CHAPTER 4

Conclusion

In this chapter, we conclude all main results of the thesis.

Let $\sigma_t \in Hyp_G(2)$. We denote

$R_1 := \{\sigma_{x_i} | x_i \in X\}$;

$R_2 := \{\sigma_t | t \notin X \text{ and } var(t) \cap X_2 = \varnothing\}$;

$CR(R_3) := \{\sigma_t| \ \ t = f(t_1, t_2) \ \text{ where } \ t_i = x_i \text{ for some } \ i \in \{1, 2\} \text{ and } var(t) \cap X_2 = \{x_i\}\} \ \cup \ \{\sigma_{f(x_1,x_2)}, \sigma_{f(x_2,x_1)}\}$.

It is easily to see that $R_1, R_2, CR(R_3)$ are pairwise disjoint and $\underline{R_1}, \underline{R_2}$ are subsemigroups of $\underline{Hyp_G(2)}$ but $\underline{CR(R_3)}$ is not a submonoid of $\underline{Hyp_G(2)}$.
So we partition $CR_3(R_3)$ by $\{CR_1(R_3), CR_2(R_3)\}$ as follow.

$CR_1(R_3) := \{\sigma_t | t = f(x_1, t') \ where \ t' \in W_{(2)}(X) \ and \ var(t) \cap X_2 = \{x_1\}\}$,

$CR_2(R_3) := \{\sigma_t | t = f(t', x_2) \ where \ t' \in W_{(2)}(X) \ and \ var(t) \cap X_2 = \{x_2\}\}$

and denote

$CR_1'(R_3) := \{\sigma_t | t = f(x_1, t') \ where \ t' \in W_{(2)}(X), \ \ var(t) \cap X_2 = \{x_1\} \ and$
$rightmost(t') \neq x_1\}$,

$CR_2'(R_3) := \{\sigma_t | t = f(t', x_2) \ where \ t' \in W_{(2)}(X) \ \ var(t) \cap X_2 = \{x_2\} \ and$
$lefttmost(t') \neq x_2\}$,

$(MCR)_{Hyp_G(2)} = R_1 \cup R_2 \cup CR_1'(R_3) \cup CR_2'(R_3) \cup \{\sigma_{id}\}$,

$(MCR_1)_{Hyp_G(2)} = R_1 \cup R_2 \cup CR_1(R_3) \cup \{\sigma_{id}\}$,

$(MCR_2)_{Hyp_G(2)} = R_1 \cup R_2 \cup CR_2(R_3) \cup \{\sigma_{id}\}$ and

$(MCR_3)_{Hyp_G(2)} = R_1 \cup R_2 \cup \{\sigma_{id}, \sigma_{f(x_2,x_1)}, \sigma_{f(x_1,x_1)}, \sigma_{f(x_2,x_2)}\}$.

Then we have:

(1) $\underline{CR_1(R_3)}, \underline{CR_1'(R_3)}, \underline{CR_2(R_3)}, \underline{CR_2'(R_3)}$ are completely regular subsemigroups of $\underline{Hyp_G(2)}$.

(2) $\{\underline{(MCR)_{Hyp_G(2)}}, \underline{(MCR_1)_{Hyp_G(2)}}, \underline{(MCR_2)_{Hyp_G(2)}}, \underline{(MCR_3)_{Hyp_G(2)}}\}$ is the set of all maximal completely regular submonoids of $\underline{Hyp_G(2)}$.

From the obtained results in $\underline{Hyp_G(2)}$, we extend and determine all maximal completely regular submonoids of $\underline{Hyp_G(n)}$.

Next, let $\sigma_t \in Hyp_G(n)$, we denote

$CR_1(R_3) := \{\sigma_t | t = f(x_{\pi(1)}, ..., x_{\pi(n)})$ where π is a bijective map on $\{1, ..., n\}\}$.

$E := \{\sigma_t | t = f(t_1, ..., t_n)$ where $t_{i_1} = x_{i_1}, ..., t_{i_m} = x_{i_m}$ for some $i_1, ..., i_m \in \{1, ..., n\}$ and $var(t) \cap X_n = \{x_{i_1}, ..., x_{i_m}\}$ and if $x_{i_l} \in var(t_k)$ for some $l \in \{1, ..., m\}$ and $k \in \{1, ..., n\} \backslash \{i_1, ..., i_m\}$, then $j - most(t_k) \neq x_{i_l}$ for all $j \neq i_l\}$.

For any $\varnothing \neq I \subset \{1, ..., n\}$, let

$CR_I(R_3) := \{\sigma_t | t = f(t_1, ..., t_n)$ where $t_i = x_{\pi(i)}$; $\pi(i) \in I$ for all $i \in I$ and π is a bijective map on I, $var(t) \cap X_n = \{x_{\pi(i)} \mid \forall\ i \in I\}\}$.

$CR'_I(R_3) := \{\sigma_t | t = f(t_1, ..., t_n)$ where $t_i = x_{\pi(i)}$; $\forall\ i, \pi(i) \in I$ and $t_k = x_{\pi(k)}; \forall k \in \{1, ..., n\} \setminus I$ and π is a bijective map on $\{1, ..., n\}\}$,

$(MCR)_{Hyp_G(n)} = R_1 \cup R_2 \cup CR_1(R_3)$,

$(MCR_1)_{Hyp_G(n)} = R_1 \cup R_2 \cup E$ and

$(MCR_I)_{Hyp_G(n)} = R_1 \cup R_2 \cup CR_I(R_3) \cup CR'_I(R_3) \cup \{\sigma_{id}\}$.

Then we have:

(1) $\underline{(MCR)_{Hyp_G(n)}}$ is a completely regular submonoid of $\underline{Hyp_G(n)}$.

(2) $\underline{(MCR_1)_{Hyp_G(n)}}$ is a completely regular submonoid of $\underline{Hyp_G(n)}$.

(3) $\underline{(MCR_I)_{Hyp_G(n)}}$ is a completely regular submonoid of $\underline{Hyp_G(n)}$.

(4) $\{\underline{(MCR)_{Hyp_G(n)}}, \underline{(MCR_1)_{Hyp_G(n)}}\} \cup \{\underline{(MCR_I)_{Hyp_G(n)}} \mid \varnothing \neq I \subset \{1, ..., n\}\}$ is the set of all maximal completely regular submonoids of $\underline{Hyp_G(n)}$.