CHAPTER 1

Introduction

Fixed point theory is a subject which aim to find a fixed point, that is, a point $x \in X$ such that $x = T(x)$ for some mapping T on a nonempty set X with some conditions on T and/or X.

The remarkable one who established the following fixed point theorem is Stefan Banach.

Theorem 1.1. *[2] (Banach contraction principle) Let* (X, D) *be a complete metric space and* $T : X \to X$ *be a mapping. If there exists* $\lambda \in [0, 1)$ *such that*

$$D(Tx, Ty) \leq \lambda D(x, y) \text{ for all } x, y \in X,$$

then T *has a unique fixed point.*

After his work, several authors have generalized, improved and extended Banach contraction principle in many directions (see [3–5, 7, 8, 12, 14, 15]). Among these works, we are interested in the work of Karapinar [12]. He presented a new contraction called generalized α-ψ-Geraghty contractive type mapping and proved some fixed point theorems in complete metric spaces.

Moreover, in 2014, Jleli, Karapinar and Samet [10] introduced JKS-contraction and proved its fixed point existence in the complete rectangular spaces.

Another way of extending Banach contraction principle is replacing metric space with some weaker abstract spaces. In 2016, Roldán and Shahzad [17] discovered a new class of spaces called RS-generalized metric spaces. This class is claimed to cover many classes of spaces such as standard metric spaces, dislocated metric spaces, b-metric spaces, B_N-spaces and JS-generalized metric spaces.

In this thesis, we obtain fixed point theorems of generalized α-ψ-Geraghty contractive type mappings and JKS contraction in the framework of RS-generalized metric spaces.