CHAPTER 2

Theory and Literature Review

2.1 Theory

2.1.1 Public Goods and Externalities

Public goods are goods and services provided by the government and result in the positive externalities. The public goods are shared by a large group of people in the society and cannot keep away from the people who do not pay. Plus, the externalities can be both costs or benefits of the transactions, which are not affected by the change in price levels, in the market. The externalities cause when the benefits or costs of the third party, people apart from the buyers and sellers of the economic transactions of the particular goods and services, happen (Hyman, 2008).

In addition, those externalities, in turn, cause the productivity and growth in the economy. One of the 10 principles of Economics states that "A Country's Standard of Living Depends on Its Ability to Produce Goods and Services" (Mankiw, 2008). Equally important, the productivity, the quantity of goods and services produced using a unit of production input, is the determination of the standard of living. In addition, the growth in productivity, in turn, determined the growth of standard of living.

More interestingly, there are four factors determining the productivity; physical capital, human capital, natural resources and technological knowledge. Focusing on the technological knowledge, the knowing how to produce goods and services in the best possible ways, the advance in the technological knowledge can lead to the improvement in quantity and quality produced which makes individual or businesses better off. The ICTs, including the Internet, is also part of the technological knowledge.

In practice, according to Jung et al. (2013), the ICT has positive impacts on not only the labour productivity but also the total factor productivity (TFP). Moreover,

the ICT has benefited economic growth greatly (Sassi & Goaied, 2013; Salahuddin & Gow, 2015).

2.1.2 Stochastic Exponential Growth Model

The ordinary differential equations (ODE) are the certain differential equations with no concerns on the random effects (Nagy, 2016), or system of linear equations formed in terms of an unknown function and its derivatives with respect to a single independent variable such as time. At the same time, the ODE is the simplest differential equation (Gershenfeld, 2014).

The ODE's form without random variables is (Khodabin et al., 2012)

$$\frac{dX_t}{dt} = \mu(X_t, t) \cdot X_t \tag{2.1}$$

with

$$X_{t_0} = x_t$$

where

X_t is the number of individuals at time t.

X_{t_0} is the initial number at time $t = 0$.

μ is the growth rate at time t.

If $\mu(X_t, t) = r(t)$ which is an accurate and non-random given function, then the exponential model is as follow

$$X_t = X_0 exp\left(\int_0^t r(s)ds\right) \tag{2.2}$$

In other cases, if $r(t) = r$, then

$$X_t = X_0 exp(rt) \tag{2.3}$$

In some cases, the ODE is inadequate when it comes to a confrontation of some random variables. Regarding the uncertainties, consequently, the stochastic

differential equation (SDE) is introduced.

The stochastic differential equations (SDE) provides a connection between probability theory and the ordinary and partial differential equations. (Goodman, 2007) Furthermore, the SDEs are the forms expressing dynamical models or time-dependent models and include both random and non-random effects. (Lee, 2013a)

The SDE holds the form

$$\frac{dX_t}{dt} = \mu(X_t, t) + \sigma(X_t, t)\delta_t \tag{2.4}$$

which is equivalent to

$$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)\delta_t dt \tag{2.5}$$

with

$$X_{t_0} = x_t \ and \ 0 \leq t \leq T$$

where

δ_t is a Gaussian White Noise.

The Gaussian white noise process can be written as the generalised derivative of the Brownian motion as follow

$$dB_t = \delta_t dt \tag{2.6}$$

and according to the rules (Øksendal, 2000)

$$dB_t \cdot dB_t = dt \tag{2.7}$$

then, the SDE takes the form

$$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dB_t \tag{2.8}$$

where (Shalizi, 2006)

μ is a drift term or a Riemann integral.

σ is diffusion term or an Ito integral.

B_t is one-dimensional Brownian motion.

A function X is a solution to the differential equation (3.8), if it satisfies

$$X_T = \int_0^T \mu(X_t, t)dt + \int_0^T \sigma(X_t, t)dB_t \tag{2.9}$$

More interestingly, a standard Brownian motion (SBM) B_t is an one-dimensional stochastic process which is a continuous-time analogue of the simple random walk (Lee, 2013b). What's more, there are some important properties of sBm as follow

1. $B_0 = q$ almost absolutely for a fixed $q \in R$.

2. All sample paths are almost absolutely continuous.

3. B_t is normally distributed with mean 0 and variance *t*.

4. $(B_{t+u} - B_t)$ is normally distributed with mean 0 and variance *u*: $(B_{t+u} - B_t) \sim N(0, u)$.

5. If the internals $[t_1, t_2]$ and $[u_1, u_2]$ are non-overlapping, then $(B_{t2} - B_{t1})$ and $(B_{u2} - B_{u1})$ are independent.

Stochastic Exponential Model

The solution of the exponential SDE driven by a standard Brownian motion (SBM) is as follow

$$S_t = S_0 \exp\left(\mu t - \frac{\sigma^2}{2}t + \sigma B_t\right) \tag{2.10}$$

where

μ and σ are constant values

Let

(2.11)

$$Y_t = \ln\left(\frac{S_t}{S_0}\right)$$

then, (3.11) is equivalent to

(2.12)

$$Y_t = \left(\mu t - \frac{\sigma^2}{2} t + \sigma B_t\right)$$

2.2 Literature Review

2.2.1 Digital Economy

Researchers have studied the effect of the Internet users, the Internet usage or the ICTs. A number of study analyses have examined the relationship between the Internet and a variety of economic consequences. Choi (2002) studied the impact of the Internet on the level of inflow foreign direct investment (FDI) with a hypothesis that the Internet improved the productivity and, in turn, increased the FDI. The finding showed that the Internet gave positive impact to the FDI volume. Moreover, Yi and Choi (2005) investigated the Internet inflation. They found that the increase in the Internet users reduced the inflation rate.

Interestingly, Naudé and Saayman (2005) proposed the paper on tourism in Africa and found that the Internet usage was one of the determinants of the tourist arrivals. Furthermore, Salahuddin and Gow (2015) studied the effect of Internet usage on the economic growth in South Africa. The result showed that the Internet usage had not only the long-run relationship but also the positive impact on the economic growth. Similarly, Elgin (2012) examined the relationship between the internet usage volume and the size of the shadow economy. Then, it revealed that the increase in the volume of the Internet usage rose the shadow economy's size.

Equally important, Jung et al. (2013) demonstrated the direct and indirect effects of technological convergence. The results revealed that the ICT had a direct positive effect on the labour productivity and the indirect positive effect on total factor productivity (TFP) growth rate. Sassi and Goaied (2013) studied the effect of financial development and ICT on the economic growth. They discovered that the ICT penetration gave the direct positive effect on the economic growth significantly, but the financial

development itself gave the negative effect. More interestingly, the reaching of the ICT development threshold can improve the financial development.

2.2.2 Population Growth Model

Population growth is the change in population numbers indexed by time. The population growth wording typically refers to human beings, but it can possibly be any species of living creatures (Khodabin, et al., 2011). In recent years, there has been an increasing amount of literature on the population growth models which indicate the theoretical ideas to predict future results or simulate models or tests. A considerable amount of literature had been published on the population growth models; see Table 3. These studies had been considered from deterministic to stochastic population models with discrete, continuous, random or non-random variables (Khodabin & Kiaee, 2014).

Table 2.1: Previous studies concerning the population growth models

No.	Title	Author(s)	Year	Model/ Description
1.	On stochastic logistic population growth models with immigration and multiple births	Matis and Kiffe	2003	The authors developed a stochastic logistic population growth model with immigration and multiple births for badger and fox populations.
2.	Itô versus Stratonovich calculus in random population growth	Braumann	2005	The author employed the general stochastic differential equation (SDE) model for the population growth in a random fluctuation environment.
3.	Stochastic Dynamical Theta-Logistic Population Growth Model	Momani and Qaralleh	2006	The author had studied the solution of a fractional population growth model in a closed system driven by the Caputo time-fractional differential equations.

Table 2.1: Previous studies concerning the population growth models (continued)

No.	Title	Author(s)	Year	Model/ Description
4.	On stochastic population model with the Allee effect	Krstić and Jovanović	2010	The authors studied the stochastic population growth model for gypsy moth and Lymantria dispar population. Furthermore, the model was driven by a one-dimensional standard Brownian motion with the Allee effect.
5.	Stochastic Dynamical Logistic Population Growth Model	Khodabin and Kiaee	2011	The authors had introduced the stochastic logistic population growth model using the non-linear stochastic differential equation for the solution.
6.	Interpolation solution in generalized stochastic exponential population growth model	Khodabin, et al.	2011	The authors proposed the generalised stochastic exponential population growth model for the Iran population.
7.	Dynamic estimation of markets exhibiting a prey-predator behaviour	Michalakelis, et al.	2012	The authors studied the fixed lines telecommunications markets over five countries in Europe based on the population concept using the Lotka–Volterra model.
8.	Stochastic Dynamical Theta-Logistic Population Growth Model	Khodabin and Kiaee	2014	The authors had proposed the non-linear stochastic differential equation on their paper, Stochastic Dynamical Theta-

Table 2.1: Previous studies concerning the population growth models (continued)

No.	Title	Author(s)	Year	Model/ Description
				Logistic Population Growth Model, so as to study the Iran population growth data.
9.	Assessment of the gap and (non-)Internet users evolution based on population biology dynamics	Neokosmidis, et al.	2014	The authors applied the population concept to study the Internet and non-Internet users' evolution Greece and Lithuania using the Lotka–Volterra model.
10.	Deterministic versus stochastic aspects of super-exponential population growth models	Grosjean and Huillet	2016	The authors have recently investigated and compared both deterministic and stochastic super-exponential population growth models.

Source: Self-illustration