CHAPTER 1

Introduction

In 1953, the notion of quasi-ideals was introduced by O. Steinfeld [18] and the notion of bi-ideals was given by R. A. Good and D. R. Hughes [5]. These notions can be found in [3] p. 84 and [18] p. 11. It is well known that every quasi-ideal is a bi-ideal.

The problems on a BQ-semigroup, a semigroup whose set of bi-ideals and quasi-ideals coincide, have been studied extensively. In 1968, J. Calais [2] characterized the BQ-semigroups. In 1969, K. M. Kapp [10] introduced the symbol BQ for the class of the semigroups whose set of bi-ideals and quasi-ideals coincide. A BQ-semigroup, which is a semigroup in BQ, was coined by B. W. Mielke [14] in 1972. He used $S \in BQ$ to denote a BQ-semigroup S.

In 1951, Green's equivalence relations on semigroups, which play a fundamental role in the algebraic theory of semigroups, was introduced by J. A. Green [6]. In 1959, B. Kolibiarová [12] defined the equivalence reletion $\mathcal{Q}$ on a semigroup S. In [12] , B. Kolibiarová showed that Green's relation $\mathcal{H}$ and the equivalence relation $\mathcal{Q}$ coincide [see [18], p. 19].

In 1969, K. M. Kapp defined the equivalence relation $\mathcal{B}$ on a semigroup S : $a\mathcal{B}b$, $a, b \in S$ means that $(a)_b = (b)_b$. In 1972, B. W. Mielke described the structure of Green's relations on BQ-semigroups : if $S \in BQ$, then $\mathcal{B} = \mathcal{Q}$.

In 2002, Y. Kemprasit studied the concept of BQ-semigroups on some transformation semigroups. The full transformation semigroup $T(X)$ belongs to BQ, see in [11]. Later in 2006, S. Nenthein and Y. Kemprasit characterized when the semigroup of transformations with invariant set, $S(X,Y)$, belongs to BQ. The semigroup $S(X,Y)$ was introduced by K. D. Magill Jr. [9] in 1966. Furthermore, the concept of BQ-semigroups was studied in other transformation semigroups, see for example [19, 20].

In 1961, S. Lajos generalized the concept of bi-ideals and quasi-ideals to that of (m,n)-ideals and (m,n)-quasi-ideals, respectively. In [13], S. Lajos showed that every (m,n)-quasi-ideal of a semigrup S is an (m,n)-ideal of S. In fact, if S is regular, an (m,n)-ideal of S is also (m,n)-quasi-ideal of S.

In this thesis, we study a semigroup whose set of (m,n)-ideal and (m,n)-quasi-ideal coincide, simply denoted by the (m,n)-BQ-semigroup.

In 1981, R. Tilidetzke defined the equivalence relation $\mathcal{B}_m^n$ on a semigroup S : $a\mathcal{B}_m^n b$, $a, b \in S$ means that $(a)_{(m,n)} = (b)_{(m,n)}$.

The purpose of this thesis is to extend the concept of BQ-semigroups to that of (m, n)-BQ-semigroups and to describe the equivalence relations on (m, n)-BQ-semigroups. Moreover, the (m, n)-BQ property will be discussed in some transformation semigroups.