can be said that the seismic data does not need to be zero-phase as long as the wavelet contains the same phase.
- The inversion result would be affected by the errors in the estimated wavelet.
- The resolution of the seismic data is enhanced.
- The inversion result is dependent on the initial model. This is why the model should be filtered to reduce this effect. So it includes the low-frequency component usually absent in the seismic data.
- It is very possible that more than one geological model exists which is consistent with the seismic response. To reduce this uncertainty, the initial model should be independent of the seismic data and must include information about the unknown geology (well logs, velocity fields, etc).

2.5 Wavelet Extraction

A wavelet extraction is required to deconvolute the seismic data to generate seismic reflectivity series. It is a harmonic wave that has an interval of amplitude, frequency and phase (Sismanto, 2006). A wavelet can be extracted in three approaches: by a synthetic wavelet extraction, an extraction from seismic data alone, and an extraction from the seismic and well data together. In this study, the wavelet is extracted from seismic data alone (statistical wavelet) and extracted from seismic and well data together.

The statistical wavelet uses the seismic data as an input and the position of the targeted time window. This extraction process starts with the auto-correlation of the seismic traces to define the length of the desired wavelet. The frequency spectrum of the auto-correlation is computed and then the square root of the frequency spectrum modulus is taken. In that time the zero hertz component is muted to zero. Finally the inverse FFT is computed to obtain the wavelet (dGB earth Sciences, 2015). Usually, the correlation factor obtained by statistical method is higher than the other methods. As this wavelet depends on the window size, it changes from trace to trace as a function of travel-time. So a single average wavelet extraction for the entire seismic section might be a practical solution (Hampson and Russell, 1999).

The wavelet extraction from the well log and seismic data together is done during the seismic-well tie process. This extraction comes from the best match of the synthetic seismogram with the seismic data. All functions of this wavelet including amplitude, phase, timing, and polarity are estimated through a matching process.

2.6 Seismic to Well Tie

It is necessary to get the matching parameters between seismic and well data to relate them as they are recorded in different domains. The seismic data is recorded in time and the well data is recorded in depth. So a time-depth curve is required to match the two data sets. The most common tool to match seismic and well data is using a check-shot survey. If the check-shot survey is not available, then a synthetic seismogram can be estimated from the statistical wavelet and reflectivity series obtained from acoustic impedance log data. The acoustic impedance log is obtained from the multiplication of the p-wave velocity and density log data. The synthetic seismogram is then matched with the seismic traces to get good matches. The match varies from 0 to 1 and if the match is good enough then a time-depth relation is found to correlate the seismic and well data.