CHAPTER 6

Constitutive model of mixed plastic waste and corn stover densification

This chapter is carried out for the constitutive model which related to stress-strain relationship or elasto-visco-plastic model. Several researchers study for the constitutive model of the characterized compaction like Kaliyan (2008), Peleg (1983), Suched (2008), Ren (Ren, 1992), Thoemen (2006) and etc. Constitutive model can explain the material deformation behavior under the reaction force. It is generality in the form of elastic, viscous and plastic.

6.1 Introduction

Stress and natural strain curve or compression curve for mixed plastic waste and cornstover can be devided into inertial deformation and elasto-visco-plastic deformation which can be described by constitutive model of material.

Kaliyan (2008) and Peleg(1983) reported that a constitutive model is a combination function of stress in terms of spring element, dashpot element, and the Coulomb friction element.

The spring element is the summation of elastic and plastic deformation as follow:

$$\sigma = E\varepsilon + R\varepsilon^{n} \qquad (1)$$

Where, σ is stress, E is elastic modulus, ε is natural strain, R is strength coefficient or plastic model, and n is strain hardening exponent.

The dashpot element is the model of viscous dissipation. The model is as follow:

$$\sigma = \eta \frac{d\varepsilon}{dt} \qquad (2)$$

Where, σ is stress, η is viscous coefficient, and $\frac{d\varepsilon}{dt}$ is natural strain rate.

The coulomb friction element is the model of frictional loss. The model is as follow:

$$\sigma = \sigma_f \qquad (3)$$

Where, σ is stress, and σ_f is frictional loss.

The total stress (σ) of the palletization process is the summation of the spring element, dashpot element, and the coulomb friction element. Therefore, the elastovisco-plastic solid model can be described by equation 6.4.

$$\sigma = E\varepsilon + R\varepsilon^n + \eta\frac{d\varepsilon}{dt} + \sigma_f \qquad (4)$$

6.2 Experimental methodology

As described the factors affecting to the densified pellet in the previous chapter, the best condition for palletization is selected. Low specific energy consumption, high density and high durability are considered. It is found that the best condition for pelletization is 75°C of preheating temperature, 10% of moisture content, 150 MPa of pressure compaction and 55:45 of plastic waste and corn stover ratio. Compression of a pellet under the best condition are tested with universal testing machine, Instron as shown in Figure 6.1. Densified plastic wastes and corn stover are compressed with the constant crosshead speed of 20 mm/min. Compressed load and distance are recorded for construction of the constitutive model of the pelletization.The constitutive model as described by Kaliyan (2008) is used in this study. The model is the summation of stresses in terms of spring element, dashpot element, and the Coulomb friction element.

Figure 6.1: Compression test of densify plastic wastes and corn stover mixed in Instron machine.

6.3 The result and discussion

6.3.1 Mixed plastic waste and corn stover constitutive model

The compression curve of stress and natural strain for mixed plastic waste and corn stover is shown as Figure 6.2. Compressive stress is in range of 0 to 150 MPa. It is found that stress is gradually rise for strain under 1.1 because of the number of void in the mold. After the void is diminished, the material is compacted and stress is sharply increased as shown. The constitutive model is constructed and compared with the experiment result as shown as Figure 6.3.

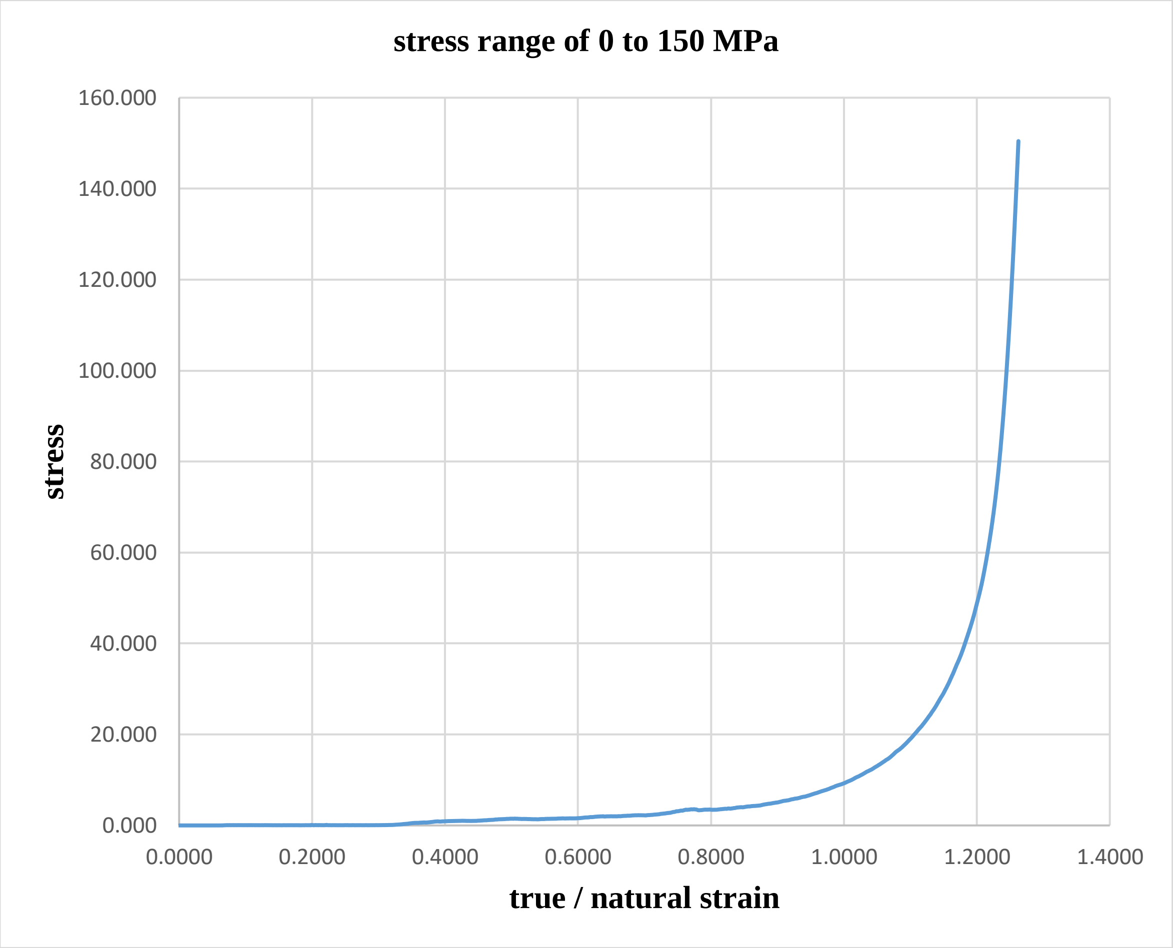


Figure 6.2: The compression curve of stress and natural strain for mixed plastic waste and corn stover at the range of 0 – 150 MPa.

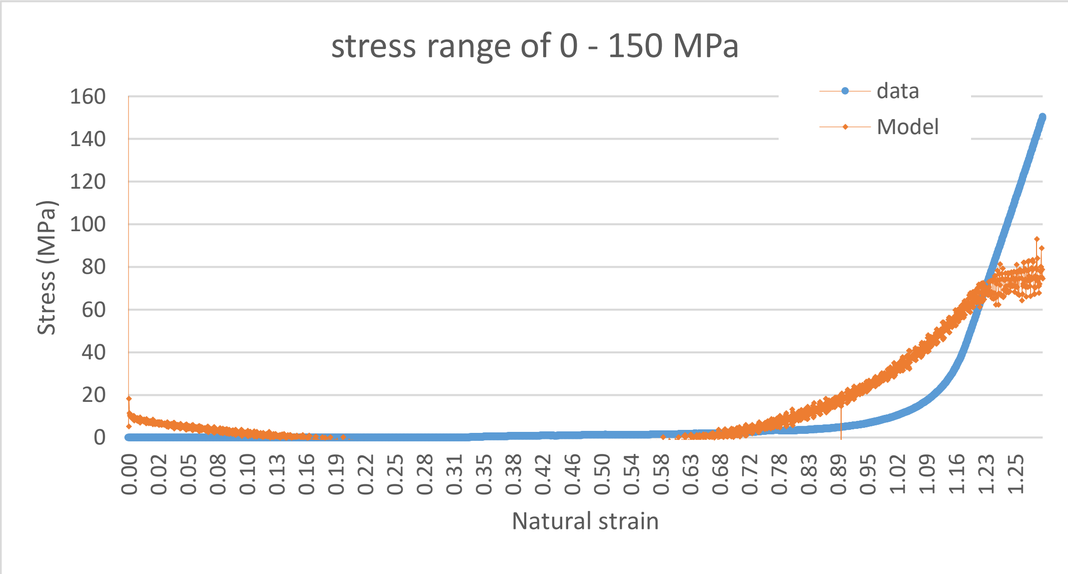


Figure 6.3: A comparison of the constitutive model with the experiment data at stress range 0-150 MPa.

The parameters of constitutive model including to E, R, n, η and σ_f are determined. The constitutive model as plotted in Figure 6.3 is $\sigma = -162.574\varepsilon + (-2094.445)\varepsilon^{0.000383} + 35145.70\frac{d\varepsilon}{dt} + 1990.372$. R-square of the model is 0.71 that the model cannot be used to fit the curve for almost stress range. Therefore, the stress ranges are separated into 5 ranges which are 0 to σ_i, σ_i to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa. This separation lets the model an be fitted to the experimental result as used by Kaliyan (2008).σ_i is the end of inertial deformation which some of particle may be elastic deformation and some of particle maybe plastic deformation. It is obtained from pressure and time curve which indicates from the last negative pressure gradient. The point of the end of inertial deformation is at 4.1 MPa as shown in Figure 6.4. And Figure 6.5(a) presents the combined plots of in ranges of 0 to 4.1(σ_i), 4.1(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa, while Figure 6.5(b) to (f) show plots of each range model, respectively. The model curve is the model of equation 6.4. The parameter of constitutive model of 0 – 150 MPa, 0 to 4.1(σ_i), 4.1(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa are showed in the Table 6.1.

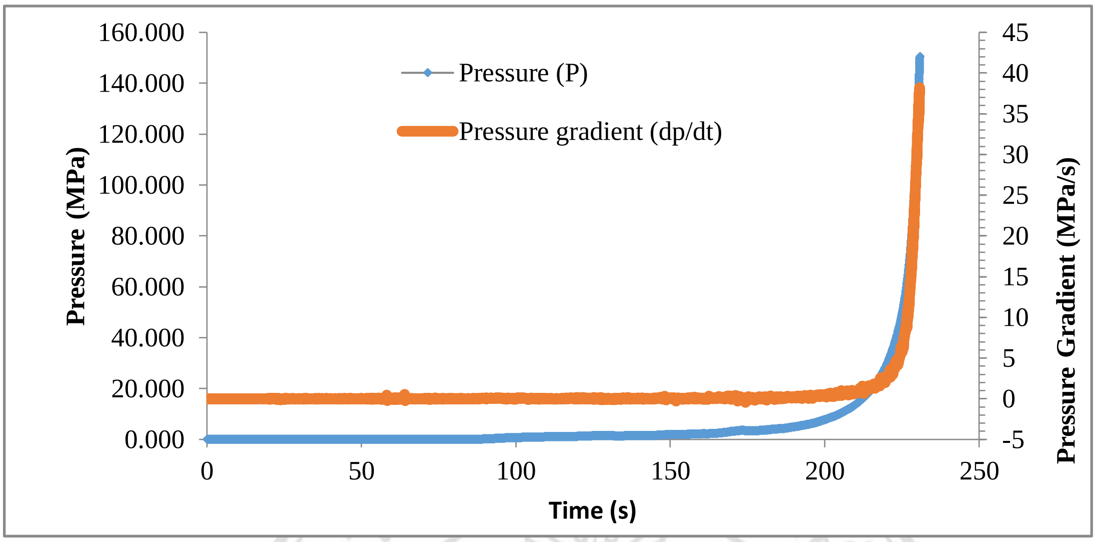


Figure 6.4 : Compression curve of 0 – 150 MPa

Figure 6.5 showed the model curve of constitutive model versus measured data at each stress range. The result found that in case of using only one relation to fit 0 – 150 MPa stress range, the model shows a low R-square of 0.71. Dividing stress range into 5 segments of 0 to 4.1(σ_i), 4.1(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa, the model gives high R-square of between 0.97 and 0.99. This is agreed with the works of (Heiko Thoemen 2006; Nalladurai & R. Vance, 2008; Ren, 1992), they reported that, the total range of stresses cannot be fitted with single equation because the structure of the densing materials is changed with applied stress, which lets changing to, their mechanical properties.

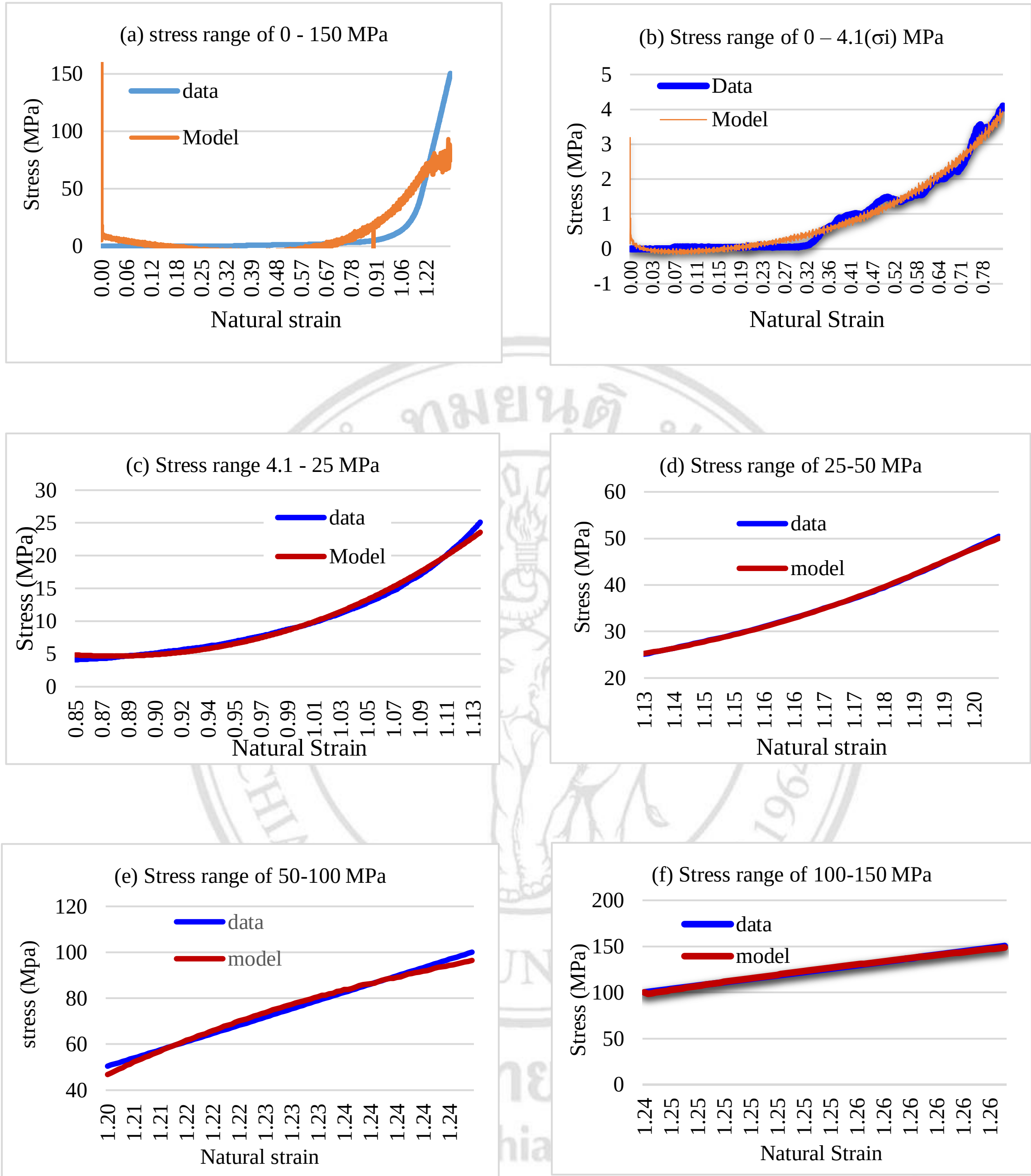


Figure 6.5: A comparison of the constitutive model with the experiment data at the total compression curve of 0 – 150, and stress ranges of 0 to σ_i, σ_i to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa.

Trying to use a single equation to fit the experiment result of stress range 4.1(σ_i) – 150 MPa as shown in Figure 6.6, the model gives R-square of 0.87. Eventhough it is higher than that of the model with stress range 0-150 MPa, it is still less than those of the each range which is showed in Figure 6.5. The graph of the constitutive model with the experiment data at stress range σ_i to 150 MPa is showed in figure 6.6.

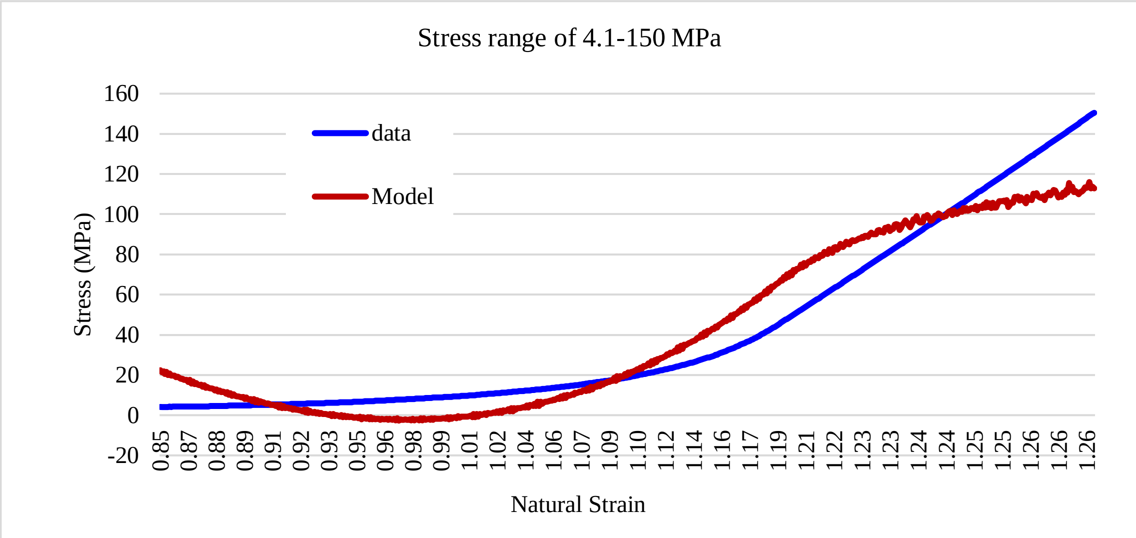


Figure 6.6 : A comparison of experimental result and a single constitutive model fitting stress range 4.1 (σi)-150 MPa.

Table 6.1 : The parameter and R-square of constitutive model of each stress range for uniaxial compression.

Stress range (MPa)	Parameter of constitutive model : $\sigma = E\varepsilon + R\varepsilon^n + \eta\frac{d\varepsilon}{dt} + \sigma_f$					R-square
	Elastic modulus, E (MPa)	Strength coefficient, R (MPa)	Strain harding, n (-)	Viscous coefficient, η (MPa-s)	Frictional loss factor, σ_f (MPa)	
0-150	-1.626 E+02	-2.094 E+03	3.830E-04	3.514 E+04	1.990 E+03	0.711
4.1-150	1.447 E+04	-1.813 E+04	7.979 E-01	8.743 E+03	3.586 E+03	0.919
0-4.1	-5.251 E+00	-1.037 E+01	1.373 E-02	2.128 E+03	3.194 E+00	0.973
4.1-25	4.872 E+03	-5.414 E+03	8.874 E-01	1.270 E+01	5.465 E+02	0.994
25-50	1.191 E+04	-2.437 E+04	5.116 E-01	1.426 E+02	1.250 E+04	1.000
50-100	2.983 E+04	-6.127 E+04	5.154 E-01	-1.078 E+03	3.156 E+04	0.986
100-150	9.907 E+03	-1.186 E+04	6.444 E-01	-1.365 E+02	1.422 E+03	0.997

Table 6.1 shows the parameter of the model of $\sigma = E\varepsilon + R\varepsilon^n + \eta\frac{d\varepsilon}{dt} + \sigma_f$ and its R-square of each stress range. The model with stress range of 0-150 and 4.1(σ_i) -150

obtains the R-square less than 0.9. In case of dividing stress range into 5 segments: 0 to 4.1(σ_i), 4.1(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa, the model gives R-square more than 0.97 which is near 1.0 like the research of Kaliyan (Nalladurai & R. Vance, 2008).

Constitutive model can explain the mechanical behaviors of biomass grinds during compression process. It is a function of sum of stress. This can be seen as the increasing of elastic modulus (R) with stress applied as showed in figure 6.7. In addition of, particle grinds of biomass would be changing to solid compact with the stress applied. Inertial deformation or particle rearrangement and air removal occurred until 4.1 MPa which is the inertial stage. The next stage, elasto-viscoplastic deformation stage was supposed at this point. In this stage, air was removed continually from the biomass or plastic grind and the particles was deformed. At the inertial stage (0 - 4.1 MPa), elastic modulus was at -5.251 MPa. Elastic modulus was increasing continuously to 2.983 E+04 MPa at the applied stress range of 50 – 100 MPa. At 150 MPa, elastic modulus was 1484 MPa. The higher modulus led to lower elastic deformation as same as Kaliyan (Nalladurai & R. Vance, 2008) research. For the strain hardening, it was decreased from about 0.8 to 0.5 as shown in figure 6.7. This result showed that plastic strain was increased with the increasing stress. The friction loss factor of this research increased with increasing pressure. This result showed that there are energy loss from friction during compression. For example, energy loss from friction loss between particle and die wall, particle and particle or the force which related to frictional effect (Li, Liu, & Rockabrand, 1996; Nalladurai & R. Vance, 2008). The viscous coefficient showed the components of material which is squeezed of the cell of material. The higher values of viscous coefficient (absolute value) made the higher compressive strength of pellet. This result is as same as Kaliyan (2008)(Nalladurai & R. Vance, 2008).

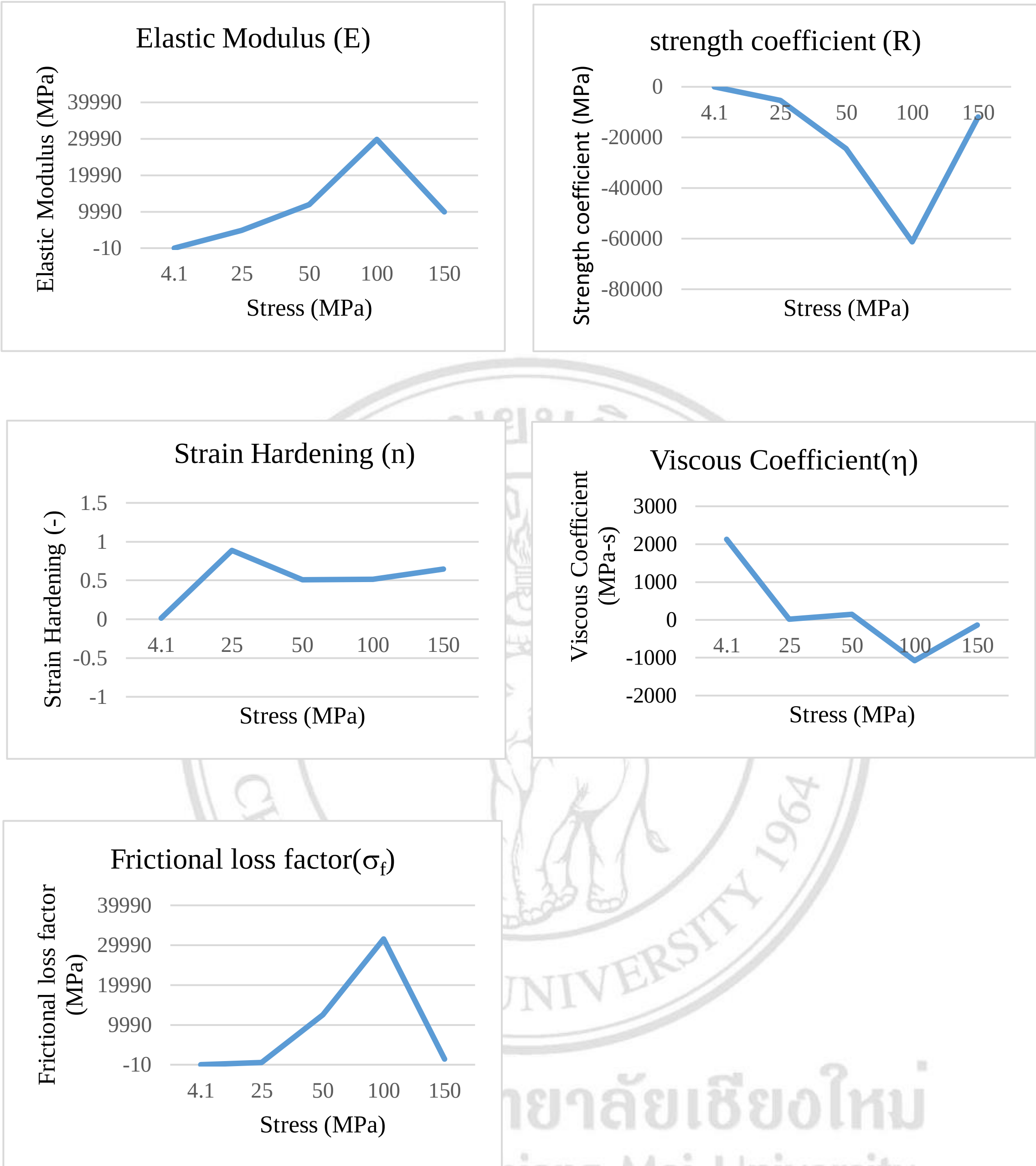
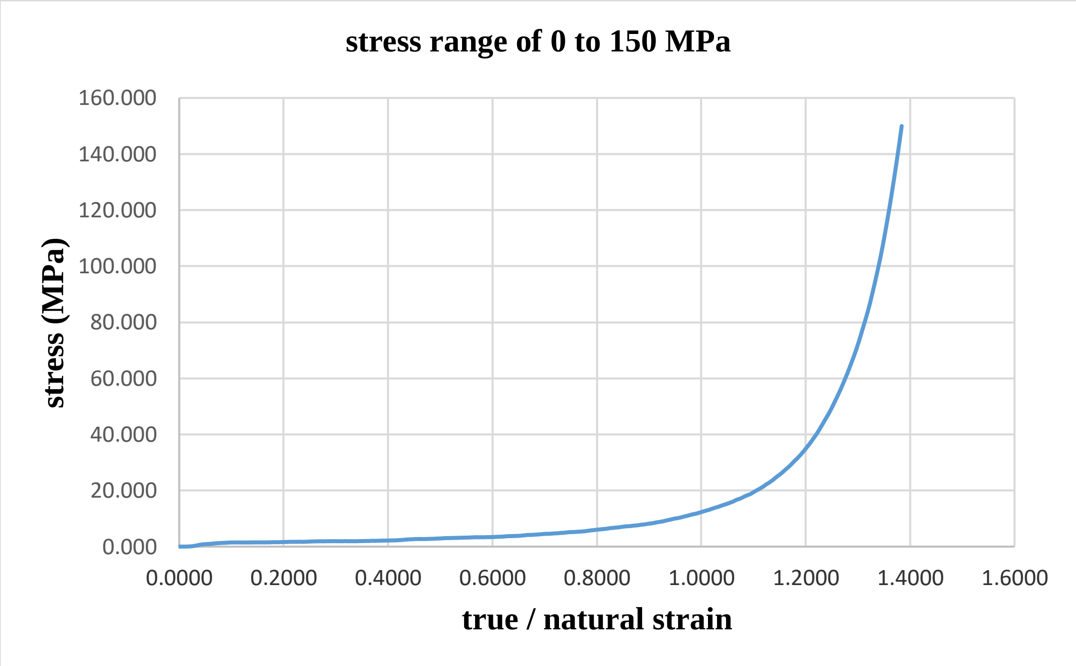


Figure 6.7: The model parameters of the constitutive model with applied stress to 150 MPa.

6.3.2 Corn stover constitutive model

The compression curve of stress and natural strain for corn stover is shown as Figure 6.8. Compressive stress is in range of 0 to 150 MPa. It is found that stress is gradually rise for strain under 2.0 because of the number of void in the mold of corn stover. After the void is diminished, the material is compacted and stress is sharply increased as shown. A comparison of the constitutive model with the experiment data at stress range 0 – 150 MPa as shown as Figure 6.9. Constitutive model of corn stover is determined as same as mixed plastic waste and corn stover. The constitutive model as plotted in Figure 6.9 is $\sigma = -145.278\varepsilon + (-1736.198)\varepsilon^{0.000332} + 30629.05\frac{d\varepsilon}{dt} + 1647.154$. R-square of the model is 0.81. The end of inertial deformation (σ_i) is obtained from pressure and time curve is at 7.5. The stress ranges are separated into 5 ranges which are 0 to 7.5(σ_i), 7.5(σ_i) to 25 MPa, 25 to 50 MPa,



50 to 100 MPa and 100 to 150 MPa.

Figure 6.8: The compression curve of stress and natural strain for corn stover at the range of 0 – 150 MPa.

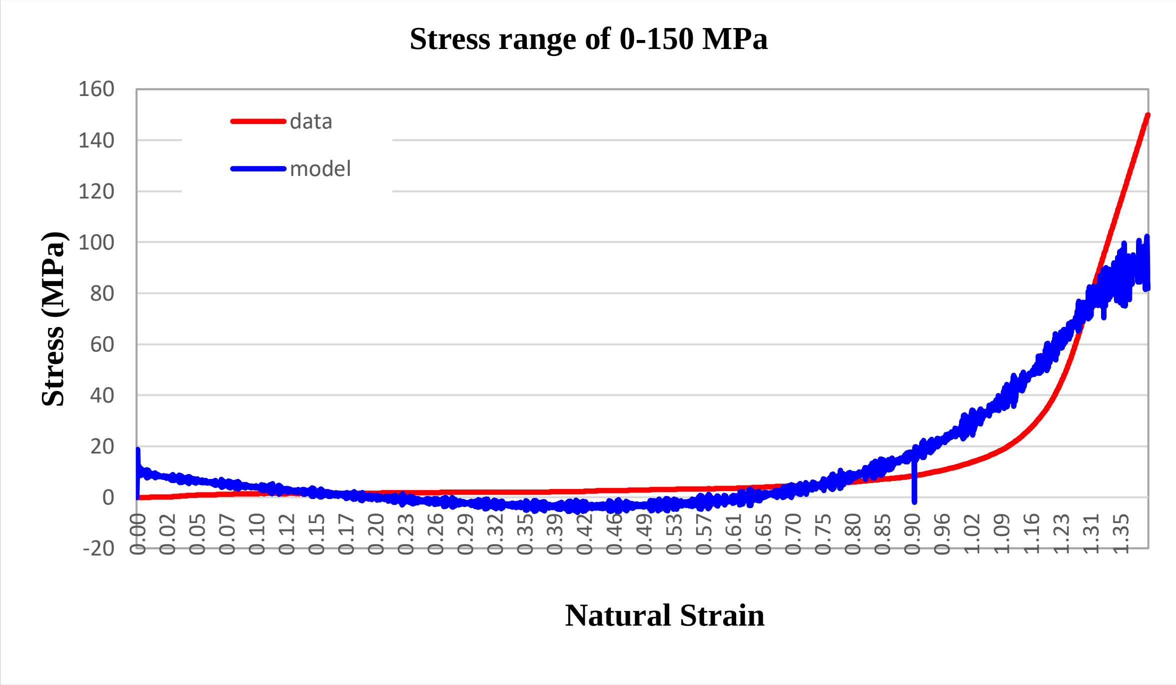


Figure 6.9: A comparison of the constitutive model with the experiment data at stress range 0 – 150 MPa for corn stover.

And Figure 6.10(a) presents the combined plots of in ranges of 0 to 7.5(σ_i), 7.5(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa, while Figure 6.5(b) to (f) show plots of each range model, respectively. The parameter of corn stover constitutive model of 0 – 150 MPa, 0 to 7.5(σ_i), 7.5(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa are showed in the Table 6.2.

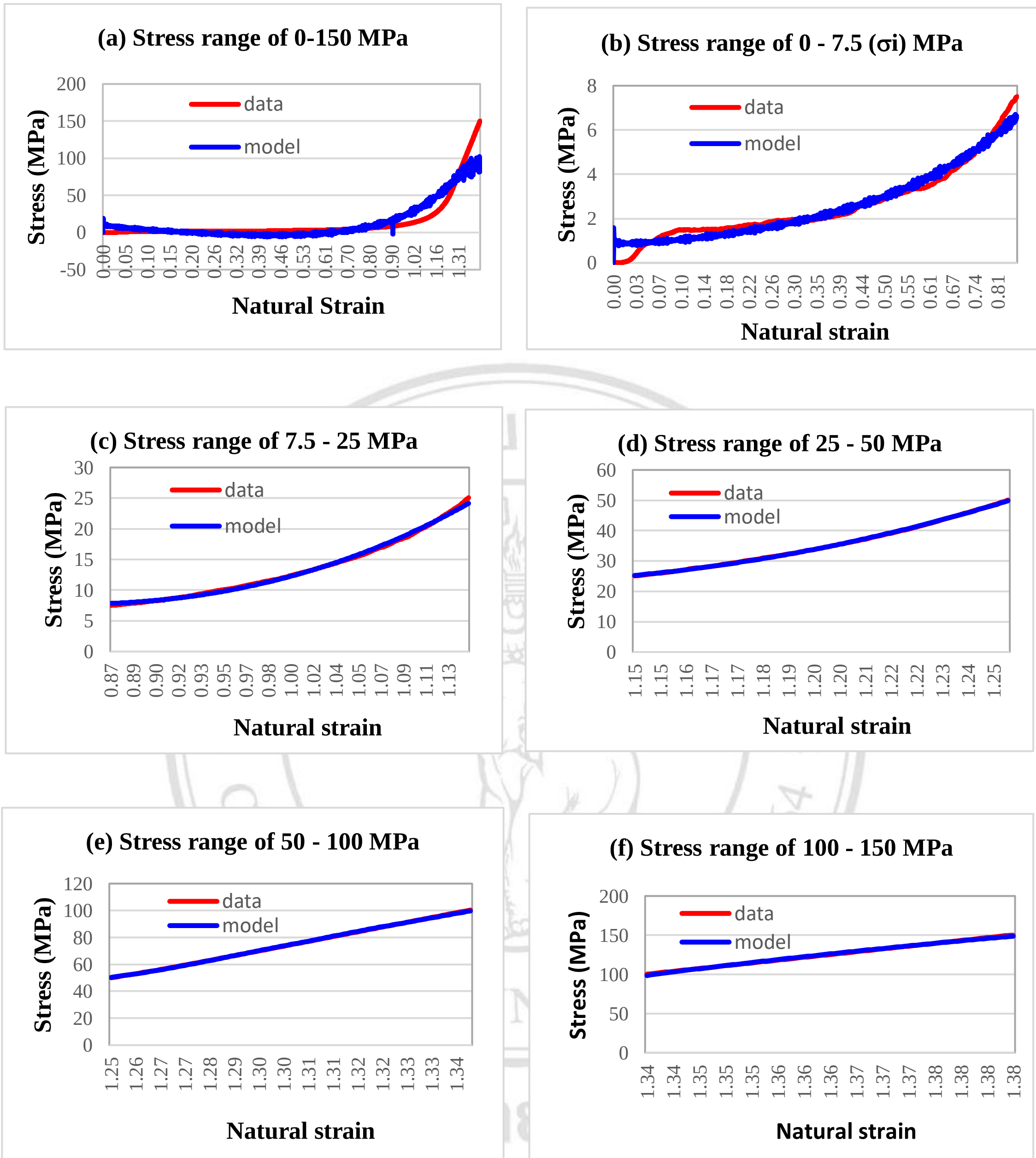


Figure 6.10: A comparison of the constitutive model for corn stover with the experiment data at the total compression curve of 0 – 150, and stress ranges of 0 to σ_i, σ_i to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa.

Table 6.2: The parameter and R-square of constitutive model of each stress range for uniaxial compression of corn stover

Stress range (MPa)	Parameter of constitutive model : $\sigma = E\varepsilon + R\varepsilon^n + \eta\frac{d\varepsilon}{dt} + \sigma_f$					R-square
	Elastic modulus, E (MPa)	Strength coefficient, R (MPa)	Strain harding, n (-)	Viscous coefficient, η (MPa-s)	Frictional loss factor, σ_f (MPa)	
0-150	-1.453 E+02	-1.736 E+03	3.320E-04	3.063 E+04	1.647 E+03	0.809
0-7.5	-4.441 E+00	-3.365 E+02	1.570 E-04	2.260 E+03	3.300 E+02	0.954
7.5-25	2.458 E+03	-2.808 E+03	8.545 E-01	8.626 E+00	3.630 E+02	0.998
25-50	5.354 E+03	-1.089 E+04	5.136 E-01	9.837 E+01	5.568 E+03	0.999
50-100	1.466 E+04	-3.072 E+04	5.196 E-01	4.195 E+01	1.623 E+04	0.999
100-150	1.739 E+03	-1.480 E+03	4.737 E-01	-1.651 E+02	-5.309 E+02	0.997

6.3.3 Mixed plastic waste constitutive model

The compression curve of stress and natural strain for mixed plastic waste is shown as Figure 6.11. Compressive stress is in range of 0 to 150 MPa. It is found that stress is gradually rise for strain under 1.5 because of the number of void in the mold of mixed plastic waste. After the void is diminished, the material is compacted and stress is sharply increased as shown. A comparison of the constitutive model with the experiment data at stress range 0 – 150 MPa as shown as Figure 6.12. Constitutive model of mixed plastic waste is determined as same as mixed plastic waste and corn stover. The constitutive model as plotted in Figure 6.12 is $\sigma = 11252.49\varepsilon + (-11345.09)\varepsilon^{0.994} + 23506.95\frac{d\varepsilon}{dt} + (-59.611)$. R-square of the model is 0.560. The end of inertial deformation (σ_i) is obtained from pressure and time curve is at 9.6. Therefore, the stress ranges are separated into 5 ranges which are 0 to 9.6(σ_i), 9.6(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa.

Figure 6.13(a) presents the combined plots of in ranges of 0 to 9.6(σ_i), 9.6(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa, while Figure 6.5(b) to (f) show plots of each range model, respectively. The parameter of mixed plastic waste constitutive model of 0 – 150 MPa, 0 to 9.6(σ_i), 9.6(σ_i) to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa are showed in the Table 6.3.

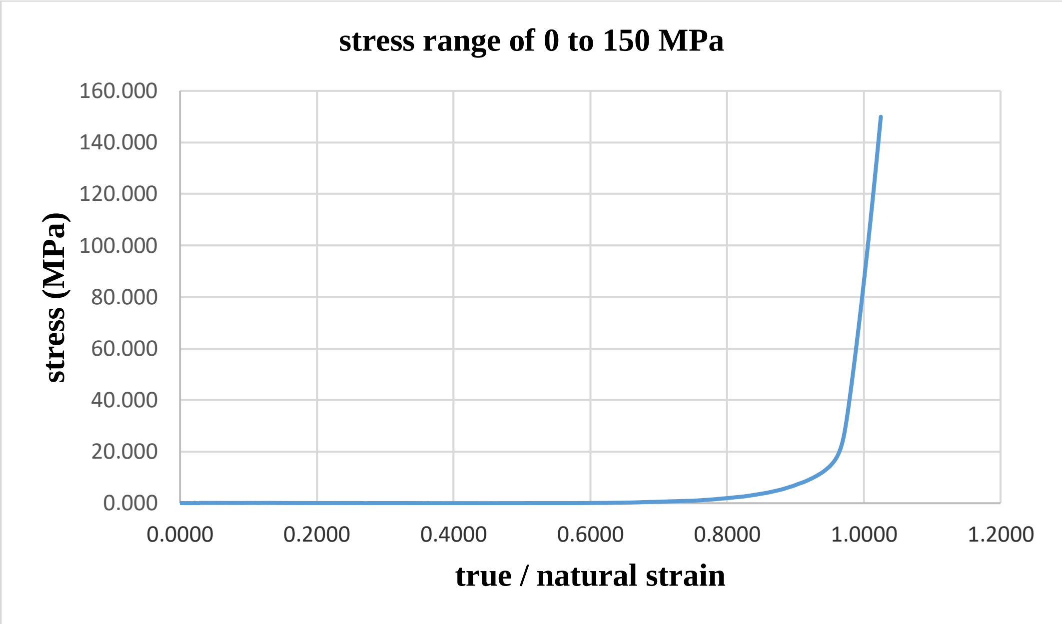


Figure 6.11: The compression curve of stress and natural strain for mixed plastic waste at the range of 0 – 150 MPa.

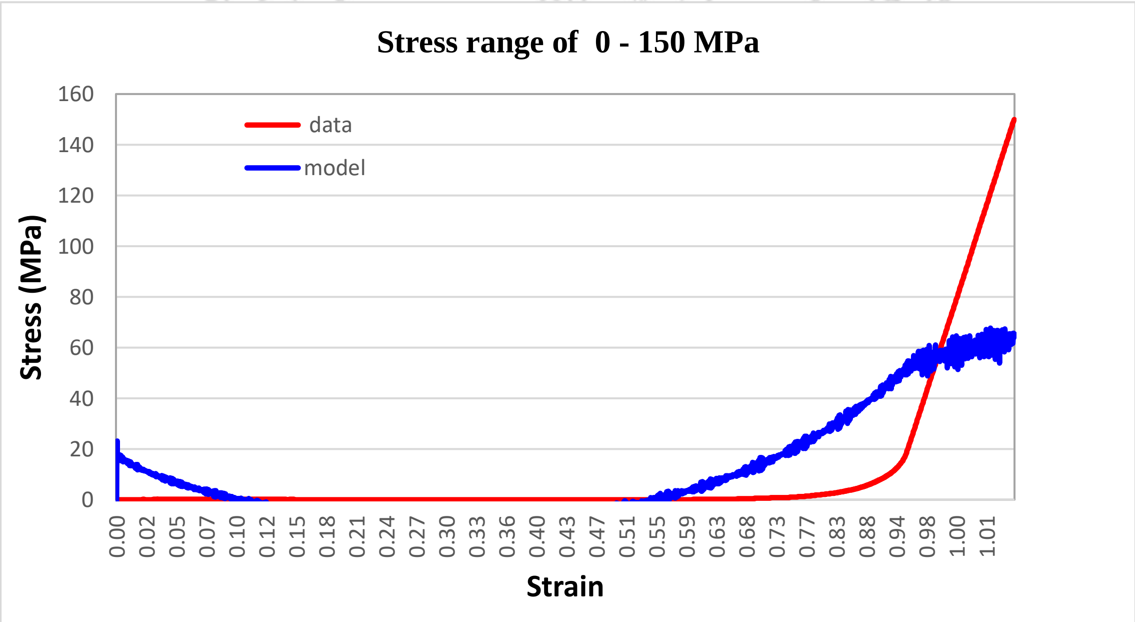


Figure 6.12: A comparison of the constitutive model with the experiment data at stress range 0 – 150 MPa for mixed plastic waste.

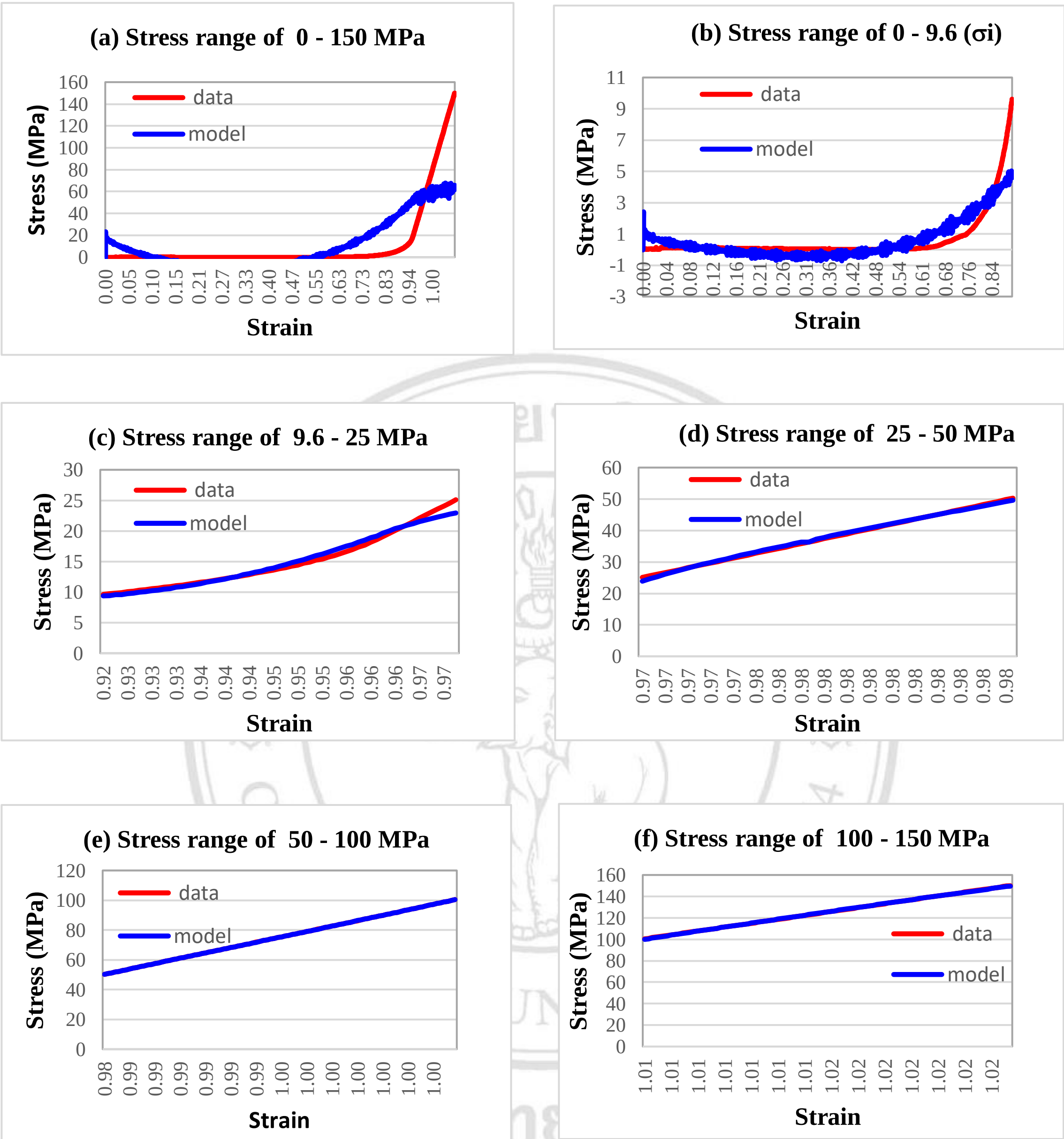


Figure 6.13: A comparison of the constitutive model for mixed plastic waste with the experiment data at the total compression curve of 0 – 150, and stress ranges of 0 to σ_i, σ_i to 25 MPa, 25 to 50 MPa, 50 to 100 MPa and 100 to 150 MPa.

Table 6.3: The parameter and R-square of constitutive model of each stress range for uniaxial compression of mixed plastic waste

Stress range (MPa)	Parameter of constitutive model : $\sigma = E\varepsilon + R\varepsilon^n + \eta\frac{d\varepsilon}{dt} + \sigma_f$					R-square
	Elastic modulus, E (MPa)	Strength coefficient, R (MPa)	Strain harding, n (-)	Viscous coefficient, η (MPa-s)	Frictional loss factor, σ_f (MPa)	
0-150	1.125 E+04	-1.134 E+04	9.937 E-01	2.351 E+04	-5.961 E+01	0.560
0-9.6	-2.136 E+01	-3.473 E+02	4.190 E-04	4.846 E+03	3.316 E+02	0.725
9.6-25	1.638 E+04	-3.151 E+04	4.968 E-01	3.896 E+02	1.517 E+04	0.977
25-50	2.781 E+03	-2.034 E+03	4.656 E-01	4.926 E+01	-6.694 E+02	0.997
50-100	3.828 E+04	-7.187 E+04	4.998 E-01	2.475 E+01	3.367 E+04	0.999
100-150	4.460 E+03	-3.702 E+03	4.943 E-01	-1.895 E+01	-6.744 E+02	0.999

The comparison of parameters of E, R, n, η, and σ_f of mixed plastic waste and corn stover pellet, corn stover pellet and mixed plastic waste pellet as shown in Figure 6.14a – 6.14f.

The result found that elastic modulus of mixed plastic waste pellet was higher than corn stover pellet, and mixed plastic waste and corn stover pellet.

The strength coefficient of mixed plastic waste and corn stover pellet was the highest, followed by corn stover pellet and mixed plastic waste and corn stover pellet, respectively. The higher strength coefficient let the higher of plastic strain in material bed.

The lower strain hardening let the higher strength coefficient. The plastic deformation equation of $\sigma = R\varepsilon^n$ can be explained in term of $\sigma^{1/n} = R^{1/n}$. The higher strength coefficient were obtained from the lower strain hardening which has the higher plastic deformation. The figure 6.14(c) found that strain hardening of mixed plastic and corn stover pellet was the lowest strain hardening which can be let the highest strength coefficient and good plastic deformation.

The value of viscous coefficient represented the squeezing of natural binding component from material cell. As Kaliyan research (Nalladurai & R. Vance, 2008), the lower viscous coefficient produced the higher compressive strength of pellet. Viscous coefficient of mixed plastic waste and corn stover pellet was the lowest value. It can be predicted that, mixed plastic waste and corn stover pellet was the highest compressive strength and durability, followed by corn stover pellet and mixed plastic waste pellet.

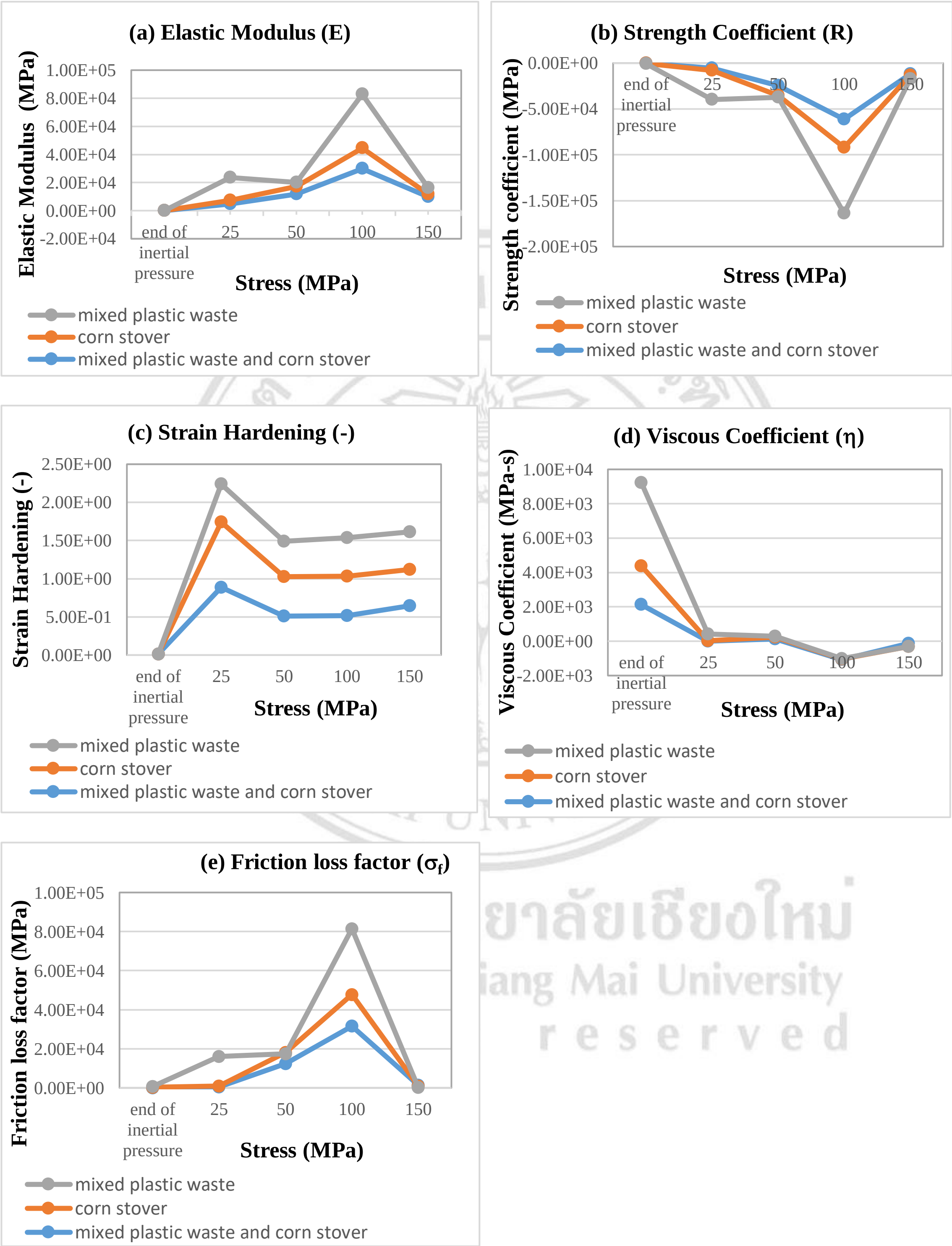


Figure 6.14: The comparison of parameters of E, R, n, η, and σ_f of mixed plastic waste and corn stover pellet, corn stover pellet and mixed plastic waste pellet.

The friction loss factors of materials were higer to compressive strength. It presented the energy loss from interparticle friction, particle and wall friction, and the adhesion of particle and wall. The Figure 6.14(f) found that, mixed plastic waste pellet has the highest viscous coefficient. Therefore, it has the highest energy loss during densification process. The viscous coefficient of mixed plastic waste and corn stover pellet was the lowest. It can be predicted that, mixed plastic waste and corn stover pellet has the lowest energy loss.

6.4 Conclusion

The constitutive model is the sum of stress functions of elastic deformation, plastic deformation and viscous dispersing. It can be used to explain the mechanical behavior of pellet densification process. It also estimate the model in term of the function parameter of stress (σ), elastic modulus (E), natural strain (ε), strength coefficient or plastic model (R), frictional loss (σ_f) and strain hardening exponent (n). Elastic modulus was increasing continuously to the applied stress. The higher modulus let to lower elastic deformation. Strain hardening can explain the plastic strain. It was increased with the increasing stress. In this study, the friction loss factor increased with increasing pressure. This is because, there are energy losses during compression including the energy loss from friction loss between particle and die wall, particle and particle or the force which related to frictional effect. The viscous coefficient showed the components of material which is squeezed out of the cell of material. The higher values of viscous coefficient (absolute value) made the higher compressive strength of pellet. The constitutive model of mixed plastic waste and corn stover pellet, mixed plastic waste pellet, and corn stover pellet can be explained that, mixed plastic waste and corn stover pellet has good mechanical properties more than corn stover pellet and mixed plastic waste pellet including to the higher plastic deformation, higher compressive strength and durability, and smaller of energy loss from densification process.

6.5 References

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