APPENDIX A

On the Linkages between Exchange rate movements, stock, bond and interest rate market in a regime-switching model: Evidence for ASIAN countries

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On the Linkages between Exchange Rate Movements Stock, Bond and Interest Rate Market in a Regime-Switching Model: Evidence for ASEAN and East Asia

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Abstract : This study analyzes the relationship among exchange rate (against US dollar), interest rate, government bond and the stock market in three ASEAN countries consisting of Thailand, Malaysia, Singapore and three East Asia countries comprising Japan, Korea, and China. The paper analyzes the question whether there exist a correlation between these variables in both high growth and low growth economy and whether there exist a similar market pattern in these countries. In this study, we estimate the correlations between these variables using the MS-VECM approach. In addition, the obtained regime probabilities allow us to detect and identify the factor or event affecting the movement of the financial markets.

Keywords : MS-VECM; Bayesian; the relationship of financial markets; regime probabilities.

2010 Mathematics Subject Classification : 47H09; 47H10.

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1 Introduction

With the development of international financial markets, the stock index, exchange rate, government bond yield and interest rate can grow more interacting through trade flow and capital flow. Volatility affecting one market may be transmitted rapidly to another by contagion effects. Estimating and understanding the dynamic linkages have important implications for asset allocation, portfolio diversification, currency risk hedging, stock and currency market return predictability. In this article, we examine whether the spill-over effects exist and take place across exchange rate (against US dollar), interest rate, government bond and the stock markets.

Why we consider these four markets in our study?

There are many approaches and evidences that confirm the relationship between those four variables. For the stock and exchange market. There are two potential theories expressing the relationship between stock prices and exchange rate (foreign exchange market: FX). The first is the flow-oriented model, which argues that the currency exchange will impact the international competitiveness and trade balance. For instance, domestic currency depreciation improves the competiveness of local firms, resulting in an upward movement of stock prices in response to the increase in expected in-coming cash flows. The second is the stock-oriented model which shows that exchange rates are affected by stock prices movements and the persistent upward trend in stock prices will attract money inflow and lead to an appreciation of the currency, or vice versa. Numerous researches have investigated the linkages between stock index and FX market and provided interesting empirical results. Diamandis and Drakos [1] used VECM model and found that the stock index and FX was positively related in Brazil, Argentina, Chile, and Mexico. Tsai [2] found that the relationship between the stock index and FX was negative in Malaysia, Singapore, South Korea, the Philippines, Taiwan, and Thailand. Tudor and Popescu-Dutaa [3] used VAR model and found the causality relationship was from FX to stock index in Brazil and Russia and no relationship between FX and stock index in China. The causality-in-variance was found to be from the stock returns to exchange rate changes in the US, in the opposite direction in the Euro area and Japan, and of bidirectional feedback in Switzerland and Canada, in the study by Caporale, Hunter, and Ali [4] who used bivariate DCC-GARCH model to study the banking crisis between 2007 and 2010. Many research papers have also been undertaken on the relationship between stock and bond markets such as those by Yang et al.[5], Andersen et al. [6], Baele [7] which commonly found positive significant relationship to exist between stock and bond markets. Another strand of the literature has brought attention to the dependency between FX and interest rates as well. The relationship between FX and interest rate is positive under the flexible prices approach [8] but under the Keynesian approach, the relationship is negative. Bautista [9] suggested a strong positive correlation between interest rate and FX during the turbulent periods in the Philippines from his dynamic conditional correlation (DCC) analysis. Conversely, Sanchez [10] found that the correlation between exchange rates and interest rates, given risk premium as condition, is negative for economic expansion and positive for economic contraction. Furthermore, we also found some evidences that stock market can be influenced by interest rate movement. There are also different views in terms of the relationship between interest rates and stock prices. For example, higher interest rates increase the opportunity cost of money, thus decreasing the return and stock prices of companies. On the other hand, lower interest rates do not have the opposite impact on stock prices. The Markov-switching vector autoregressive (MS-VAR) model, is utilized by Kal, Arslaner, and Arslaner [8] for investigating whether the deviation of a currency from its fundamentally determined rate of return affects the relationship between interest rates and stock market yields. From some evidences cited above motivated us to explore to explore the relationship between the four financial variables of our interest. Our study will cover six East and Southeast Asian countries because financial markets in Asia have become more attractive for foreign capital investment and these countries in particular have grown more export-dominant in recent decades. Hence, the goal of this paper is aimed at examining profoundly the various relationships between these four financial variables and providing the explanation for the different economic condiction. To achieve our purpose, the Markov Switching Vector error correction model (MS-VECM), which was introduced in Krolzig, Marcellino, and Mizon [11], is employed in this study. The model has an ability to estimate the cointegrated structure of variables and capture the long-run relationship of the variables in the financial model and it can also explain the non-linearity embedded in the relationship of financial markets in each country. To estimate the parameters in the model, we select a Bayesian estimation technique because the computation in the conventional maximum likelihood method may be difficult in our case where we have a large number of unknown parameters in the model. Moreover with the Bayesian prior for our estimated parameter, it is possible to reduce the estimation uncertainty and to obtain accurately the inference[12] . The rest of the paper is organized as follows. Section 2 discusses the MS-VECM model and the Bayesian prior and posterior estimation. The data description and the estimation results are presented in Sections 3 and 4. Finally, Section 5 summarizes and concludes the paper.

2 Methodology

2.1 Markov Vector Error Correction Model

To understand our approach, consider the following Markov-switching VECM (MS-VECM):

$$\Delta y_t = c_{S_t} + \sum_{i=1}^{p} \beta_{i,S_t} \Delta y_{t-i} + \Pi_{S_t} \Delta y_{t-1} + u_{t,S_t} \tag{2.1}$$

where Δy_t denotes a k-dimensional vector of differenced variables of interest, c_{S_t} is a vector of state dependent intercept term, β_{i,S_t} is state dependent autoregressive parameter matrices of vector Δy_{t-i}, Π_{S_t} are the state dependent error correction terms defined by the r k matrix of co-integrating vectors and is u_{t,S_t} error variance which is allowed to change across the regimes with normally distributed,$u_{t,S_t} \sim N(0, \Sigma_{S_t})$. S_t denotes the unobserved state variable which evolves according to a H-state and thus, allowing intercept term, co-integrating term, autoregressive term and variance-covariance matrix to switch across regimes. In this study the state variable is assumed to follow the first-order Markov switching process with the transition probabilities,$P_{ij}(S_t = i|S_{t-1} = j), \quad i, j = 1, ..., H$

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{H1} \\ p_{12} & p_{22} & \cdots & p_{H2} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1H} & p_{2H} & \cdots & p_{HH} \end{bmatrix} \tag{2.2}$$

where P_{ij} is the probability change from regime i to regime j. In this study, the two-regimes MS-VECM is assumed, following the popular practice in many studies. Consequently, we can extend Eq. (2.1) as follows:

$$\Delta y_t = \begin{cases} c_{(S_t=1)} + \beta_{1(S_t=1)}\Delta y_{t-1} + ... + \beta_{i(S_t=1)}\Delta y_{t-p} + \Pi_{(S_t=1)}\Delta y_{t-1} + u_{t,(S_t=1)} \\ c_{(S_t=2)} + \beta_{1(S_t=2)}\Delta y_{t-1} + ... + \beta_{i(S_t=2)}\Delta y_{t-p} + \Pi_{(S_t=2)}\Delta y_{t-1} + u_{t,(S_t=2)} \end{cases} \tag{2.3}$$

2.2 Prior Distributions and Likelihood

In this study, we choose a prior density for our parameters following the estimation by Doan in RATS software. The selected Flat prior density is applied in the estimation of MS-VECM model where intercept term (c_{S_t}), autoregressive term (β_{i,S_t}), co-integrating vector (Π_{S_t}) are assumed to have informative prior, flat prior, variance-covariance matrix (Σ_{S_t}) to have Inverted Wishart prior, and Beta prior for the transition probabilities (P_{ij}).

Let $\theta = \{c, \beta, \Pi\}$,have the least informative priors, i.e., flat prior, where the prior is simply a constant. Thus, the posterior is constant times the likelihood,

$$P(\theta_{S_t}, \Sigma_{S_t}, P_{ij}|\Delta y_t) = pr(\theta_{S_t}) \bullet 1(\theta_{S_t}, \Sigma_{S_t}, P_{ij}|\Delta y_t) \tag{2.4}$$

where $pr(\theta_{S_t})$ is a flat prior with uniform distribution $(-\infty, +\infty)$. Thus, the likelihood of the model will generate more effect on the posterior distribution. For Σ_{S_t}, the inverted Wishart priors are used.

$$\Sigma_{S_t} \sim IW(\Phi_{S_t}, v_{S_t}) \tag{2.5}$$

where $\Phi_{S_t} \in R^{n\times n}$ is the prior error variance for variance-covariance parameters for both two regimes and v_{S_t} is the degree of freedom of the Wishart densities. As prior for transition probabilities p_{ij} ; $i = 1, 2; j = 1, 2$, we define the prior for the P_{ij}, to be P_{ij} : $\beta(m_{ij} + 1, m_{ii} + 1)$ where m_{ij} is the number of prior transitions. Summarizing, the likelihood function for c_{S_t}, Π_{S_t}, β_{S_t}, Σ_{S_t}, P_{ij} and S_t is given by,

$$L\left(c_{S_t}, \beta_{S_t}, \Sigma_{S_t}, P_{ij}, S_t|\Delta y_t\right) \propto$$
$$\prod_{i=1}^{n}\left\{\sum_{S_t=1}^{H=2}\left((2\pi)^{-\frac{1}{2}}|\Sigma_{S_t}|^{-\frac{1}{2}}\right)exp\left(-\frac{1}{2}tr\left[\left\{vec(u_{S_t})'\left(\Sigma_{S_t}\bigotimes I\right)(vec(u_{S_t}))\right\}\right]\right)\right\} \tag{2.6}$$

where $u_{S_t} = \Delta y_t - c_{S_t} - \beta_{1,S_t}\Delta y_{t-1} - ... - \beta_{i,S_t}\Delta y_{t-p} - \Pi_{S_t}\Delta y_{t-1}$.

2.3 Posterior Estimation

The posterior densities were obtained from the priors times the likelihood functions. Katsuhiro [13] proposed two steps of posterior estimation via Gibb sampling. First, using Hamiltons filter method to estimate the state variable $S_t = \{s_1, ..., s_t\}$, $S_t \in (1, 2)$, then we estimate the posterior densities for the intercept term, co-integrating term, autoregressive term and variance-covariance matrix.

To sample the state (or regime) variable (S_t), Hamiltons filter [14] is used to filter the state variable S_t from the following conditional distribution

$$P\left(S_t|S_{t+1}, \Theta, \Delta y\right) = \frac{P\left(S_{t+1}|S_t, \Theta, \Delta y\right) P\left(S_t|\Theta, \Delta y\right)}{P\left(S_{t+1}|\Theta, \Delta y\right)} \tag{2.7}$$

where $\Theta = \{c_{S_t}, \beta_{S_t}, \Sigma_{S_t}, \Pi_{S_t}, p_{11}, p_{22}, p_{12}, p_{21}\}$. After drawing the$S_t$, we then, generate the transition probabilities, $P = \{p_{11}, p_{12}, p_{21}, p_{22}\}$which are also derived from the previous estimation algorithm. Note that they are drawn from posteriors formed from beta-conjugate distributions. Then, to estimate$\Theta$the Multi-move Gibbs sampling can be used to generate sample draws which involve the repeated generation of variates from their full conditional densities, as follows:

1) Specify the staring values for P^0,${c_{S_t}}^0$,${\Pi_{S_t}}^0$,${\beta_{S_t}}^0$ and ${\Sigma_{S_t}}^0$.
2) Generate ${S_t}^j = \left\{{s_1}^j, {s_2}^j ..., {s_t}^j\right\}'$ from $P\left(S_t|\Theta^0, \Delta y\right)$.
3) Generate the transition probabilities P^j from $P\left(P_{11}, P_{12}, P_{21}, P_{22}|{S_t}^j, \Theta^0, \Delta y\right)$.
4) Generate ${c_{S_t}}^j$ from $P\left(c_{S_t}|{\beta_{S_t}}^0, {c_{S_t}}^0, {\Sigma_{S_t}}^0{S_t}^j\Delta y\right)$.
5) Generate ${\beta_{S_t}}^j$ from $P\left(\beta_{S_t}|{\beta_{S_t}}^0, {c_{S_t}}^j, {\Sigma_{S_t}}^0{S_t}^j\Delta y\right)$.
6) Generate ${\Sigma_{S_t}}^j$ from $P\left(\Sigma_{S_t}|{\beta_{S_t}}^j, {c_{S_t}}^j, {\Sigma_{S_t}}^0{S_t}^j\Delta y\right)$.
7) Repeating step 2-6 to generate P^{j+1},${\beta_{S_t}}^{j+1}$,${\Sigma_{S_t}}^{j+1}$, and ${S_t}^{j+1}$.

In this study, 10,000 iterations samples were generated using the MCMC Gibbs sampling estimation procedure as described in the steps above. The first 1,000 samples were discarded and the remaining 9,000 samples were used to describe the joint parameter density. As a result, we can obtain the posterior means and standard deviations of these remaining samples.

3 Dataset

In this study to analyze the relationship between the stock index, exchange rate, government bond yield and interest rate. The data were collected from Thomson DataStream; the selected variables consist of exchange rate, stock price, interest rate and bond yield from Thai, Malaysian, Singapore, Japanese, South Korean, and Chinese financial markets. The data are weekly time series for the period from March 2009 to February 2016, covering totally 362 observations. we transformed these variables into logarithms before computing in the model.

4 Empirical Results

4.1 The Results of Unit Roots Test

Prior to conducting the Markov-switching with co-integration analysis, it is important to determine the order of integration for all variables in order to ensure that there are not integrated at the zero order. In this study, we employed the Bayes factor unit root test of Wang and Ghosh [15] to identify the order of integration of our variables.

In this study, we specify the null hypothesis of unit root as $H_0 = P\left(\phi = 1|\Delta y_t\right)$ and the alternative hypothesis as $H_a = P\left(0 < \phi < 1|\Delta y_t\right)$. The null hypothesis can be determined as the marginal likelihood of AR(1) model $\Delta y_t = a + (\phi - 1)\,\Delta y_{t-1} + \varepsilon_t$ where $\phi = 1$ while $0 < \phi < 1$ for an alternative marginal likelihood of AR(1) model. In this test, Bayes factor is the posterior odd ration $P\left(\phi = 1|\Delta y_t\right)/P\left(0 < \phi < 1|\Delta y_t\right)$ and the null hypothesis is rejected if Bayes factor is less than 1. The results of the Bayes factor are presented in Table 1, which showed that the logarithm of all variables are I(1) and I(2).

4.2 Lag Length Selection

In this section, we have to specify the lag length for the MS-VECM model in order to choose the shortest lags which produce serially uncorrelated residuals. We employed the vector error correction lag length criteria to find the best number of lag lengths. For the VECM lag length criteria based on BIC, the results are reported in Table 2 and revealed that the BIC values for lag=1 are the lowest. Therefore, in this study, we chose the appropriate lag length p=1 to estimate our model.

4.3 Test for Number of Co-Integration

To determine the rank or the number of co-integration vectors, Bayesian information criteria (BIC) is conducted and the results are shown in Table 3.

into a high income economy by the year 2020, launched on September 25, 2010. There are some costs for the Economic Transformation Program, and also some risk for these programs, such as declining in oil price and the volatility in capital flows from the normalization of US monetary policy. In Figure 2, we can see all of these risks resulting in the low growth regime from early of 2010 to middle of 2014.

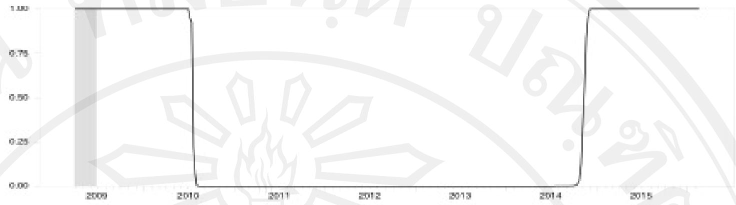

Figure 2: Regime 1 probabilities of Malaysia's Market

The regime probabilities of Singapore for regime 1 are presented in Figure 3. From the estimated results of Singapore, we interpret regime 1 as low growth economy and regime 2 as high growth economy. Singapore has become the largest foreign exchange trading center in Asia and ranks second in interest rate derivatives trading. Singapore is a leading global financial center in the world, particularly in Southeast Asia. Singapore is highly vulnerable to the global economic environment given its open economy. Therefore the world economic crisis can bring a huge impact on Singapores economy. As we can observe from Figure 3, the low growth regime was during 2009-2016. Over that period, there were severe crises in United States of America (USA) and Euro zone called hamburger crisis and European debt crisis, respectively. We expect that Singapores economy would be influenced by those crises from aboard and probably slowed down an economic growth along our sample period. There are some economic reports that could reflect the four recession periods in the graph. In the first period, 2009-2010, we found that it was corresponding to the hamburger crisis in the USA. The second period in 2011 was corresponding to the beginning of European (EU) debt crisis. In the third period, between 2013 and 2014, the government reported that Singapore's unemployment rate was around 1.9% and the country's economy had a lowered growth rate, when compared with the year 2010. Finally, the last period was corresponding to the announcement of the tightened policy and constrained exports of EU that contracted the export of Singapore. Overall, Singapores economy stays in low growth economy more than in high growth economy.

The MS-VECM of Japan provides regime 1 probabilities in Figure 4. Similar to Singapore, from the estimated results of Japan, we interpret regime 1 as low growth economy and regime 2 as high growth economy. In Figure 4, we can see the low growth regime exhibit in the middle of 2012. In those period, Japan's economy contracted since the first quarter of 2012, due to the slowing global growth and tensions with China. Moreover, the high pressure of deflation in Japans economy

Table 1: Bayes factor unit root test

Variable	Bayes factor	Integrated order
SET	0.9969	I(2)
THB	0.7862	I(2)
THI	0.9978	I(2)
THBY	0.9979	I(2)
KLSE	0.9997	I(1)
MYR	0.1926	I(1)
MYI	0.9993	I(1)
MYBY	0.9972	I(1)
STI	0.9979	I(1)
SGD	0.9999	I(1)
SGI	0.9976	I(2)
SGY	0.9945	I(2)
Nikkei	0.9993	I(1)
JPY	0.9934	I(2)
JPI	0.9999	I(2)
JPBY	0.999	I(1)
KOSPI	0.9978	I(1)
KWR	0.2799	I(2)
KI	0.9986	I(1)
KBY	0.9583	I(1)
SSE	0.5438	I(1)
CHY	0.9343	I(2)
CHI	0.9996	I(1)
CHBY	0.9994	I(1)

Source: Calculation Note: SET, KLSE, STI, Nikkei, KOSPI, and SSE denote as a stock market of Thailand, Malaysia, Singapore, Korea, and China, respectively. THB, MYR, JPY, KWR, CHY denote as currency of Thailand, Malaysia, Singapore, Korea, and China, respectively. THI, MYI, SGI, JPI, KI, and CHI denote as interest rate of Thailand, Malaysia, Singapore, Korea, and China, respectively. THBY, MYBY, SGY, JPBY, KBY, and CHBY denote as interest rate of Thailand, Malaysia, Singapore, Korea, and China, respectively.

We select the rank of the long-run relationship using BIC which was obtained from VECM with a conjugate prior. In this study, we specified a tightness parameter, a decay parameter, and a parameter for the lags of the variables as 0.10, 0.10, and 0.50, respectively. Based on the results of co-integration selection shown in Table 3, the result show that models of Thailand, Malaysia, Japan, and Korea present the lowest value of BIC at one co-integrating vector, while Singapore and China has two and zero number of cointregration, respectively. Therefore, the study chose r = 1 for Thailand, Malaysia, Japan, and Korea, r=2 for Singapore, and r=0 for China (MS-VAR).

Table 2: VECM Lag length criteria

Country	Lag	BIC
Thailand	1	4.668841*
	2	4.868588
	3	5.09762
	4	5.342994
Malaysia	1	-1.674580*
	2	-1.44877
	3	-1.23569
	4	-1.00201
Singapore	1	1.165207*
	2	1.315242
	3	1.459583
	4	1.576193
Japan	1	14.02791*
	2	14.08883
	3	14.21729
	4	14.35055
Korea	1	12.88807*
	2	13.1272
	3	13.33543
	4	13.50933
China	1	1.954617*
	2	2.147083
	3	2.334305
	4	2.51713

Source: Calculation

Table 3: Co-integration rank selection

BIC	r=0	r=1	r=2	r=3
Thailand	-20.3081	-20.3634	-20.3571	-20.3337
Malaysia	-23.5417	-23.5848	-23.5785	-23.5663
Singapore	-12.9893	-13.0257	-13.3597	-12.8657
Japan	-11.6093	-11.6653	-11.6005	-11.5077
Korea	-20.5098	-20.5454	-20.5446	-20.5218
China	-23.9792	-23.922	-23.8084	-23.6927

Source: Calculation Note : r = Cointegration rank

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China	-23.9792	-23.922	-23.8084	-23.6927

Source: Calculation Note : r = Cointegration rank

MYBY show a significant adjustment in the short-run deviation. However, the values of the ECT(1) of these equations are all positive, meaning they diverge from the long run equilibrium. For regime 2, we can see that the coefficients of KLSE and MYI equations demonstrate that the lagged MYI and MYBY seem to significantly influence KLSE and MYI, respectively. Consider the ECT(1) of this regime, the similar result is obtained except for the ECT(1) of MYBY equation. The error correction term of MYBY is statistically significant negative and lies between 0 and -1, meaning only Malaysian bond yield is co-integrated with Kuala Lumpur Stock Exchange, Malaysia ringgit and interest rate, respectively.

Table 6: Estimated MS(2)-VECM(1) : Singapore

	STI	SGD	SGI	SGBY
	Regime-dependent intercepts			
R1	$0.984(0.26)^a$	$-1.386(0.85)$	$-40.196(12.15)^a$	$-9.906(2.24)^a$
R2	0.267(0.21)	$3.116(0.82)^a$	16.740(11.40)	$-1.006(1.64)$
	Regime-dependent Autoregressive Lag 1			
	Regime 1			
STI	0.228(0.12)	0.066(0.18)	0.700(3.78)	-1.601(0.86)
SGD	-0.066(0.23)	0.447(0.36)	-0.627(7.41)	-1.192(1.64)
SGI	0.002(0.003)	0.005(0.005)	0.106(0.10)	-0.003(0.02)
SGBY	-0.024(0.03)	-0.002(0.05)	-0.073(1.19)	0.313(0.26)
ECT(1)	0.002(0.002)	-0.003(0.002)	-0.027(0.04)	-0.001(0.01)
ECT(2)	$0.087(0.003)^a$	0.016(0.01)	$0.419(0.12)^a$	$0.124(0.02)^a$
	Regime 2			
STI	-0.047(0.09)	-0.046(0.13)	-1.873(2.73)	-0.547(0.34)
SGD	-0.325(0.26)	0.767(0.43)	7.029(8.75)	0.226(0.84)
SGI	0.004(0.003)	0.002(0.005)	0.042(0.09)	$0.019(0.010)^a$
SGBY	-0.044(0.04)	0.032(0.066)	1.512(1.36)	$0.464(0.127)^a$
ECT(1)	0.001(0.001)	-0.002(0.002)	0.040(0.04)	$-0.009(0.004)^a$
ECT(2)	$0.095(0.002)^a$	$-0.036(0.01)^a$	-0.183(0.11)	0.013(0.020)
	p1	p2	Duration	Observations
R1	0.968	0.023	43.478	215
R2	0.032	0.977	31.25	146

Source: Calculation () is standard deviation and a is Bayesian statistic significant R1 and R2 are regime 1 and regime 2, respectively

Consider the matrix of transition probability parameters, which are also presented in Table 5. The result shows that regime 1 and regime 2 are persistent since the probabilities of switching between these two regimes are around 1.3-2.1% while remaining in their own regime has approximately 98% probability. Whereas the high growth regime has duration of approximately 76.923 weeks, the low growth regime has duration of 47.619 weeks. This means that the Malaysian economy stays mostly in high growth state rather than in low growth situation.

Table 7: Estimated MS(2)-VECM(1) : Japan

	Nikkei	JPY	JPI	JPBY
Regime-dependent intercepts				
R1	9.217(0.01)[a]	4.334(0.01)[a]	−2.300(0.001)[a]	0.027(0.041)
R2	9.623(0.04)	4.605(0.02)[a]	−2.100(0.001)[a]	0.149(0.081)
Regime-dependent Autoregressive Lag 1				
Regime 1				
Nikkei	0.367(0.22)	0.396(0.19)[a]	0.001(0.001)	0.285(0.616)
JPY	0.036(0.41)	-0.490(0.36)	-0.001(0.001)	-1.240(1.170)
JPI	0.001(0.00)	0.001(0.001)	0.001(0.001)	0.001(0.001)
JPBY	0.086(0.14)	0.101(0.13)	0.001(0.001)	0.911(0.412)[a]
ECT(1)	0.019(0.001)[a]	−0.037(0.00)[a]	-0.001(0.001)	-0.025(0.014)
Regime 2				
Nikkei	0.282(0.67)	0.161(0.37)	0.001(0.001)	-0.624(1.256)
JPY	-1.289(1.46)	-0.584(0.82)	-0.001(0.001)	6.260(2.755)[a]
JPI	0.001(0.001)	0.001(0.001)	0.001(0.001)	0.001(0.001)
JPBY	0.076(0.11)	0.048(0.06)	0.001(0.001)	-0.355(0.209)
ECT(1)	-0.020(0.01)	−0.032(0.01)[a]	-0.001(0.001)	0.371(0.032)[a]
	p1	p2	Duration	Observations
R 1	0.985	0.011	66.667	167
R 2	0.015	0.989	90.909	194

Source: Calculation () is standard deviation and a is Bayesian statistic significant R1 and R2 are regime 1 and regime 2, respectively

Table 6 presents the estimated results of Singapore financial market. The values of the intercept term in regime 1 are mostly lower than regime 2 thus we can interpret regime 1 as low growth economy and regime 2 as high growth economy. Consider regime 1, for all equations, there are no significant adjustment to be observed in case of a short-run deviation from their equilibrium thus suggesting that these variables are weakly exogenous. In addition, the error correction term (ECT(2)) of STI, SGI, and SGBY show a significant adjustment in the short-run deviation; however, the values of the ECT(2) of these equations are positive, meaning they diverge from the long run equilibrium. For regime 2, we can see that the coefficients of SGBY equation demonstrate that SGI seems to significantly influence SGBY. Consider the ECT(1) of this regime, the error correction term of SGBY is statistically significant negative and lies between 0 and -1, meaning only Singapore bond yield is co-integrated with SingaporeStraits Times Index, Singapore dollar and interest rate. Consider the ECT(2) of regime 2, the error correction term of SGD is negative at statistically significant level and lies between 0 and -1, meaning only Singapore dollar is co-integrated with SingaporeStraits

Times Index, Singapore bond yield and interest rate. The results, furthermore, show that SGBY is significantly affected by its own lag in regime 2.

Consider the matrix of transition probability parameters, which are also presented in Table 6. The result shows that regime 1 and regime 2 are persistent since the probabilities of switching between these two regimes are around 2.3-3.2% while remaining in their own regime is approximately 97%, meaning that the two regimes are persistent. While the high growth regime has duration of approximately 31.25 weeks, the low growth regime has duration of 43.478 weeks. This means that Singapore economy stays in low growth economy longer than in high growth economy.

Table 8: Estimated MS(2)-VECM(1) : Korea

	KOSPI	KRW	KI	KBY
		Regime-dependent intercepts		
R1	9.701(1.741)[a]	0.624(1.283)	6.076(6.405)	−27.490(5.770)[a]
R2	−12.609(2.658)	11.435(1.275)[a]	−5.812(4.888)	2.753(3.208)
		Regime-dependent Autoregressive Lag 1		
		Regime 1		
KOSPI	0.380(0.172)[a]	-0.224(0.184)	0.610(1.010)	-0.518(0.820)
KRW	0.326(0.272)	0.436(0.304)	-2.597(1.744)	−3.541(1.423)[a]
KI	0.011(0.115)	-0.013(0.129)	0.620(0.746)	0.533(0.608)
KBY	0.141(0.095)	−0.242(0.101)[a]	0.680(0.568)	0.577(0.461)
ECT(1)	-0.008(0.007)	0.024(0.005)[a]	-0.020(0.024)	0.109(0.022)[a]
		Regime 2		
KOSPI	-0.778(0.421)	0.539(0.200)[a]	-1.022(0.888)	-0.029(0.531)
KRW	-0.540(0.707)	0.556(0.331)	0.220(1.507)	-1.028(0.917)
KI	-0.013(0.537)	0.011(0.251)	0.420(1.147)	-0.097(0.692)
KBY	-0.492(0.405)	0.197(0.191)	-0.482(0.871)	0.707(0.520)
ECT(1)	0.076(0.010)[a]	−0.017(0.005)[a]	0.026(0.019)	0.016(0.012)
	p1	p2	Duration	Observations
R1	0.986	0.024	71.428	157
R2	0.014	0.976	41.667	204

Source: Calculation () is standard deviation and a is Bayesian statistic significant R1 and R2 are regime 1 and regime 2, respectively

Table 7 presents the estimated result of Japan. The values of the intercept term in regime 1 are mostly lower than regime 2 thus we can interpret regime 1 as low growth economy and regime 2 as high growth economy. Consider regime 1, we can see that the coefficients of JPY equations demonstrate that Nikkei seems to significantly influence the lagged values of JPY. In addition, the error correction term (ECT(1)) of JPY shows that the error correction term of JPY is statistically significant negative and lies between 0 and -1, meaning only JapaneseYen is co-integrated with Nikkei index, Japan bond yield and interest rate. Consider the error correction term (ECT(1)) of Nikkei, a significant adjustment takes place when there is a short-run deviation; however, the value of the ECT(1) of Nikkei is positive, meaning they diverge from the long run equilibrium. For regime 2, we can see that the coefficients of JPBY equation demonstrate that JPY seems to significantly influence the lagged JPBY. Similar to regime 1, there is only JPY that has a statistically significant long run relationship and short-run adjustment dynamics. However, the results show that JPY adjusts more rapidly in the low growth markets since the speed of adjustment to long-run equilibrium of ECT(1) in regime 1 is faster than in regime 2. Consider the error correction term (ECT(1)) of JPBY, there is a significant adjustment in the short-run deviation; however, the value of the ECT(1) of JPBY is positive, meaning they diverge from the long run equilibrium. The results furthermore show that JPBY is significantly affected by its own lag in regime 1.

Consider the matrix of transition probability parameters, the result shows that regime 1 and regime 2 are persistent since the probabilities of switching between these two regimes are around 1.1-1.5% while that of remaining in their own regime is approximately 99%. Since the high growth regime has duration of approximately 90.909 weeks while the low growth regime has duration of 66.667 weeks, we can say that the Japanese economy stays in high growth economy longer than in low growth economy.

Table 8 presents the estimated results of Korea. It is difficult to identify the regime for Korea case. However, we can look at the sign of the intercept term and it shows that the negative signs mostly take place in regime 2. Thus, we can interpret regime 2 as low growth economic state and regime 1 as high growth one. Consider regime 1, we can see that the coefficients of KRW and KBY equations demonstrate that the lagged KBY and KRW seem to have significant bidirectional influence (KRW and KBY, respectively) In addition, the error correction term (ECT(1)) of KRW and KBY shows a significant adjustment after the short-run deviation; however, the values of the ECT(1) of these equations are positive, meaning they diverge from the long run equilibrium. For regime 2, we can see that the coefficients of KRW equation demonstrate that KOSPI seems to significantly influence KRW. Consider the ECT(1) of this regime, the error correction term of KRW is statistically significant negative and lies between 0 and -1, meaning only Korean Won is co-integrated with South Korea KOSPI Index, Korean bond yield and interest rate. In addition, the error correction term (ECT(1)) of KOSPI indicates a significant adjustment in the short-run deviation; however, the value of the ECT(1) of KOSPI is positive, meaning they diverge from the long run equilibrium. The results furthermore show that KOSPI is significantly affected by its own lag in regime 1.

Consider the matrix of transition probability parameters in Table 8. The result shows that both regime 1 and regime 2 are persistent since the probabilities of staying in their regimes are approximately 98%. Whereas the high growth regime has duration of approximately 71.428 weeks, the low growth regime has duration of 41.667 weeks meaning that Korea economy mostly stays in high growth economy more than in low growth economy.

Table 9: Estimated MS(2)-VAR(2): CHINA

	SSE	CHY	CHI	CHBY
	Regime-dependent intercepts			
R1	0.0005(0.005)	0.0052(0.023)	0.0246(0.026)	0.0231(0.032)
R2	−0.0007(0.005)	−0.0029(0.024)	0.0377(0.026)	0.001(0.032)
	Regime-dependent Autoregressive Lag 1			
	Regime 1			
SSE	$1.002(0.004)^a$	0.0001(0.017)	-0.0132(0.019)	$-0.444(0.224)^a$
CHY	-0.012(0.017)	$0.9084(0.077)^a$	0.0024(0.086)	-1.218(0.765)
CHI	0.017(0.015)	0.085(0.071)	$1.107(0.078)^a$	0.332(0.205)
CHBY	-0.014(0.019)	0.005(0.081)	-0.081(0.090)	$0.571(0.180)^a$
	Regime 2			
SSE	$0.9943(0.004)^a$	0.025(0.017)	-0.0318(0.019)	0.007(0.023)
CHY	$0.0379(0.019)^a$	$0.8433(0.074)^a$	0.0496(0.085)	0.0004(0.106)
CHI	0.002(0.017)	0.074(0.069)	$1.036(0.078)^a$	0.091(0.098)
CHBY	-0.0217(0.019)	-0.043(0.081)	0.0412(0.091)	$0.828(0.118)^a$
	p1t	p2t	Duration	Observations
R1	0.9703	0.0298	33.67	290
R2	0.0297	0.9702	33.557	71

Source: Calculation () is standard deviation and a is Bayesian statistic significant R1 and R2 are regime 1 and regime 2, respectively

Table 9 presents the estimated results of MS(2)-VAR(1) model which is different from the other cases since there is no cointegration term in this model. Table 9 provides a result of China financial market for two regimes and found that the values of the intercept term in regime 1 are mostly higher than in regime 2 thus we can interpret regime 1 as high growth state and regime 2 as low growth state. Consider regime 1, we can see that the coefficients of CHBY equations demonstrate that SSE seem to significantly influence CHBY. For regime 2, we can see that the coefficients of SSE equation demonstrate that CHY seems to significantly influence SSE. The results furthermore show that all these four variables are significantly affected by their own lag in both regime 1 and regime 2.

Consider the matrix of transition probability parameters. The similar result is obtained from the MS(2)-VAR(1) model. the probabilities switching between these two regimes are around 2.97-2.98% while remaining in their own regime approximately 97%, this means that the two regimes are persistent. Whereas the high growth regime has duration of approximately 33.67 weeks, the low growth regime has duration of 33.557 weeks. This signifies that Chinas economy stays in low growth economy and high growth economy for virtually equal length of time.

4.5 Regime Probabilities

The estimated MS-VECM model also produces smoothed probabilities, which can be understood as the optimal inference on the regime using the full-sample information. We plot the regime probabilities for the six countries, in Figures 1-6. Each Figure shows the smooth probability, which is the probability of staying in either regime 1 or regime 2, during the period of 2009 - 2016.

Figure 1 shows that the model is consistent with the hypothesis that high growth and low growth represent different financial outcomes. Regime 1 of the model is plotted in Figure 1. We interpreted this regime as the era of the expansion. According to this result, we can observe that from the late 2011 to 2012, the Thai economy stayed in low growth regime. Apparently at that period of time, Thailand was in trouble with the flood crisis. Word Bank estimated damages to have reached THB 1,440 billion due to the closure of multiple factories. The economy continued to be in a delicate position as the flood impact had reduced investors and insurance companies confidence, which would ultimately lead to an increase in unemployment and poor economy. Tourism, another substantial revenue generator in the economy, suffered a loss of THB 3.71 billion and a fall of 3.2 million tourists according to the Tourism Ministry. We can see this flooding resulted in the low growth regime from late of 2011 to middle of 2012. In addition, Domestic political crisis which gave rise to a period of political instability in Thailand from the late 2013 onward also became another factor causing the Thai economy to slow down. Subsequently, anti-government protests took place between November 2013 and May 2014; and the Royal Thai Armed Forces staged a coup d'tat unseating the government on 22 May 2014. Some country urged tourists to cancel trips and halted non-essential visits by its governmental officers. The Ministry of Tourism and Sports said on 27 May 2014 that the arrival of "foreign tourists dropped by 20%" resulting in a low growth regime after November 2013.

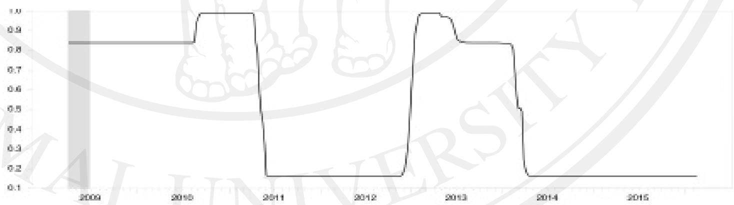

Figure 1: Regime 1 probabilities of Thailand's Market

Figure 2 presents the probabilities for the MS-VECM of Malaysia, which is a single MS chain of two regimes. Malaysia is a highly open, upper-middle income economy. In 2010, Malaysia launched the New Economic Model (NEM), which aims for the country to reach high income status by 2020. The Economic Transformation Program is an initiative by the Malaysian government to turn Malaysia into a high income economy by the year 2020, launched on September 25, 2010. There are some costs for the Economic Transformation Program, and also some risk for these programs, such as declining in oil price and the volatility in capital flows from the normalization of US monetary policy. In Figure 2, we can see all of these risks resulting in the low growth regime from early of 2010 to middle of 2014.

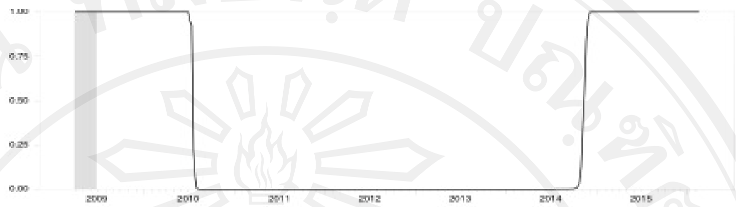

Figure 2: Regime 1 probabilities of Malaysia's Market

The regime probabilities of Singapore for regime 1 are presented in Figure 3. From the estimated results of Singapore, we interpret regime 1 as low growth economy and regime 2 as high growth economy. Singapore has become the largest foreign exchange trading center in Asia and ranks second in interest rate derivatives trading. Singapore is a leading global financial center in the world, particularly in Southeast Asia. Singapore is highly vulnerable to the global economic environment given its open economy. Therefore the world economic crisis can bring a huge impact on Singapores economy. As we can observe from Figure 3, the low growth regime was during 2009-2016. Over that period, there were severe crises in United States of America (USA) and Euro zone called hamburger crisis and European debt crisis, respectively. We expect that Singapores economy would be influenced by those crises from aboard and probably slowed down an economic growth along our sample period. There are some economic reports that could reflect the four recession periods in the graph. In the first period, 2009-2010, we found that it was corresponding to the hamburger crisis in the USA. The second period in 2011 was corresponding to the beginning of European (EU) debt crisis. In the third period, between 2013 and 2014, the government reported that Singapore's unemployment rate was around 1.9% and the country's economy had a lowered growth rate, when compared with the year 2010. Finally, the last period was corresponding to the announcement of the tightened policy and constrained exports of EU that contracted the export of Singapore. Overall, Singapores economy stays in low growth economy more than in high growth economy.

The MS-VECM of Japan provides regime 1 probabilities in Figure 4. Similar to Singapore, from the estimated results of Japan, we interpret regime 1 as low growth economy and regime 2 as high growth economy. In Figure 4, we can see the low growth regime exhibit in the middle of 2012. In those period, Japan's economy contracted since the first quarter of 2012, due to the slowing global growth and tensions with China. Moreover, the high pressure of deflation in Japans economy

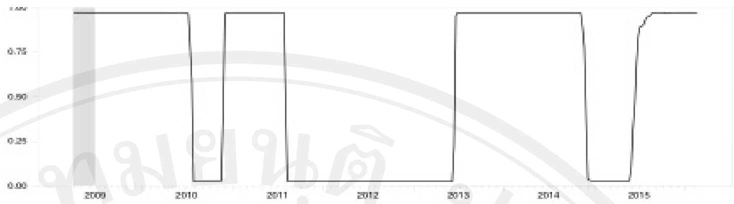

Figure 3: Regime 1 probabilities of Singapore's Market

and the high debt to GDP are also the factor that are generate the negative effect to Japans economy. Thus, these brought the world's third-largest economy into recession. As we observed in the Figure 3, the smoothed probabilities of low regime is mostly took place along our sample periods.

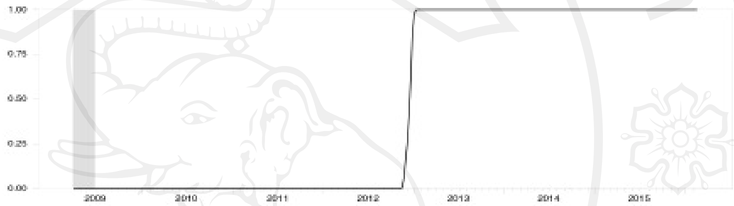

Figure 4: Regime 1 probabilities of Japan's Market

The regime probabilities of Koreas economy are illustrated in Figure 5. From the estimated results of Korea, we can interpret regime 1 as high growth economic state and regime 2 as low growth one. The economy of South Korea is the global leader of consumer electronics, Mobile Broadband and Smartphone. South Korea was one of the few developed countries that were able to avoid a recession during the global financial crisis. The International Monetary Fund complimented the resilience of the South Korean economy against various economic crises, citing low state debt, and high fiscal reserves. In Figure 5, we can see the high growth regime to present from 2009 to 2011.

Despite its economy's high growth potential and apparent structural stability, South Korea has suffered perpetual damage to its credit rating in the stock market due to the belligerence of North Korea in times of deep military crises, which has an adverse effect on the financial markets of South Korean economy. North Korea has continued to test weapons systems since 2012, including the launch of the long-range Unha-3 rocket in December 2012 and a nuclear test in February 2013. Pyongyang threatened a fourth test in November 2014, following the adoption of a resolution by the UN General Assembly condemning North Korean human rights abuses. In addition, the slowdown in the world economy during these times also the factor that pushed the high pressure on the Korean economy and resulting in the low growth regime since 2012.

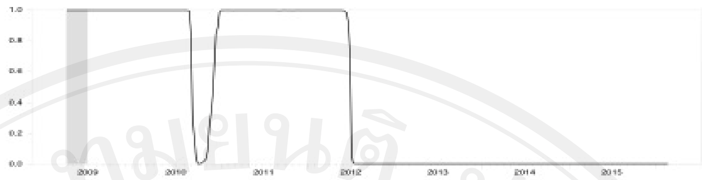

Figure 5: Regime 1 probabilities of Korea's Market

The regime probabilities of Chinese economy are illustrated in Figure 6. From the estimated results of China, we can interpret regime 1 as high growth state and regime 2 as low growth one. We can observe that the Chinas economy is likely to stay in high growth regime during 2009-2011. We found that the State Council unveiled a CNY 4.0 trillion (USD 585 billion) stimulus package in an attempt to shield the country from the worst effects of the financial crisis during that time. Apparently, China exited the financial crisis in good shape, with low inflation and a sound fiscal position. According to the International Monetary Fund, the Chinese economy grew more than 9% per year between 2009-2011. However, the global downturn and the subsequent slowdown in demand did severely affect the Chinese economy. In Figure 5, we can see the low growth regime taking place during 2011. The fifth generation came to power in 2012, when President Xi Jinping and Premier Li Keqiang took the reins of the country. The new Xi-Li administration unveiled an ambitious reform agenda in an attempt to change the countrys economic fundamentals and ensure a sustainable growth model. In Figure 5, we can see the high growth regime occurring from 2012 to the middle of 2015. However, we observe that the Chinese economy tended to switch to low growth regime after the mid-2015. This corresponds to the speech of Premier Li Keqiang delivered at the opening of the National Peoples parliament in China. He mentioned that the government had cut its growth target for that year to a range of 6.5% to 7%, down from 7%. China's financial system had a high debt levels at both banks and local authorities and the concern over Yuan devaluation in the previous year has caused the high negative pressure on Chinese economy until present day.

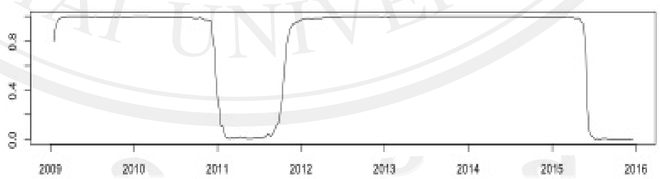

Figure 6: Regime 1 probabilities of China's Market

5 Conclusion

In this paper, we analyze the relationship between the stock index, exchange rate (against US dollar), government bond yield and interest rate of six Asian countries in the Markov-Switching VECM framework. The study conducted a Bayesian estimation technique to estimate the mean of parameters of the model. Based on the results of co-integration test, the models of Thailand, Malaysia, Japan, and Korea have one co-integrating vector, while Singapore has two and China has zero co-integrating. The results of this study show that in Malaysias low growth regime, its interest rate and government bond yield seem to significantly influence its stock market and interest rate, respectively; in Singapores high growth regime, its interest rate seems to significantly influence its government bond yield; in Japans low growth regime, Nikkei seems to significantly influence its exchange rate movement, and in Japans high growth economy regime, its exchange rate movement seems to significantly influence its government bond yield; in Koreas high growth economy regime, its government bond yield and its exchange rate movement seem to significantly influence mutually, and in Koreas low growth economic regime, its stock market KOSPI seems to significantly influence its exchange rate movement; in Chinas high growth economic regime, its stock market SSE seems to significantly influence it government bond yield, and in Chinas low growth economy regime, its exchange rate movement seems to significantly influence its stock market SSE. We also find evidence that the smooth probability, which is the probability of staying in either regime 1 or regime 2, is different in each country. This can be attributed to global capital inflows and outflows among other possible sources. Investors, fund and portfolio managers, and policy-makers should thus give heed to these regime-specific interactions when they make capital budgeting decisions and implement regulation policies.

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APPENDIX B

The Multi-Asset Portfolio Returns: A Markov Switching Copula-Based Approach

Kongliang Zhu, Woraphon Yamaka, and Songsak Sriboonchitta

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Multi-Asset Portfolio Returns: A Markov Switching Copula-Based Approach

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Abstract : The motivation for undertaking this paper stems from doubt that whether investors should keep the same strategy on the portfolio over periods of market regime shift. This paper investigates portfolio risk structure for multi-asset allocation issue using a Markov Switching copula-based approach. With this method we focus on returns in the different regime to improve the performance of portfolios. We conduct a Markov Switching with high dimension copula in order to measure a dependency of the variables, thus the model is flexible and can capture the economic behaviour change over time. The conditional Value at Risk is taken into account in the economic change and we employ Bayesian estimation method to estimate parameters of the model.

Keywords : GARCH; Markov Switching Multivariate Copula; Value-at-Risk; Expected Shortfall.
2010 Mathematics Subject Classification : 47H09; 47H10.

1 Introduction

The Chinese stock market crash has occurred since June 2015. Notably, not only was Shanghai main share index down 8.49 percent of its value on 24 August, the markets in Japan, Europe and America also suffered the meltdown. Furthermore, the Bloomberg Commodity Index has hit a low for more than 15 years. There appears to be some correlation between stock markets and commodity futures. Should investors include commodities in their portfolios to reduce risk or

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increase returns? There exists a large body of literature documenting this issue. Daskalaki and Skiadopoulos [1] , found that commodities offer in-sample diversification benefits only in the case where higher order moments are taken into account and these benefits are not preserved in the out-of-sample framework. Bessler and Wolff [2] investigated individual commodities and commodity groups separately as well as alternative commodity indices. They found that aggregate commodity indices, industrial and precious metals as well as energy improved the performance of a stock-bond portfolio for most asset allocation strategies but hardly traced positive portfolio effects for agricultural and livestock commodities.

So far, many studies have worked on stock and commodity portfolio returns using conventional model, such as minimum-variance portfolio optimization strategy and sample-based mean-variance optimization model. The dependence between financial asset returns is explained by those conventional models, which only can explain dependence between random variables in the linear regression. In an investment environment, there are no outliers. An incorrect model for portfolio optimization can lead to significant loss of investment. Embrechts, Lindskog and McNeil [3] noticed that linear correlation can often be quite misleading and should not be taken as the canonical dependence measure. In order to capture heavy tail information regarding the financial market, we use the copula-based GARCH model to get value at risk (VaR) and Expected ShortfallES). The copula-based GARCH model can be used to analyze asymmetric or tail dependence structure (see Patton [4] and Wu, Chung and Chang [5]). There are already several papers that show its advantages. For example, Autchariyapanitkul, Chanaim and Sriboonchitta [6] and Ayusuk and Sriboonchitta [7] investigated multivariate t-copula and Vine copula based on GARCH model to explain portfolio risk structure for high-dimensional asset allocation issue. But most still worked on strong assumption of no economic change. We need to relax this assumption since many papers presented the different structure of dependency for a long time. So the dependency may be represented as two regimes, i.e., high dependence regime and low dependence regime[8]. Thus we need Markov Switching technique. Markov Switching models have become popular for modeling non-linearities and regime shifts. Why is it interesting to focus on a dynamic asset allocation context? Because high and low regime can affect asset pricing and focusing on the different regime can remove some short-term impacts in market price dynamics and distortion of performance of portfolios. Ntantamis and Zhou [9] investigated the relation between different market states (bull and bear markets) to examine whether being in a different market phases for a given commodity can provide information about whether the corresponding commodity stocks or stock market indices are in a comparatively market states. Moreover, most investigators used MLE as an estimator. In this paper we employ a Bayesian estimation since the likelihood function is difficult to estimate in the discrete margins case [10]. Moreover, if estimation of the copula parameters is undertaken jointly with the parameters of the marginal models, the maximum likelihood estimator is difficult to reach the global maximum and is not easy to be converged.

This study contributes to the literature in several aspects. First, the high dimensional copula is extended to Markov Switching and conduct a Markov Switching with high dimensional copula in order to measure a dependency of the variables, thus the model is flexible and can capture the economic behaviour change over time. Second, the conditional Value-at-Risk is taken into account in the economic change, thus it will be the more accurate risk measure than the conventional method, which is measured under the one dimension.

Our empirical results confirm that rice futures found useful in investors portfolios. Furthermore, we consider the stock and commodity returns in high dependence regime and low dependence regime. We found that rubber futures add more value than rice and oil futures in stock and commodity portfolios.

The remainder of this study is organized as follows. In section 2 we present the multivariate copula and Markov Switching model. Section 3 describes our dataset of commodity futures and stock indices. In section 4 we discuss our empirical results. Section 5 concludes.

2 Methodology

2.1 Basic Concepts of Copula

Copula is a multivariate probability distribution that is used to describe the dependence between random variables. Sklar's Theorem [11] states that any multivariate joint distribution can be written in terms of univariate marginal distribution functions and a copula which describes the dependence structure between the variables. Consider the multivariate case with n random variables, given n variables $x_1, ..., x_n$ with marginal distribution $F_1(x_1), ..., F_n(x_n)$, Sklars theorem [11] introduced a linkage between distributions of $x_1, ..., x_n$ and bind their marginals using copula function. That is $H(x_1, ..., x_n) = C(F_1(x_1), ..., F_n(x_n)) = C(u_1, ..., u_n)$ where $u_1, ..., u_n$ are uniform in the [0,1] interval. If marginals $F_1(x_1), ..., F_n(x_n)$ are continuous distribution functions, then there is a unique copula function C but if$F_1(x_1), ..., F_n(x_n)$ are discrete then C is not unique. For multivariate case, the copula density c is obtained by

$$c(F_1(x_1), ..., F_n(x_n)) = \frac{h\left({F_1}^{(-1)}(u_1), ..., {F_n}^{(-1)}(u_n)\right)}{\prod_{i=1}^{n} f_i\left({F_i}^{(-1)}(u_i)\right)} \tag{2.1}$$

where

h= the density function associated to H

f_i=the density function of each marginal distribution

c= the copula density.

There are two famous classes of copula functions, namely Elliptical and Archimedean. However, this study will focus on the Elliptical class. Elliptical copula function is a variance-covariance structure similar to the multivariate normal family, but is essentially richer because its marginal tails are allowed to decrease to zero exponentially, according to power, or at many other rates and also has symmetrical tail dependence. The dependence structure, related to this function, is the Pearsons correlation which has the value of its parameter in the [-1,1] interval. The copula functions in Elliptical class are the Gaussian and the Student-t copulas.

2.1.1 Gaussian Copula

The Gaussian, or Normal copula is a linear correlation with symmetric function because the upper and the lower tail dependences are equal, and so it has no tail dependence in this function. In the multivariate case, let $\Phi()$ be standard normal cumulative distribution, thus Gaussian copula density can be written as

$$f_{(n)}\left(R\right) = \frac{1}{\left|R^{1/2}\right|} exp\left\{\frac{-1}{2}\gamma\left(R^{-1} - I\right)\gamma'\right\}\left(\prod_{i=1}^{n} exp\left\{\frac{-1}{2}\gamma_i^2\right\}\right)^{-1} \tag{2.2}$$

2.1.2 Student-t Copula

The Student-t copula has a linear correlation coefficient and has symmetrical tail dependence. However, it can capture some tail dependence. Thus the multivariate Student-t copula density can be written as

$$f_{(t)}\left(X\right) = \frac{\frac{\Gamma[(v+n)/2]\left|R^{1/2}\right|}{\sqrt{v^n\pi^n\Gamma(v/2)}}}{\left\{1 + \frac{R^{-1}(x-\mu)'(x-\mu)}{v}\right\}^{\frac{v+n}{2}}} \prod_{i=1}^{n}\left\{1 + \frac{\left(x-\mu\right)'\left(x-\mu\right)}{v}\right\}^{\frac{v+n}{2}} \tag{2.3}$$

Where, v is degree of freedom parameter and Γ is gamma function.

2.2 ARMA(p,q) GARCH Models for Univariate Distributions

To model the marginal distribution of each random variable, we employ a univariate ARMA(p,q)-GARCH(m,n) specification that can be described as

$$y_t = \phi_0 + \sum_{i=1}^{p}\phi_i y_{t-i} + \sum_{j=1}^{q}\theta_j\varepsilon_{t-j} + \varepsilon_t \tag{2.4}$$

$$\varepsilon_t = h_t\eta_t \tag{2.5}$$

$$h_t^2 = \alpha_0 + \sum_{i=1}^{m}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{n}\beta_j h_{t-j}^2 \tag{2.6}$$

where (2.4) and (2.6) are the conditional mean and variance equation, respectively. ε_t is the residual term which consists of the standard variance, h_t, and the standardized residual, η_t, which is proposed to have a Gaussian distribution, a Student-t distribution, a generalized error distribution (GED), a skewed GED and a skewed-t distribution. The best-fit ARMA(p,q)-GARCH(m,n) will give the standardized residuals to be transformed into a uniform distribution in (0,1).

2.3 Value at Risk with Copula

Value at Risk (VaR) and conditioned Value at Risk or Expected Shortfall (ES) has been widely used to measure risk since the 1990s. The VaR of portfolio can be written as

$$VaR_{\alpha} = inf\left\{l \in R : P\left(L > l\right) \leq 1 - \alpha\right\} \tag{2.7}$$

where, α is a confidence level with a value [0,1] which presents the probability of Loss L to exceed l but not larger than $(1 - \alpha)$. While an alternative method, ES, is the extension of the VaR approach to remedy two conceptual problems of VaR ([12]). Firstly, VaR measures only percentiles of profit-loss distribution with difficulty to control for non-normal distribution. Secondly, VaR is not sub-additive. ES can be written as

$$ES_{\alpha} = E\left(L|L > VaR_{\alpha}\right). \tag{2.8}$$

To find the optimal portfolios, Rockafellar and Uryasev [13] introduced the portfolio optimization by calculating VaR and extend VaR to optimized ES. The approach focused on the minimizing of ES to obtain the optimal weight of a large number of instruments. In other words, we can write the problem as in the following The objective function is to

$$\text{Minimize} \quad ES_{\alpha} = E\left(L|L > inf\left\{l \in R : P\left(L > l\right) \leq 1 - \alpha\right\}\right) \tag{2.9}$$

Subject to

$$R_p = \sum_{i=1}^{n} \left(w_i \bullet r_i\right)$$
$$\sum_{i=1}^{n} \left(w_i\right) = 1$$
$$0 \leq w_i \leq 1, \quad i = 1, 2, ..., n$$

where R_p is an expected return of the portfolios, w_i is a vector of weight portfolio, and r_i is the return of each instrument.

2.4 Regime Switching Copula

In general, financial time series exhibit different behaviour and lead to different dependencies over time; for this reason, the dependence structure of the variables may be determined by a hidden Markov chain with two regimes or more. Therefore, it is reasonable to extend the copula to Markov Switching [14] and obtain Markov Switching copula. Thus the model becomes more flexible since it allows the dependence copula parameter $(R_{c,t}^{S_t})$ to be governed by an unobserved variable at time t (S_t). Let S_t be the state variable, which is assumed to have two states (k=2), namely high dependence regime and low dependence regime. The joint distribution of $x_1, ..., x_n$ conditional on S_t, is defined as

$$\left(x_{1,t}, ..., x_{n,t}|S_t = i\right) \sim C_t^{S_t}\left(u_{1t}, ..., u_{nT}|\theta_{c,t}^{S_t}, R_{c,t}^{S_t}\right), \quad i = 1, 2. \tag{2.10}$$

The unobservable regime (S_t) is governed by the first order Markov chain, which is characterized by the following transition probabilities (P):

$$P_{ij} = Pr\left(S_{t+1} = j|S_t = i\right) \quad \text{and} \quad \sum_{j=1}^{k} P_{ij} = 1 \quad \text{for} \quad i = 1, 2 \tag{2.11}$$

where P_{ij} is the probability of switching from regime i to regime j, and these transition probabilities can be formed in a transition matrix P, as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} = 1 - P_{11} \\ p_{21} = 1 - P_{22} & p_{22} \end{bmatrix} \tag{2.12}$$

The Gaussian copula density function from Eq.(2.2) can be rewritten in the likelihood function form as

$$L_{(n)}\left(u_1, .., u_n|\theta_1, .., \theta_n, R\right) = \frac{1}{\left|R^{1/2}\right|}\prod_{i=1}^{T}\left(exp\left\{\frac{-1}{2}\gamma\left(R^{-1} - I\right)\gamma'\right\}\prod_{j=1}^{n} f_i\left(x_{ij}; \theta_j\right)\right) \tag{2.13}$$

where $f_i\left(x_{ij}; \theta_j\right)$ is the density function obtained from the ARMA-GARCH step and we assume this function to be fix. Similarly, the Student-t copula density function from Eq.(2.3) can be rewritten in the likelihood function form as

$$L_{(t)}\left(u_1, .., u_n|\theta_1, .., \theta_n, R, v\right) = \prod_{i=1}^{T}\left(\frac{\frac{\Gamma[(v+n)/2]\left|R^{1/2}\right|}{\sqrt{v^n\pi^n\Gamma(v/2)}}}{\left\{1 + \frac{R^{-1}(x-\mu)'(x-\mu)}{v}\right\}^{\frac{v+n}{2}}}\prod_{j=1}^{n} f_i\left(x_{ij}; \theta_j, v\right)\right). \tag{2.14}$$

In this study, the method of Kim's filtering algorithm [15] is conducted to filter the state variable S_t and let $L_{(t)}$ and $L_{(n)}$ be $L(T)$ and $L(N)$ respectively, thus we can write the two regime Markov Switching copula log likelihood as

$$\log L_N\left(\theta_{N,S_t}, R_{N,S_t}, P\right) = \sum_{S_t=1}^{2} \log L(N) Pr\left[S_t|\theta_{N,S_{t-1}}, R_{N,S_{t-1}}, P\right] \quad \text{Gaussian} \tag{2.15}$$

$$\log L_T\left(\theta_{T,S_t}, R_{T,S_t}, P\right) = \sum_{S_t=1}^{2} \log L(T) Pr\left[S_t|\theta_{T,S_{t-1}}, R_{T,S_{t-1}}, P\right] \qquad \text{Student} - t. \tag{2.16}$$

To evaluate the log-likelihood in Eq. (2.15) and Eq. (2.16), we need to calculate the weight $Pr\left[S_t|\theta_{n,S_{t-1}}, R_{n,S_{t-1}}\right]$ and $Pr\left[S_t|\theta_{t,S_{t-1}}, R_{t,S_{t-1}}, v_{S_{t-1}}\right]$ for S_t=1,2 because the estimation of the Markov Switching copula needs inferences on the probabilities of S_t,

$$Pr\left(S_t = 1|w_t\right) = \frac{logL\left(\theta_{S_t=1}, R_{S_t=1}, P\right) Pr\left(S_t = 1|w_{t-1}\right)}{\sum_{S_t=1}^{2} logL(\theta_{S_t}, R_{S_t}, P) Pr\left[S_t = S_t|w_{t-1}\right]} \tag{2.17}$$

$$Pr\left(S_t = 2|w_t\right) = 1 - Pr\left(S_t = 1|w_t\right) \tag{2.18}$$

where w is all the information set of the model.

2.5 Prior Distributions

In the Bayesian approach we need to specify the prior distribution for all parameters sets in the model consisting of transition matrix parameters and dependence parameters to obtain the posterior distribution. We define the distribution of our parameters following Smith [10] and Smith, Gan and Kohn [16]. The uniform prior $Unif(-1, 1)$ is given for the dependence parameters $R_{c,t}^{S_t}$ while the Dirichlet distribution with the hyper-parameters (α_1, α_2) is assumed to be our prior since the transition matrix parameter is the probability [0,1] and suitable for make the persistence of the probability of staying in their own regime. For v , we use a uniform prior on [2, 50]. Since the marginal models are application specific, so are the priors on the marginal parameters, we adopt non-informative priors in our empirical work. Thus, the log posterior distribution of Markov Switching copula becomes

$$Pr\left(\Theta, P|u_1, ..., u_n\right) = \sum_{k=1}^{2} \log L(N) Pr\left[S_t|\Theta_{t-1}\right] + \log\left(Pr\left(\Theta, S\left(t\right)\right)\right) \quad \text{Gaussian} \tag{2.19}$$

$$Pr\left(\Psi, P|u_1, ..., u_n\right) = \sum_{k=1}^{2} \log L(T) Pr\left[S_t|\Psi_{t-1}\right] + \log\left(Pr\left(\Psi, S\left(t\right)\right)\right) \quad \text{Student} - t \tag{2.20}$$

where $\log\left(Pr\left(\Theta, S\left(t\right)\right)\right)$ and $\log\left(Pr\left(\Psi, S\left(t\right)\right)\right)$ are the log prior distribution for Gaussian and Student-t copulas respectively.

To sample all of these parameters based conditional posterior distribution, we employ the Markov chain Monte Carlo, Metropolis Hasting algorithm. To draws these parameters, first of all, the target distribution function is set as a truncated normal [-1,1] for dependence parameters and truncated normal [0,1] for transition matrix. We run the Metropolis Hasting sampler for 10,000 iterations where the first 2,000 iterations serve as a burn-in period. For Metropolis Hasting algorithm, we apply it to find all parameter sets together where the acceptance ratio is

$$r = \frac{Pr\left(\theta^{*}|u_1,...,u_n\right)Pr\left(\theta_{i-1}|\theta^{*}\right)}{Pr\left(\theta_{i-1}|u_1,...,u_n\right)Pr\left(\theta^{*}|\theta_{i-1}\right)} \tag{2.21}$$

where θ is $\Theta = \left\{\theta_{n,S_{t-1}}, R_{n,S_{t-1}}, P\right\}$ or $\Psi = \left\{\theta_{t,S_{t-1}}, R_{t,S_{t-1}}, v_{S_{t-1}}, P\right\}$.
If $r \geq 1 \Rightarrow \theta = \theta^{*}$.
if $r < 1 \Rightarrow$ draw Uniform [0,1].
if $U \leq 1 \Rightarrow \theta_i = \theta^{*}$ else $\theta_i = \theta_{i-1}$.

3 Dataset and Estimation

In this study, we use the data set comprising the Stock Exchange of Thailand index (SET), Hang Seng Index (HSI), Brent oil spot price (OIL), rubber commodity price (Rubber), and rice commodity price (RICE). For the period July, 2008 to April, 2015 , totally 1766 observations. The data are collected from Thomson and Reuter DataStream, Chiang Mai, University. All the series have been transformed into the difference of the logarithm. We would like to focus on Thailand market and to mix stock market and commodity market. We choose oil, rubber and rice as representation of commodity market. There are several reasons. First, the rice price has a significant effect on quantity of rubber production in Thailand with an estimated elasticity of -2.6 (see [17]). Second, Li and Yang [18] using A Copula-based GARCH model approach found that the rubber price is affected by the price of oil. Thailand has become the largest rubber exporter in the world. Thai rubber rank second in value of agricultural export after rice.

Table 1 provides the summary statistics for each rate of returns. As previously found in other studies, these return rates demonstrate excess kurtosis and negative skewness except HSI. In addition, from the results of Jarque-Bera test, we may state that they do not exhibit Gaussian distribution.

In the estimation of copula with Markov switching, the method consists of three steps. The first step is the estimation of the ARMA-GARCH to obtain the standardized residual for each stock and transform it into uniform[0,1]; the second step involves maximizing the Markov Switching copula log-likelihood in order to get the starting value of dependence parameters. Finally, the Bayesian estimation is conducted to estimate the posterior mean of the parameter sets in the model. Note that Gaussian and Student-t copulas are two families that we employ to join the marginal distribution in this study.

Then, the obtained final mean posterior parameter of dependence between all variables will be extended to compute the VaR and the ES in two different regimes, using the following method. First, the Monte Carlo simulations are used to simulate the joint-dependent distribution uniform from the fitted Markov Switching copula model.

We simulate 10,000 replications of the portfolio returns for each regime and, then we multiply the inverse of the marginal distribution with the random variable

Table 1: Data Descriptive Statistics

	SET	HSI	OIL	RUBBER	RICE
Mean	0.0002	0.00006	-0.00021	-0.00012	0.00001
Median	0.0002	0.00002	0	0	0
Maximum	0.03409	0.05821	0.05518	0.02879	0.0266
Minimum	-0.05037	-0.05902	-0.04429	-0.03803	-0.06982
Std. Dev.	0.0063	0.00702	0.00917	0.00729	0.00427
Skewness	-0.67331	0.12004	-0.0258	-0.41097	-2.79167
Kurtosis	9.86278	13.734	7.09583	5.8445	50.19978
Jarque-Bera	3599.050	8483.831	1234.618	645.086	166224.198
Probability	0	0	0	0	0
Sum	0.35651	0.10509	-0.36391	-0.20691	0.02093
Sum Sq. Dev.	0.06995	0.08694	0.14853	0.09373	0.03223
Observations	1766	1766	1766	1766	1766

to obtain ε_{it}^{k}. To find the return of each variable ($r_{it}^{(k)}$), we perform the estimation using the following formula:

$$r_{it}^{(k)} = \widetilde{u_{it}} + \sqrt{h_{it}} \cdot \varepsilon_{it}^{(k)}$$

where $\widetilde{u_{it}}$ is the simulated mean form ARMA equation. To compute the portfolio return in each regime, we specify an equally weighted portfolio return, that is, $X_{pt} = 0.2SET_t + 0.2HSI_t + 0.2BRENT_t + 0.2Rubber_t + 0.2Rice_t$. In this computation, we compute all the risk measures at 1%, 5%, and 10% levels. Then, The study conducts two backtesting of Kupiec [19] measure the accuracy of the obtained VaR and the ES estimates (See [12]).

4 Empirical Result

4.1 ARMA-GARCH Results

We used ARMA-GARCH process to appropriately analyze the volatility and estimate the marginal. We selected the optimal lag and marginal distribution assumption for ARMA(p, q)-GARCH(1,1,) by using AIC and found that the returns on SET, HSI , OIL, RUBBER and RICE satisfied ARMA(1,1), ARMA(3,4), ARMA(5,5), ARMA(1,1), and ARMA(2,1) with GARCH(1,1) respectively. In addition, we compared various margins assumption and the lowest Akaike Information criterion (AIC) is preferred. We found that the margins of SET, HSI and OIL are GED and the margins of RUBBER and RICE are normal distributed. The parameters of each are all significant as shown in Table 2. The estimated ARCH effects equal 0.097, 0.066, 0.054, 0.076 and 0.059. These results indicate that a shock to the growth rate of return has short-run persistence in all cases.

Table 2: Estimates of ARMA-GARCH parameters for raw returns

	SET	HSI	OIL	RUBBER	RICE
C	0.000217	0.00004	0.00001	0.000017	-0.00001
	(0.000054)	(0.000042)	(0.000)	(0.00008)	(0.00002)
AR(1)	4.889	0.4057	-0.1448	0.5811	0.685
	(0.04503)	(0.1338)	(0.00001)	(0.1238)	(0.1523)
AR(2)		0.5062	-0.4316		0.08698
		(0.1585)	(0.00001)		(0.0296)
AR(3)		-0.3447	0.01003		
		(0.04722)	(0.00001)		
AR(4)			0.09061		
			(0.00001)		
AR(5)			-0.5544		
			(0.00001)		
MA(1)	-0.4821	-0.3981	0.09564	-0.4294	-0.7283
	-0.05096	-0.1321	(0.00002)	(0.1380)	(0.1519)
MA(2)		-0.5066	0.4451		
		(0.1580)	(0.00002)		
MA(3)		0.3334	-0.06321		
		(0.05054)	(0.00002)		
MA(4)		-0.00263	-0.0685		
		(0.01808)	(0.00002)		
MA(5)			0.5581		
			(0.00002)		
α_0	0	0	0	0.000002	0
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ARCH(1)	0.09768	0.06619	0.05497	0.07657	0.05904
	(0.01709)	(0.01189)	(0.0106)	(0.01038)	(0.00692)
GARCH(1)	0.894	0.9287	0.9446	0.8849	0.927
	(0.01692)	(0.01212)	(0.0102)	(0.01616)	(0.00683)
SHAPE	1.197	1.212	1.325		
	(0.05665)	(0.06707)	(0.06127)		
LogL	6784.036	6741.797	6230.19	6364.204	7274.951
normalized	3.84147	3.8176	3.527854	3.603739	4.119452
BERK-test	0.8249	0.9882	0.5459	0.9989	0.9987
ARCH-LM	0.4467	0.5332	0.1002	0.2778	0.9975

Source: Calculation

The values of the GARCH coefficient are 0.894, 0.928, 0.944, 0.884, and 0.927 that illustrate each growth rate of return has a long-run persistence of volatility. Testing for marginal distribution that satisfies the two preconditions: uniformity and serial independence is a critical step in constructing multivariate models using copula. We used the Berkowitz test to confirm the marginal has uniform distribution and ARCH-LM Test to ensure residuals are i.i.d random variables and no autocorrelation.

4.2 Model Selection

In this section, we compare two copula functions, namely Gaussian and Student-t copulas. The Deviance Information criterion (DIC) is employed to compare the performance of our purposed models. Table 3 provides an evidence that MS-copula with Student-t function presents a lower DIC than Gaussian copula. Thus, we adopt MS-copula with Student-t function to be inference in our study. Moreover, the acceptance rate is considered here about how often was a proposal rejected by the Metropolis Hastings acceptance criterion. In the general, acceptance rates between 20% and 40% are optimal since these will confirm the good mixing between the proposal function and the target distribution. In the present study the acceptance is 40.27% for marginal parameters in our Markov Switching Student-t copula model.

Table 3: My caption

	Acceptance	DIC
Gaussian	0.4456	-2423.259
Student-t	0.4027	-2761.691

Source: Calculation

4.3 Markov Switching Student-t Copula

Table 4 shows the solutions of multivariate Student-t copula parameters with regime switching. We can use these values to construct efficient portfolio and find optimal plans for best expected returns with minimum loss which will be reported in the last section. Table 4 reports the estimated parameters of the Markov Switching Student-t copula. The results show that the value of the matrix dependence parameter in regime 1 is higher than regime 2. Thus, we can interpret regime 1 to be the high dependence regime, while regime 2 is the low dependence regime. Moreover, recently, the studies of Karimalis and Nimokis [20], found an evidence that the dependence among assets during market upturns is less than that during market downturns. Thus, this confirms the high dependence regime as the market downturn regime and the low dependence regime as the market upturn regime. Next, we take into consideration the estimated dependence parameters for 2 regimes, we observe that all of pair copula parameters present a positive dependence in both regimes except for RICE-OIL pair in regime 2. The presence of a positive dependence among these commodity prices gives us some economic inference that these prices are moving in the same direction and that the scope for the diversification of these commodity prices to reduce risk is more limited. In addition, the transition probability matrix of these commodity prices are also reported in Table 4. The $Pr(S_t = 1)$ is 91.09% and $Pr(S_t =)$ is 94.5% while the probabilities of regime switching between these two regimes are less than 10%.

Table 4: Empirical copula parameters

Regime 1					
	SET	HSI	OIL	RUBBER	RICE
SET	1	0.6804	0.4722	0.3267	0.1359
		(0.0013)	(0.005)	(0.0023)	(0.0023)
HSI	0.6804	1	0.5349	0.422	0.0821
	(0.0013)		(0.0033)	(0.0023)	(0.0023)
OIL	0.4722	0.5349	1	0.309	0.0166
	(0.005)	(0.0033)		(0.0033)	(0.004)
RUBBE	0.3267	0.422	0.309	1	0.0437
	(0.0023)	(0.0023)	(0.0033)		(0.0035)
RICE	0.1359	0.0821	0.0166	0.0437	1
	(0.0023)	(0.0023)	(0.004)	(0.0035)	

Regime2					
	SET	HSI	OIL	RUBBER	RICE
SET	1	0.4519	0.0539	0.2986	0.1301
		(0.0017)	(0.0015)	(0.0014)	(0.0015)
HSI	0.4519	1	0.0566	0.3103	0.0929
	(0.0017)		(0.0018)	(0.0017)	(0.0013)
OIL	0.0539	0.0566	1	0.0501	-0.0124
	(0.0015)	(0.0018)		(0.0014)	(0.0018)
RUBBE	0.2986	0.3103	0.0501	1	0.1623
	(0.0014)	(0.0017)	(0.0014)		(0.0017)
RICE	0.1301	0.0929	-0.0124	0.1623	1
	(0.0015)	(0.0013)	(0.0018)	(0.0017)	
	Regime1	Duration	Regime2	Duration	
Regime1	0.9109	11.224	0.0891	1.0978	
Regime2	0.055	1.0583	0.945	18.1669	

Source: Calculation

The results indicate that both regimes are persistent because of the high values obtained from the probabilities. Moreover, the duration of stay is short for both the regimes, with the duration equal to 11.24 days for the high dependence regime and 18.16 days for the low dependence regime. This result, apparently, indicates that the dependence between these returns has high fluctuation.

4.4 Regime Probabilities

As, we mentioned before, regime 1 can be interpreted as high dependence regime while regime 2 is interpreted as low dependence regime.

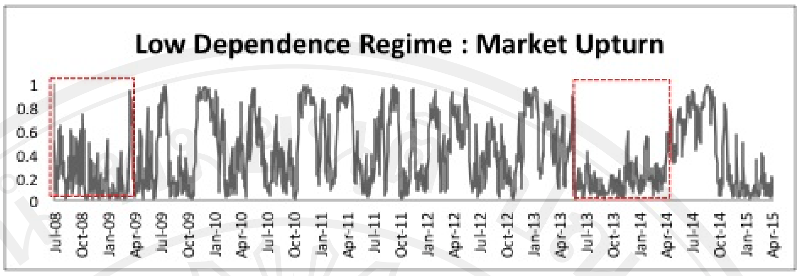

Figure 1: Filtered Probabilities of Market upturn regime.

Figure 1 plots the posterior mean regime at each time point for low dependence regime or market upturn. In this section, we analyse the evolution of the regime probabilities at each time period and find two interesting periods. First, we can observe that the 2 main sub periods (box plot line) consist of the July 2008 to April 2009 and June 2013 to April 2014 mostly take place in market downturn. These periods correspond to the US. Financial crisis in 2008 and the European Crisis in 2013-2014. We found that these two periods created a large negative effect on the world economy. The demand in commodity market shrunk and thereby lowering price of the commodities. The model seems to capture the financial behaviour well since it could detect the two great crises in our samples.

4.5 Value at Risk and Expected Shortfall Estimation

Table 5: Risk Measurement

Regime 1		
	VaR%	ES%
1%	-4.51	-5.57
5%	-2.91	-3.95
10%	-2.09	-3.21
Regime 2		
	VaR%	ES %
1%	-4.62	-5.58
5%	-3.02	-4.02
10%	-2.11	-3.28

Source: Calculation

Further estimation results on the expected VaR and ES are reported in Table 5. We calculated the expected values of 1%, 5%, and 10% VaR and ES on an equally weighted portfolio based on the Markov Switching Student-t copula.

Table 6: Result of Kupiec and Christoffersen Tests for VaR and ES

			Regime1	Regime2
Copula		α	Kupiec	
Student-t	VaR	1%	-14.9126	-14.95024
		5%	-9.2124	-4.0825
		10%	-5.6669	-1.5461
	ES	1%	-0.1081	-0.5694
		5%	-1.7473	-4.3055
		10%	-6.1858	-7.5136

Source: Calculation

Table 7: Optimal Portfolios weight

Regime 1							
Port	SET	HSI	OIL	RUBBER	RICE	Ret%	Risk%
1	0.4255	0.1608	0.1608	0.3220	0.0151	0	4.57
2	0.4659	0.1464	0.0725	0.3104	0.0046	0.01	4.58
3	0.4304	0.2246	0.044	0.3005	0	0.02	4.62
4	0.4834	0.2308	0.0060	0.2796	0	0.02	4.71
5	0.5642	0.1997	0	0.2360	0	0.03	4.84
6	0.6417	0.1748	0	0.1833	0	0.04	5
7	0.7242	0.1421	0	0.1335	0	0.04	5.18
8	0.8018	0.1171	0	0.0809	0	0.05	5.41
9	0.8603	0.1220	0	0.0175	0	0.06	5.67
10	1	0	0	0	0	0.06	5.96
Regime 2							
Port	SET	HSI	OIL	RUBBER	RICE	Ret%	Risk%
1	0.3587	0.2227	0.0812	0.3309	0.0063	0	4.6
2	0.3762	0.2185	0.0745	0.3306	0	0	4.6
3	0.4304	0.2246	0.044	0.3005	0	0.01	4.61
4	0.4834	0.2308	0.0060	0.2796	0	0.01	4.67
5	0.5642	0.1997	0	0.2360	0	0.01	4.78
6	0.6417	0.1748	0	0.1833	0	0.02	4.92
7	0.7242	0.1421	0	0.1335	0	0.02	5.12
8	0.8018	0.1171	0	0.0809	0	0.03	5.39
9	0.8603	0.1220	0	0.0175	0	0.03	5.7
10	1	0	0	0	0	0.03	6.13

Source: Calculation

For regime 1 or market downturn, the estimated VaR values are 4.51%, 2.91%, and 2.09%, respectively, while the estimated ES values are, respectively, 5.57%, 3.95%, and 3.21%. In the case of VaR, we can indicate that it might be 1%, 5%, and 10% sure that this portfolio will fall more than 4.51%, 2.91%, and 2.09%. If we take ES into account, it might be 1%, 5%, and 10% sure that this portfolio will fall more than 5.57%, 3.95%, and 3.21%. For regime 2 or market upturn, the result from VaR shows that it might be 1%, 5%, and 10% sure that this portfolios will fall more than 4.62%, 3.02%, and 2.11% while ES shows that it might be 1%, 5%, and 10% sure that this portfolio will fall more than 5.58%, 4.02%, and 3.28%. We observe that the probability of loss in regime 2 is higher than regime 1. This result confirms that the investor will face higher risk during the market upturn.

The study conducts two backtesting of Kupiec [11] measure the accuracy of the obtained VaR and the ES estimates (See [12]). The backtest at 99%, 95%, and 90% confidence levels are shown in Table 6. We can observe that our portfolio, at 1%, 5%, and 10% levels, are not statistically significant at 10% level.Thus, it is not possible to reject the null hypothesis that the expected proportion of violation is equal to the VaR confidence level (α). Therefore, the Markov Switching Student-t copula was concluded as the appropriate model to estimate the VaR and the ES in both two regimes.

Figure 2 illustrates the efficiency frontier for two regimes embracing the 10 portfolios in the Table 7. In this section, we also provide the optimal weight investment for these stock and commodities price in the market upturn and market downturn. The results can be interpreted separately for the two regimes. For example, in regime 1 or market downturn, these investors who are risk lover and want to gain high returns can allocate their investment in SET 86.04%, HSI 12.21%, and Rubber 1.75% in order to get the highest return at 0.06% and risk at5.67%. In contrast the investors who are risk averse and afraid of risk, they can invest in SET 46.60%, HSI 14.64%, OIL 7.26% and Rubber 31.04% rice 0.46% to face with the lowest risk (4.57%). Similar investors response are advised for to regime 2 or market upturn. In addition, we observed that SET index presents the best choice of investing when compare with other stock and commodity prices while rice presents the worse choice.

5 Conclusion and Future Works

This paper offers portfolio risk structure for multi-asset allocation issue using a Markov Switching copula-based approach. We intendedly deal with two different regimes to improve the performance of portfolios. We focus on Thai market and use the data set comprising the stock index of SET, HIS, and commodity price of OIL, Rubber and RICE for the period 2008:07-2015:04. There are three main findings. The first is that we found evidence that MS-copula with Student-t function present lower DIC than Gaussian copula. Thus, we adopt MS-copula with Student-t function to be inference in our study. The second finding is that the results of multivariate Student-t copula parameters with regime switching confirm the high dependence regime as the market downturn regime and the low dependence regime as the market upturn regime. This model also could capture the

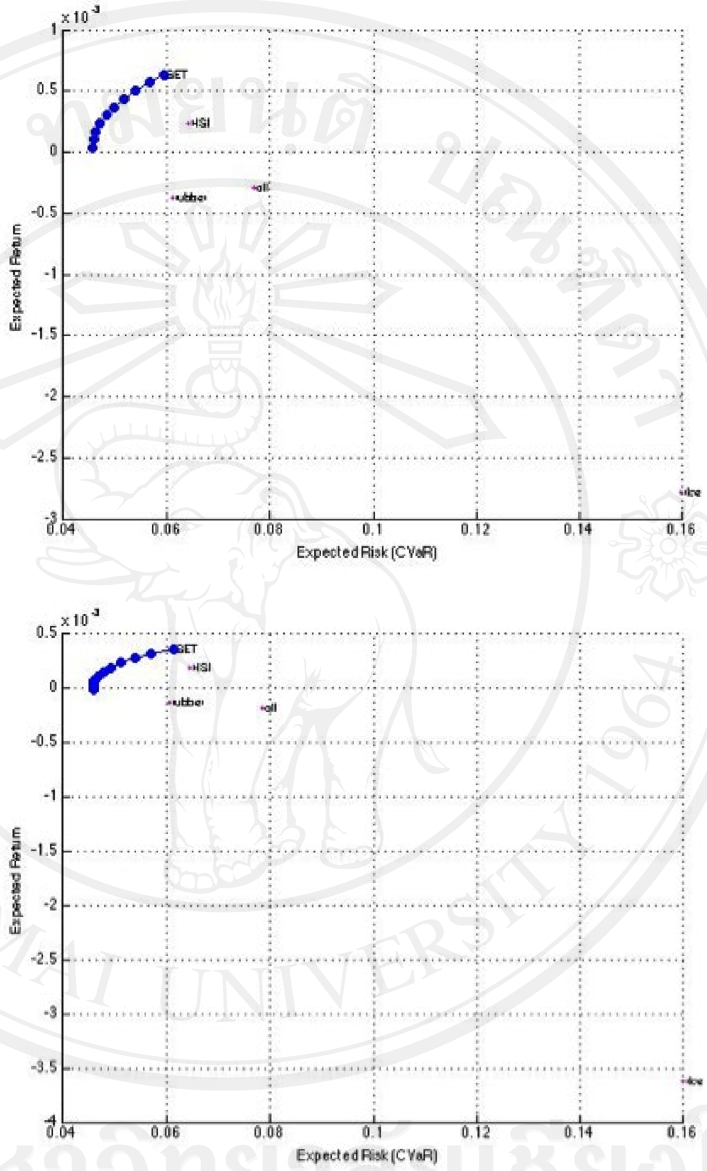

Figure 2: Efficient frontier for two regimes.

financial behavior well since it could detect the two great crises in our samples. Finally, the estimation of Expected Shortfall (ES) confirms that the investors will face higher risk with markets upturn. We also obtained the optimal weight for the portfolios which varies with the ES in the market upturn and market downturn. Further researches on this work can be pursued from different angles. Since there are two main classes of copula functions, namely Elliptical and Archimedean, our study only focuses on the Elliptical class which has symmetrical tail dependence. It would be interesting to see whether Archimedean copulas benefit from these advantages in multi-asset allocation issue using a Markov Switching approach when the data set has asymmetrical tail dependence. Additionally, in our paper, we assume that the dependence of copula parameters does not change over time. It would be interesting to extend dynamic portfolio risk for multi-asset allocation issue using a Markov Switching with time-varying copulas.

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APPENDIX C

Pairs Trading Rule
with Swiching Regression GARCH Model

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Pair Trading Rule with Switching Regression GARCH Model

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Abstract. Pairs trading strategy is a famous strategy and commonly taken by many investors. There are various approaches to define the pairs trading signal which is the important part of the strategy. This study aims to propose an alternative approach, Markov Switching Regression GARCH model, to specify the trading signal for stock pair taking into account the structural change in the pair return. We applied our proposed model to the Stock Exchange of Thailand and the result shows our pairs trading strategy is relatively more effective for financial investment management compared with the single mean return from individual stock method.

Keywords: Pairs trading · Markov switching · GARCH · SET50 Index

1 Introduction

Today, pairs trading continues to remain an important quantitative method of speculation strategy since its invention at Morgan Stanley in 1987. Pairs trading is a trading strategy which is work by identifying two stocks whose prices have high correlation. The key advantage of this strategy is that it can be used to gain profit under different market conditions, including periods when the equity market goes up, down, or oscillating between a relatively narrow range, along with low or high volatilities [16]. When the price relation is broken, short the winner and buy the loser. If the past is a good mirror of the future, the prices of two stocks will converge to a mean and the arbitrageur will profit. Like the statistical arbitrage strategy, pairs trading is a market-neutral strategy that matches a long position and a short position of a two stocks that are correlated.

There exists a wide range of different researches on pairs trading such as a distance method, co-integration approach and stochastic spread method. These are three main methods applied in pairs trading strategy. Firstly, the distance method involves calculating the sum of squared deviations between two normalized stock prices as the criteria to select pairs and form trading opportunities. It was first used in the study by Gatev et al. [14] who found average annualized excess returns over 10 % based on the daily data from 1962 to 2002 in the US market. Later, Perlin [13] extended the analysis to investigate the profitability and risk of the pairs trading strategy for Brazilian stock market. Do and Faff [5]

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positive conditional variance, ${h^2}_{t,s(t)}$. In this variance equation, the state dependent unconditional variance can be computed by $\omega_{s(t)}/(1-\alpha_{i,s(t)}-\beta_{j,s(t)})$. In addition, some distributions, such as normal, student-t, generalized error distribution (GED), skewed GED, skewed normal, and skewed student-t distributions are adopted for innovation $v_{j,s(t)}$.

The feature of the Markov switching model is the estimate parameters in both mean and variance equations can switch across different regimes or are state dependent according to the first order Markov process. This means that all parameters are governed by a state variable $s(t)$ which is assumed to evolve according to $s(t-1)$ with transition probability, p_{ij}, thus

$$p(s(t)=j|s(t-1)=i)=p_{ij}, \qquad \sum_{j=1}^{h} p_{ij}=1, \qquad for \quad i=1,...,h \tag{3}$$

Usually these probabilities can be formed as transition matrix (Q)

$$\begin{matrix} p(s_t=1|s_{t-1}=1)=p_{11} \\ p(s_t=1|s_{t-1}=2)=p_{12} \\ p(s_t=2|s_{t-1}=1)=p_{21} \\ \vdots \\ p(s_t=i|s_{t-1}=j)=p_{ij} \end{matrix} = \begin{bmatrix} p_{ij} & p_{11} & \cdots & p_{1j} \\ \vdots & p_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ p_{i1} & \cdots & \cdots & p_{ij} \end{bmatrix} \tag{4}$$

3 Maximum Likelihood Estimator for Markov Switching Regression GARCH

In this study, the Markov switching regression GARCH(1, 1) is considered since it is able to reproduce the volatility dynamics of financial data and most commonly employed in many studies. To estimate the parameter set in this model, the maximum likelihood method is used and the general form of the likelihood can be defined as

$$L(\theta_{s(t)}|y,X)=f(\theta_{s(t)}|y,X)Pr(s(t)=j) \tag{5}$$

where $f(\theta_{s(t)}|y,X)$ is the density function, $\theta_{s(t)}$ is state dependent parameter set of the model and $Pr(s(t))$ is the filtered probabilities in each regime. Note that the study adopts 6 different distributions for innovation $v_{j,s(t)}$, namely normal, student-t, generalized error distribution (GED), skewed GED, skewed normal, and skewed student-t distributions. Thus the density function $f(\theta_{s(t)}|y,X)$ in Eq. 5 can be written differently according to the distribution of the $v_{j,s(t)}$.

To estimate the filtered probability, $Pr(s(t)=j)$, the Hamiltons filter as proposed in Hamilton (1989) is employed where the formula of the filter can be written as

$$Pr(s(t)=j|\theta_t)=\frac{f(y,X_t|s(t)=j,\theta_{t-1})Pr(s(t)=j|\theta_{t-1})}{\sum_{j=1}^{h} f(y,X_t|s(t)=j,\theta_{t-1})Pr(s(t)=j|\theta_{t-1})} \tag{6}$$

where $f(y,X_t|s(t)=j,\theta_{t-1})$ is the density function of each regime (see, Perlin 2004).

extended the original analysis of Gatev et al. [14] to June 2008 and found to be profitable for a long period of time, albeit at a declining rate. Secondly, Vidyamurthy [19] suggested a co-integration approach and described how to apply this method to pairs trading. If two stocks are co-integrated with each other, they should theoretically have a narrow spread in long-term equilibrium; and investors can attempt changing their portfolio to take a profit when co-integrated assets depart from their equilibrium. Miao [16] developed high frequency and dynamic pairs trading system using the two-stage correlation and co-integration approach. Chiu and Wong [4] derived the optimal trading strategy in a closed-form solution by investigating time-consistent mean-variance portfolio strategies for co-integrated assets in a continuous-time economy. Thirdly, Elliott et al. [7] proposed the stochastic spread method which applies a Kalman filter to estimating a parametric model of the spread, in which the spread is assumed to follow the Vasicek model. Do et al. [5] extended the stochastic spread method into the stochastic residual spread method to overcome the defects of the former method. Although these methods are likely to have appropriate result, there is a question on the linear model assumption. Many studies mentioned that the financial data might have a non-linear behaviour and that they are often found to switch between different regimes. Thus, the conventional linear method for pair spread data might fail to identify potential arbitrage opportunities [6] and might cause simple pairs trading signals to be wrong [3]. In the most recent literature, we found some studies proposed to use non-linear models such as threshold model and Markov Switching model. Many previous studies have suggested that both Markov-switching model and threshold model provide a better performance to the stock returns when compare with the linear models. Bock and Mestel [3] develop a useful trading rules for pairs trading to solve the problem related to phases of imbalance when the deviation of the price stock spread may temporarily or persistently endure. They also mentioned that the regime-switching rule for pairs trading generate a positive returns and hence it can be employed as an alternative model to traditional pairs trading rules. Yang et al. [20] combined the Markov regime-switching and Vasicek models with a mean-reverting strategy, and compare the model with conventional methods using 12 months of S&P 500 index daily price data. They found that his proposed method provide the best performance in a simple portfolio and that the shorter the trading period and the higher the performance is obtained. In the case of threshold model, Chen et al. [6] proposed a three-regime threshold GARCH (generalized autoregressive conditional heteroskedasticity) model to capture asymmetries in the average return, volatility level, mean reversion in the pair spread and also proposed to use the threshold value to determine the pairs trading strategy, say used as trading entry and exit signals. They found that the model can detect the regime change in the stock price and also provide a good trading signal, leading to reap adequate profits from the Dow Jones 30 stocks. Based on these previous studies, we also expect our data to have a non linear behavior; therefore, the non-linear model should be used for specifying the trading signal. In this study, we aim to extend the Threshold GARCH of Chen et al. [6] into Markov Switching regression GARCH since the threshold models have some limitations as discussd in Kuan [11]. Kuan [11] noted that first the non-linear optimization algorithms are difficult to find the global or the optimal solution in the parameter space. Second, the threshold models are proposed to describe certain nonlinear patterns of data and hence may not be so flexible. Moreover, the threshold model allows the parameter to change the regime on only occasion and exogenous changes. However, Kuan [11] suggested that the Markov switching model is more suitable for explaining correlated data that exhibit different behavior in unusual economic condition. Thus, instead of using Threshold GARCH model, this study proposed a Markov Switching regression GARCH as an alternative tool for determining pairs trading signals. Based on our best knowledge, Markov Switching regression GARCH has not been applied to explain pairs trading strategies before and thus one of our contributions will be an alternative model for pairs trading strategy.

The remainder of this study proceeds as follows. Section 2 introduces the Markov switching regression GARCH with different error distributions. Maximum likelihood estimation is also briefly discussed in Sect. 3. Section 4 explains pairs trading strategy and identifies pairs trading signals. The preliminary empirical results are provided in Sect. 5. Conclusions are presented in Sect. 6.

2 Markov Switching Regression GARCH

Over long period, the financial series exhibit different behavior; signifying depression, recession, bull market, and bear market and resulting in a regime change. [8] proposed the Markov switching model to capture the behavior change in the data where the regime probabilities are obtained by the proposed Hamilton-filter ([8,9]). Furthermore, Bollerslev [2] who introduced GARCH (Generalized Autoregressive Conditionally Heteroskedasticity) model noted that the time series data present variable volatility over time, thus tending to show GARCH effects in the model. Therefore, the Markov switching model has been extended to GARCH in many studies, such as those by Haas et al. [10] and Marcucci [12]. Bauwens et al. [1] in order to gain more ability to capture some stylized facts of financial time series namely volatility of the data. The general form of the Markov switching regression GARCH(m, q) model can be written as

$$y_t = \varphi_{0,s(t)} + \sum_{i=1}^{k} \varphi_{1,s(t)} X_t + \varepsilon_{t,s(t)} \tag{1}$$

$$\varepsilon_{t,s(t)} = h_{t,s(t)} v_{t,s(t)}$$

$$h^2{}_{t,s(t)} = \omega_{s(t)} + \sum_{i=1}^{m} \alpha_{i,s(t)} \varepsilon^2{}_{t-i,s(t)} + \sum_{i=1}^{q} \beta_{j,s(t)} h^2{}_{t-j,s(t)} \tag{2}$$

where Eqs. 1 and 2 are the mean and variance equations, respectively, and they are allowed to switch across regime. y_t is a dependent variable and X_t is a matrix of independent variables. $h^2{}_{t,s(t)}$ is the state dependent conditional variance and state dependent $\omega_{s(t)} \geq 0$, $\alpha_{i,s(t)} \geq 0$, and $\beta_{j,s(t)} \geq 0$ to ensure the

4 Pairs Trading

To select the pair stock, Chen et al. [6] proposed to select the pair stock using the lowest value of the Minimum Squared Distance method (MSD) which is given as follows.

$$MSD = \sum_{t=1}^{n} (P_t^1 - P_t^2)^2 \tag{7}$$

where P_t^1 and P_t^2 are the normalized stock price

$$P_t^i = (P_t^i - \bar{P}_t^i)/sd_i$$

where, sd_i is the standard deviation of stock i. The selected pairs are then used to calculate the spread return, rS_t, using the Markov switching regression GARCH(1, 1) which can be written as.

$$stock_t^1 = \varphi_{0,s(t)} + \varphi_{1,s(t)} stock_t^2 + \varepsilon_{t,s(t)} \tag{8}$$

$$\varepsilon_{t,s(t)} = h_{t,s(t)} v_{t,s(t)}$$

$$h^2{}_{t,s(t)} = \omega_{s(t)} + \sum_{i=1}^{m} \alpha_{i,s(t)} \varepsilon^2{}_{t-i,s(t)} + \sum_{i=1}^{q} \beta_{j,s(t)} h^2{}_{t-j,s(t)} \tag{9}$$

The in-sample return stock will be used to compute a simple hedge ratio which can be defined by the coefficient of $stock_t^2$, $\varphi_{1,s(t)}$, and then we will apply this hedge ratio to compute the spread return. The spread return of the stock pair is constructed by

$$rS_t = stock_t^1 - \sum_{s(t)=j}^{2} \left[\varphi_{0,s(t)=j} + \varphi_{1,s(t)=j} stock_t^2\right]' \bullet [Pr(s(t) = j|\theta_t) \times Q] \tag{10}$$

where $Pr(s(t) = j|\theta_t) \times Q$ is the multiplying of filtered probability and transition matrix.

To define the trading rule, the obtained $(rS_1, ..., rS_T)$ from Eq. 10 is used to compute the mean (u) and standard deviation (sd) in order to get the threshold value where the upper and lower threshold values can be defined as $Uthres = u + sd$ and $Uthres = u - sd$, respectively. Note that when the pair spread return exceeds our upper threshold $(Uthres)$, we sell $stock_t^1$ and buy$stock_t^2$. Once the spread drops below our lower threshold $(Lthres)$, we buy $stock_t^1$ and sell $stock_t^2$.

Finally, the average return of pairs trading can be computed by

$$r_1 = \frac{1}{D}\left[-ln\frac{P_{sell}^1}{P_{buy}^1} + ln\frac{P_{sell}^2}{P_{buy}^2}\right], \quad r_2 = \frac{1}{D}\left[ln\frac{P_{sell}^1}{P_{buy}^1} - ln\frac{P_{sell}^2}{P_{buy}^2}\right]$$

where D is number of holding days.

5 Estimate Results

5.1 Data Description

The daily close prices of 30 stocks in the Stock Exchange of Thailand (SET) SET50 Index are used as an illustration. The data are obtained from Thomson Reuter data stream, Faculty of Economics, Chiang Mai University over a 12-year time periods, from January 1, 2004 to February 17, 2016, totally 3165 observations. The in-sample period is from December 18, 2015 to January 29, 2016. Before the estimation or our model, we transform all the daily data to be log-return and the Augmented Dickey Fuller test (ADF) is employed for stationary test and we found that all log-returns are stationary at the level.

Notice how we defined in-sample range. We will use the in-sample data to compute a simple hedge ratio and then we will apply this hedge ratio to find a spread return. In this study, we select 30 companies comprising Advanced Info Service(ADVANC), Banpu(BANPU), Bangkok Bank(BBL), Bangchak Petroleum(BCP), Bangkok Dusit Med.Svs(BDMS), Bumrungrad Hospital(BH), Central Plaza Hotel(CENTEL), CH KarnChangCH(CK), Charoen Pokphand Foods(CPF), Central Pattana(CPN), Delta Electronics(DELTA), Electricity Generating(EGCO), Intuch Holdings(INTUCH), IRPC(IRPC), Italian-Thai Developement(ITD), Jasmine International(JAS), Kasikorn Bank(KBANK), Krung Thai Bank(KTB), Minor InternationalMINT), PTT Exploration & PRDN(PTTEP), Robinson Department store(ROBINS), Siam Comercial Bank (SCB), Siam Cement(SCC), Siam City Cement(SCCC), Tipco Asphalt(TASCO), Thanachart Capital(TCAP), TMB BANK (TMB), TPI Polene (TPIPL), True Corporation(TRUE), THAI Union Frozen PRDS(TU). And the five best candidate stock pairs are selected for further investigation using the lowest MSD between two normalized stock prices.

Prior to illustrating the pairs trading strategy, we calculate the MSD for all possible pair stocks. The MSD is conducted here to select the first five stock pairs that provide the lowest MSD. We found the five pais trading candidates as presented in Table 1.

Table 1. Pair selection

Pair	Stock 1	Stock 2	MSD
1	SCB	KBANK	84.8114
2	CPN	CENTEL	128.0145
3	INTUCH	ADVANCE	164.37
4	CENTEL	BDSM	197.3506
5	CPN	BDSM	205.1679

We then fit a Markov switching regression model with GARCH effect to these five selected pair returns. Once the model is fitted, the upper and lower threshold values, which are calculated from the standard deviation of spread return of the stock pair, are used as trading entry and exit signals. In this study, we follow a line of literatures in the pairs trading strategy by specifying that if spread return is above or below the upper or lower threshold value, we then either short or long one stock and either long or short the other stock. Once the position is open and the spread falls back to the standard deviation line, the position is closed.

5.2 Model Selection

As we mentioned before, the study conducted six different error distributions, thus we compared these six distributions, namely Normal, Skew-normal, Student-t, Skew-T, GED and Skew-GED, in both two- and three- regime model. To select the best fit distribution for our models, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are employed to compare the performance of our proposed models. Table 2 provides evidence that student-t is the best fit distribution for all pairs. However, the heterogeneous results are obtained for regime selection. We found that CPN-CENTEL, CENTEL-BDSM AND BDSM-CPN pairs prefer 2-regime Markov switching regression GARCH$(1,1)$ while 3-regime Markov switching regression GARCH$(1,1)$ provides the best fit to SCB-KBANK and INTUCH-ADVANCE pairs.

5.3 Estimation of MS-reg-GARCH Model

Table 3 shows the estimated results of two and three regimes MS-reg-GARCH$(1,1)$ when the error term has student-t distribution for five stock pairs. The model provides two equations namely, mean equation and variance equation for two and three regimes. Consider the mean equation, we interpret $\theta_{1,s(t)=i}$ as the hedge ratio and the result shows that the hedge ratio of these pairs changes when the regime changes. This confirms our expectation that there exist a regime change and non-linear structure in the stock pair returns. The results in Table 3 also show that SCB-KBANK and INTUCH-ADVANCE follow a three-regime model while the two- regime model fits the other pairs. When we compared the value of $\theta_{1,s(t)=i}$, we observed that the value of hedge ratio of all pairs decreases when the pair moves to the higher regime, except for SCB-KBANK pair. This indicates that those pairs tend to have a weaker movement when the pair returns shift to the higher regime. The SCB-KBANK pair on the contrary will have a stronger co-movement as it shifts to the higher regime (from regime 1 to regime 3). Then, let we consider the variance equation in order to interpret the meaning of each regime. It is important to identify which of these regimes presents a high volatility and which regime presents a low volatility. To answer this question, we consider the persistence of volatility shocks for each regime. Generally, the volatility persistence can be measured by the sum $\alpha_{s(t)=i}+\beta_{s(t)=i}$ and the higher value of $\alpha_{s(t)=i}+\beta_{s(t)=i}$ corresponds to the higher unconditional variance of the process. According to Table 3, we obtain a different regime interpretation and the result from these variance equations can be interpreted in two cases. In the first case, the value of $\alpha_{s(t)=i}+\beta_{s(t)=i}$ in each regime decreases when the regime is

Table 2. Model selection

1 Regime					
AIC/BIC	SCB-KBANK	CPN-CENTEL	INTUCH-ADVANCE	CENTEL-BDSM	BDSM-CPN
Normal	−21115.38	−20549.8	−20011.5	−21016.88	−19812.83
	−21085.03	−20549.8	−19975.07	−20986.53	−19776.41
student-t	−21113.38	−20566.98	−21952.04	−21504.72	−20566.9
	−21076.95	−20530.55	−21909.54	−21468.29	−20530.47
skew-T	−21351.44	−20735.94	−21756.88	−21425.48	−20735.44
	−21308.94	−20693.44	−21714.38	−21395.13	−20692.94
Skew-normal	−20450.44	−15819.16	−16276.87	−15057.39	−14737.1
	−20413.97	−15782.73	−16234.38	−15020.96	−14694.6
GED	−19761.44	−19670.74	−20392.5	−20025.82	−19669
	−19725.02	−19640.39	−20356.07	−19995.47	−19638.64
skew GED	−19798.56	−20079.5	−18278.24	−20373	−20079.82
	−19756.07	−20037	−18235.74	−20330.5	−20037.32
2 Regime					
AIC/BIC	SCB-KBANK	CPN-CENTEL	INTUCH-ADVANCE	CENTEL-BDSM	BDSM-CPN
Normal	−21350.9	−20810.66	−21872.36	−21493.02	−20810.66
	−21278.05	−20737.81	−21799.51	−21420.17	−20737.81
student-t	−21340.32	**−21828.4**	−22746.72	**−22868.84**	**−21228.36**
	−21261.4	**−21743.41**	−22661.73	**−22783.85**	**−21143.37**
skew-T	−21359.24	−20631.48	−22522.7	−21535.06	−20370.98
	−21262.11	−20534.35	−22437.71	−21437.93	−20273.85
skew-normal	−19096.79	−20631.86	−21583.52	−21487.52	−20631.84
	−19017.86	−20546.87	−21498.53	−21390.39	−20546.85
GED	−21396.7	−20798.12	−21831.74	−21437.2	−20798.12
	−21317.78	−20713.13	−21746.75	−21352.21	−20713.13
skew GED	−15029.48	−20627.86	−21583.52	−21487.52	−20627.86
	−14944.49	−20530.73	−21498.53	−21390.39	−20530.73
3 Regime					
AIC/BIC	SCB-KBANK	CPN-CENTEL	INTUCH-ADVANCE	CENTEL-BDSM	BDSM-CPN
Normal	−22313.72	−20601.18	−21599.46	−20981.16	−20592.94
	−22186.23	−20473.69	−21471.97	−20853.67	−20465.45
student-t	**−23951.52**	−21024.4	**−23283.14**	−21748.22	−21003.84
	−23805.82	−20878.7	**−23137.44**	−21602.52	−20858.14
skew-T	−21138.62	−21720.68	−23279.78	−21813.8	−21094.72
	−20974.71	−21556.77	−23115.87	−21649.89	−20930.81
skew-normal	−22294.4	−20558.28	−21502.06	−20993.22	−20603.96
	−22148.7	−20412.58	−21356.36	−20847.52	−20458.26
GED	−22244.98	−20529.62	−21533.18	−20192.46	−20572.98
	−22099.28	−20383.92	−21387.48	−20046.76	−20427.28
skew GED	−20599.96	−19932.18	−21616.8	−20192.46	−20133.58
	−20436.05	−19768.26	−21452.89	−20046.76	−19969.67

higher. We found that CPN-CENTEL, INTUCH-ADVANCE, CENTEL-BDSM, and CPN-BDSM stock pair returns are in this case. In the second case, the value of $\alpha_{s(t)=i} + \beta_{s(t)=i}$ in each regime increases when the regime is higher and there is only one pair, namely SCB-KBANK that corresponds to this second case. Thus, we can interpret the first regime of CPN-CENTEL, INTUCH-ADVANCE, CENTEL-BDSM, and CPN-BDSM stock pair returns as the highest persistence of volatility shock regime while for the second or third regime, we interpret as

Table 3. Estimation result of MS-reg-GARCH for the five pair returns

Parameter	SCB-KBANK	CPN-CENTEL	INTUCH-ADVANCE	CENTEL-BDSM	BDSM-CPN
$\theta_{0,s(t)=1}$	0.0002	0.0001	0.0001	0.0001	0.0002
$\theta_{1,s(t)=1}$	0.4392***	0.1614***	0.9227***	0.3434***	0.2532***
$\omega_{s(t)=1}$	0.0001*	0.0001**	0.0001*	0.0001**	0.0001**
$\alpha_{s(t)=1}$	0.0001	0.1006***	0.2125	0.1007	0.1008
$\beta_{s(t)=1}$	0.5436***	0.8001	0.6192***	0.8142	0.7451
$v_{s(t)=1}$	2.1000***	3.0993	5.4264***	3.0963***	3.0995***
$\theta_{0,s(t)=2}$	0.0001	0.0001	0.0003	−0.0001	0.0001
$\theta_{1,s(t)=2}$	0.0701*	0.0806***	0.5246*	0.1716	0.1265
$\omega_{s(t)=2}$	0.0001*	0.0001	0.0001*	0.0001	0.0001
$\alpha_{s(t)=2}$	0.0001	0.005	0.2584	0.0507	0.0054
$\beta_{s(t)=2}$	0.5436***	0.4004***	0.3491***	0.4004***	0.4005***
$v_{s(t)=2}$	2.1000***	2.1142***	2.8354***	2.1372***	2.1431***
$\theta_{0,s(t)=3}$	0.0002		0.0001		
$\theta_{1,s(t)=3}$	0.6058***		0.3883***		
$\omega_{s(t)=3}$	0.0005***		0.0001***		
$\alpha_{s(t)=3}$	0.0917***		0.3184***		
$\beta_{s(t)=3}$	0.7896*		0.3026*		
$v_{s(t)=3}$	2.841***		2.1613***		
p_{11}	0.8108***	0.9000***	0.6242***	0.9003***	0.9001***
p_{22}	0.8106***	0.9	0.8096***	0.8998***	0.9
p_{33}	0.9464***		0.9712***		

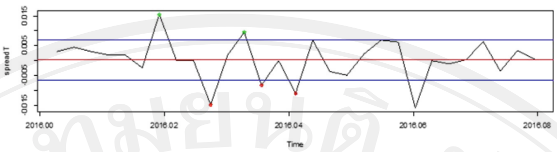
moderate or low volatility regime. On the contrary, in the case of SCB-KBANK pair, we can interpret the regime in the opposite direction to those four other pairs.

In a nutshell, our empirical analysis provides evidence of: (1) positive hedge ratio for all pairs in every regime; (2) the hedge ratio likely to be high in the high volatility regime and vice versa. Thus, we can say that our stock pairs exhibit a stronger movement when the market exhibits a high volatility. This evidence seems to be in line with those found in previous works undertaken for example by Tofoli et al. [18] and Karimalis and Nimokis [15], and Pastpipatkul et al. [17]. These studies reported an interesting result about the high co-movement between financial assets in the market downturn regime. This evidence is very important for investors because putting an investment in different period seems to face with a different market situation.

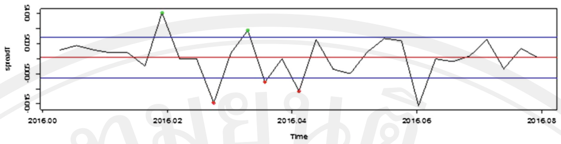
Moreover, the Table also provides the result of the transition matrix and shows that the regimes in all pair are persistent because the probability of staying in their own regime is larger than 80 %, while the probability of switching between these regimes is less than 20 %, except for p_{11} of INTUCH-ADVANCE pair. This indicates that only an extreme event can switch the pair returns to change between regimes.

5.4 Pairs Trading Strategy

In this section, we illustrate a trading signal of our 5 pair returns in Figs. 1, 2, 3, 4 and 5 and also provide a summary result of the returns from pairs trading



threshold line. From Table 4, we found that



ching regression GARCH. We found that CPN-CENTEL, CENTEL-BDSM and BDSM-CPN are preferred for 2-regime Markov switching regression

GARCH(1, 1) while 3-regime Markov switching regression GARCH(1, 1) provides the best fit to SCB-KBANK and INTUCH-ADVANCE pairs. The spreads are used for computing the mean and standard deviation in order to get the threshold value where the upper and lower threshold values can be defined as $Uthres = u+sd$ and $Uthres = u-sd$, respectively. Following the trading rule, we find that there are 7.9 round trips trading on average in the 30 trading days from the period December 18, 2015 to January 29, 2016. The average 5 pairs profit is 6.20 % where INTUCH and ADVANCE pair performs the highest return. In future research, Copula approach with regime switching can be applied to the pairs trading strategy and it would be useful for capturing the marginal distributions as well as the dependency structure between the stock returns. With a better understanding of the joint distribution of the two stocks, practitioners could gain preferential entry positions and have more trading opportunities.

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