CHAPTER 2

THEORY

2.1 Gamma Ray

The gamma ray log, which is normally annotated as GR, is a type of geophysical log that measures the total natural gamma radiation emitting from a formation. Such emission comes from potassium and uranium, radium and thorium (Glover, 2000). The geological significance of radioactivity lies in the distribution of those elements. The majority of rocks on earth are radioactive to some level, including igneous, metamorphic, and sedimentary rocks. Shale often has high gamma-ray emission. Some heavy minerals in sand can also have high gamma-ray readings (Figure 2.1).

Turning to its principal uses, this log can be useful both qualitatively and quantitatively (Rider, 2000).

Qualitative uses:

1. Facies and sequence correlation.
2. Lithology identification (shalyness).
3. Indication of clay mineral types.
4. Indication of depositional environment
5. Fracture identification
6. Source rock localization

Quantitative uses:

1. Shale volume calculation
2. Radioactive material calculation

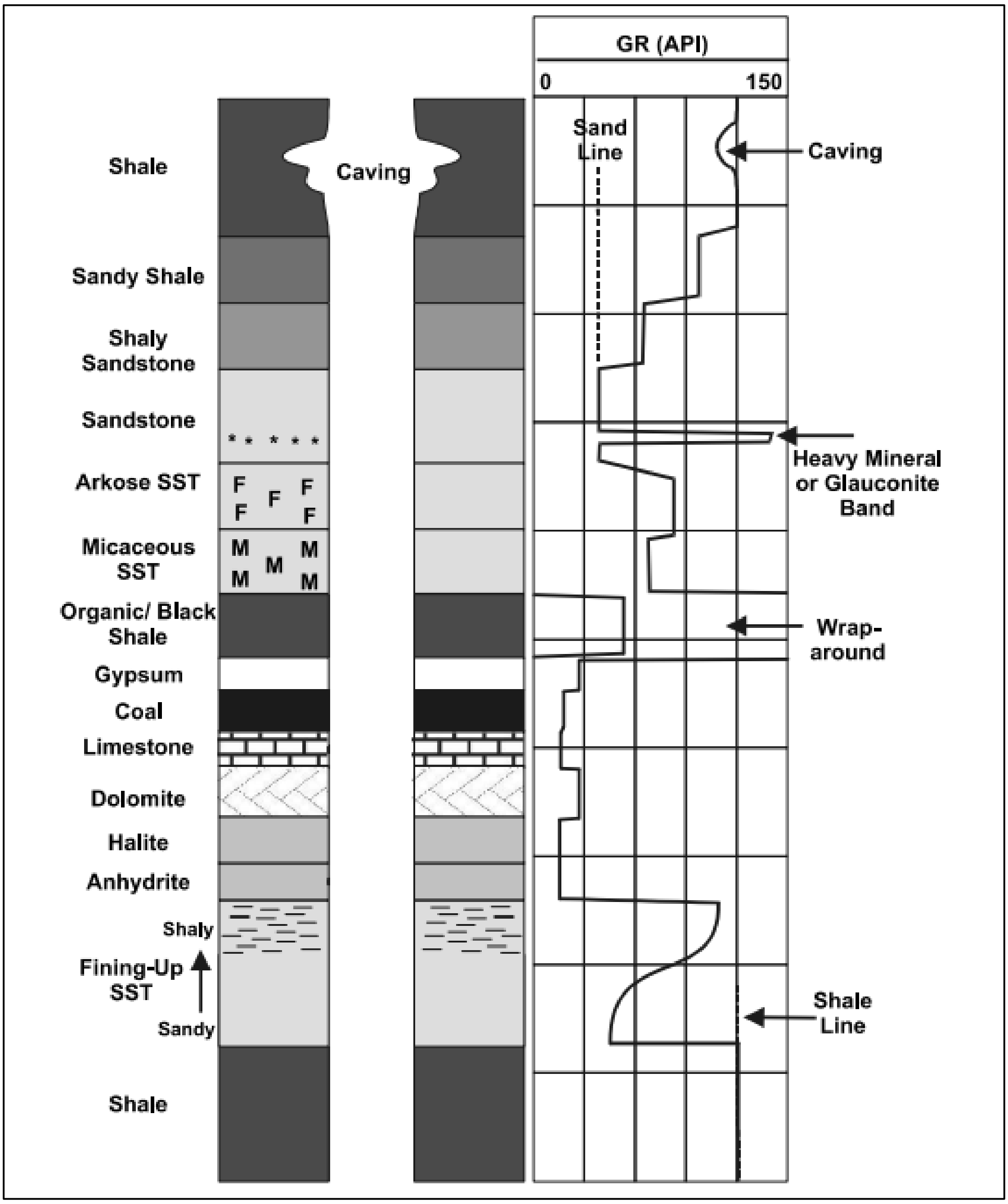

Figure 2.1. Gamma ray log common response in certain lithologies (modified from Glover, 2000).

2.2 Sonic

The sonic log, which is known also as an acoustic log, measures an elastic wave travel time through a rock formation. The sonic log tool measures the time needed for an elastic wave (pulse) to propagate from a transmitter to 2 different receivers. The characteristic of the outgoing pulse is high amplitude and very short duration. The pulse propagates through the formation while experiencing attenuation and dispersion. At the time the energy pulse touches the receiver after passing through the rock, the different forms of sound wave (V_P, V_S, tube wave) arrive at different times. This happens because different types of wave propagate at different velocities in the rock (Glover, 2000).

The principal uses of this log are as follows (Rider, 2000)

Qualitative uses:

1. Lithology indication
2. Source rock identification
3. Normal compaction trend indication
4. Overpressure indication
5. Lithology correlation
6. Fracture identification, to some extent.

Quantitative uses:

1. Evaluating porosity
2. Giving interval velocities and velocity profile as an aid to seismic interpretation
3. When multiplied by density, it produces an acoustic-impedance log which will be used to make a synthetic seismic trace.

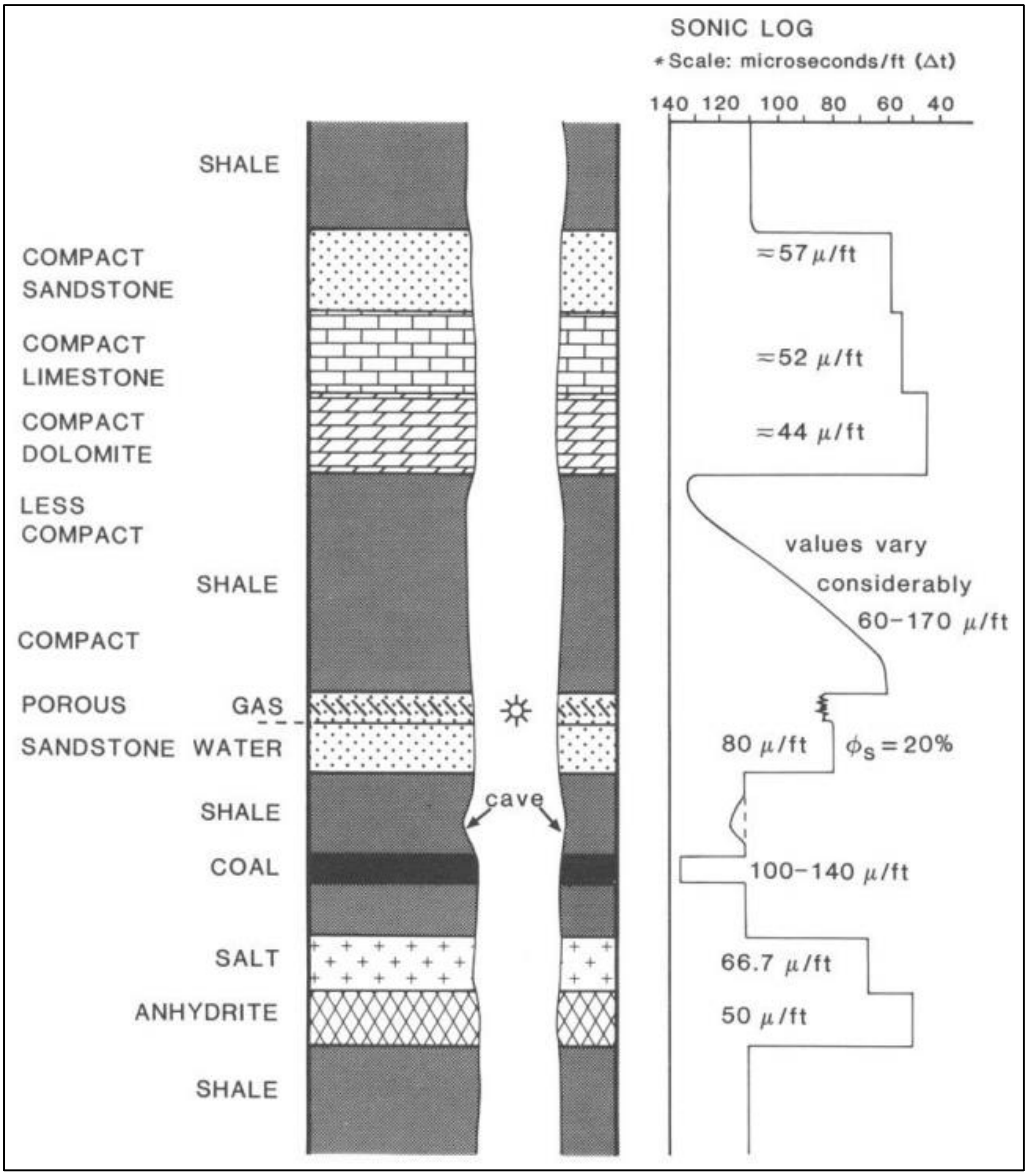

Figure 2.2. Typical responses of the sonic log for certain lithologies (modified from Rider, 2000).

2.3 Density

As per its name, the density log measures the bulk density of a formation. Such measurement represents the overall density of a rock, including the solid matrix and the fluid filling the pores. The tool works by bombarding the formation with radiation, then measures the amount radiation returning to a sensor.

Turning to details, the gamma rays propagate through the rock and experience Compton scattering due to the interaction with electrons in the atoms. This phenomenon lowers the gamma-ray energy in a step-wise manner then scatters in all directions. In the case of gamma rays whose energy is below 0.5 MeV, there is a possibility of photo-electric absorption happening by interaction with the atomic electrons. The amount of gamma ray flux that arrives at each of the two detectors is, therefore, attenuated by the formation, and the amount of attenuation is dependent upon the density of electrons in the formation (Grover, 2000).

The principal uses of this log are as follows (Rider, 2000).

Qualitative uses:

1. Lithology indicator
2. Certain mineral identification
3. Organic matter content assessment of source rock
4. Overpressure identification
5. Fracture porosity identification

Quantitative uses:

1. Porosity calculation
2. Indirect hydrocarbon density calculation
3. Acoustic impedance calculation for generating synthetic seismogram

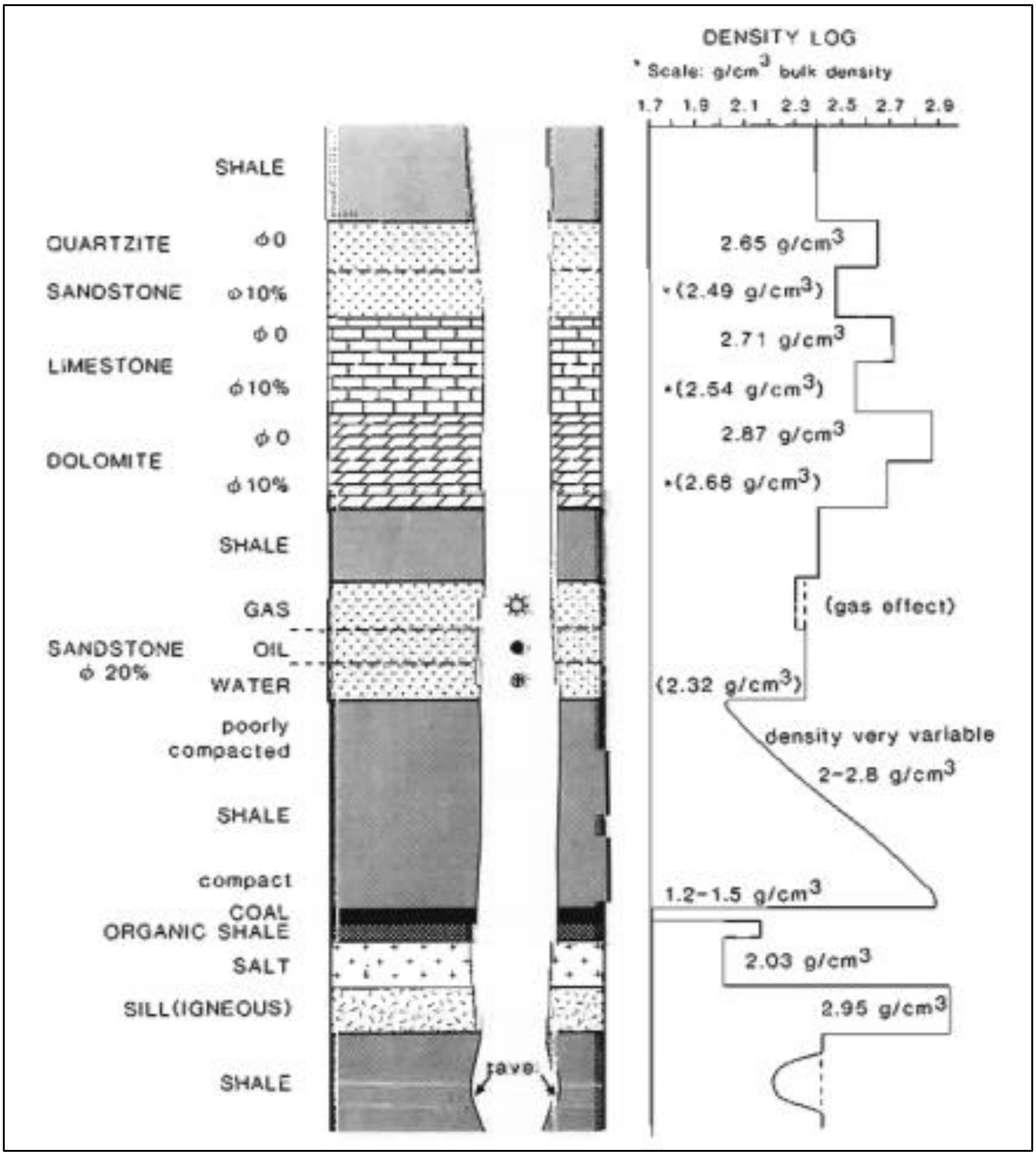

Figure 2.3. Typical responses of the density log towards certain lithologies (modified from Rider, 2000).

2.4 Resisitivity

The resistivity log measures the resistivity of the formation, which is its resistance to the passage of an electric current. The majority of rock materials are insulators while their enclosed formation water is a conductor, and hydrocarbon is an insulator. Where the pores in the formation are filled by brine, low resistivity will be recorded. In contrast, if the pores are filled by hydrocarbon, the resistivity will be high.

Formation resistivity often ranges from 0.2 to 1000 ohm-m. In permeable formations, it is rare to have resistivity above 1000 ohm-m. However, this case may be observable in impervious, very low porosity formations such as evaporites (Rider, 2000).

The principal uses of the resistivity log are as follows (Rider, 2000).

Quantitative uses:

1. Calculating fluid saturations with the purpose of knowing formation water resistivity (Rw), mud-filtrate resistivity (Rmf), porosity (ϕ), and temperature of formations.

Semi-quantitative and qualitative uses:

1. Understanding the facies and bedding characteristics to know gross lithology
2. Understanding the compaction, overpressure, and shale porosity by looking at the normal compaction trend
3. Source rock identification and maturation.

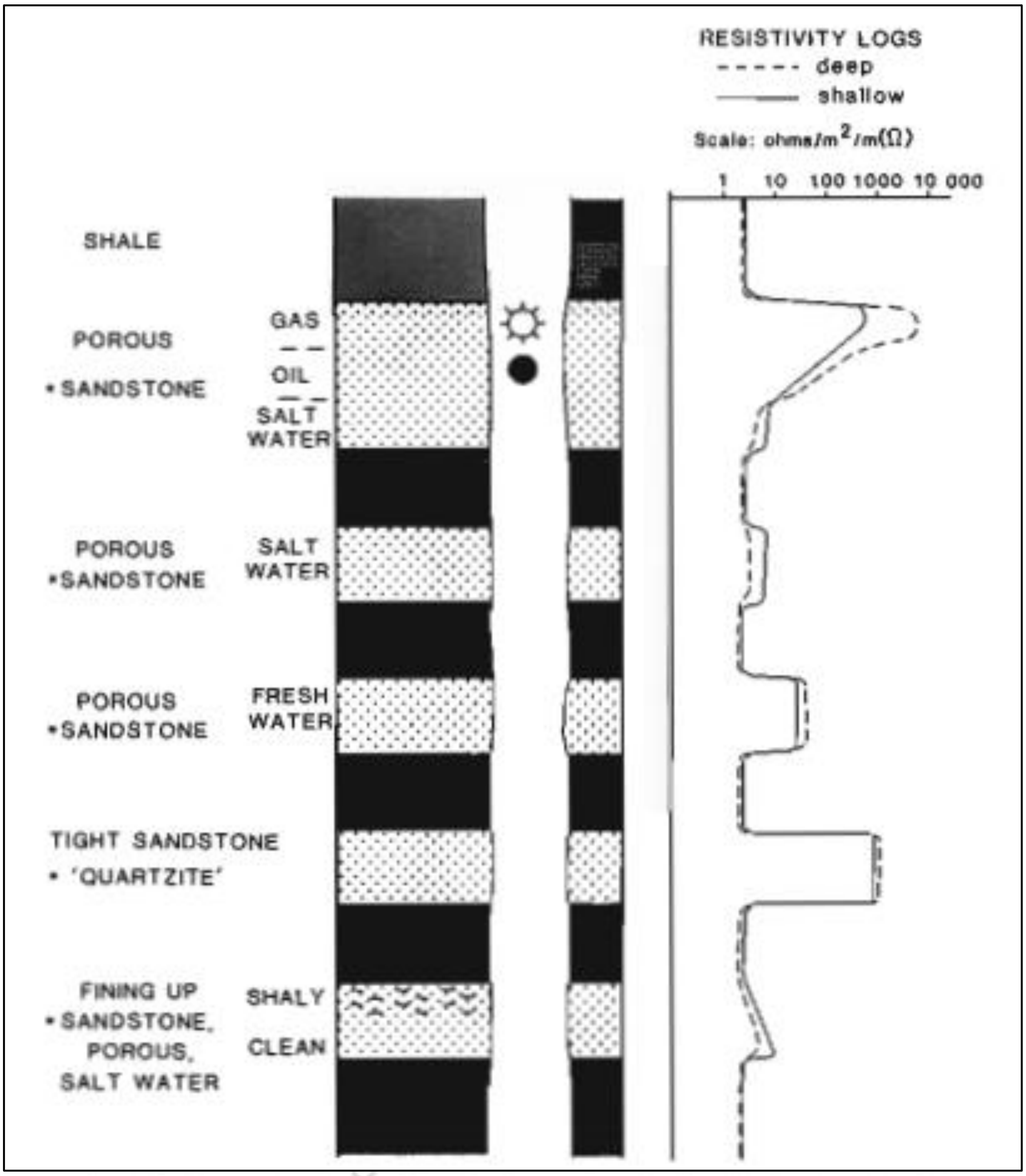

Figure 2.4. Typical responses of the resistivity log towards various fluid types in sandstone (modified from Rider, 2000).

2.5 Empirical Wyllie's Relationship

A number of empirical rock physics models have been based on experimental results. One of the most frequent empirical models used is Wyllie's equation. Wyllie et.al (1958) derived a relationship between velocity and porosity fitting the data from well-consolidated sandstone and limestone. This relation is substantially intuitive rather than referring to physical principles. It is commonly written as

$$t = \Phi t_{fl} + (1 - \Phi) t_0 \qquad (2.1)$$

Where,

ϕ : Porosity

t : Interval transit time in the rock

tfl : Interval transit time in the fluid

t0 : Interval transit time in the mineral matrix

2.6 Well-to-seismic tie

Well log data is in the depth domain while seismic data is in time. With the purpose of extracting information from those two, it is essential to integrate well and seismic data. This process is called well seismic tie.

The primary reason to tie the well is to determine the time-depth function. Additionally some typical objectives of tying wells to seismic data are listed below (Simm and Bacon, 2014).

1. Zero phasing: to check the phase of the seismic data whether it is zero phase and adjust the phase if needed.
2. Identifying horizon: to correlate the marker from well with the right event on the seismic data.
3. Wavelet extraction for seismic inversion or synthetic seismogram generation.
4. Offset scaling: to check the amplitude behavior for the purpose of AVO analysis.

The typical workflow of performing well seismic tie is shown in Figure 2.5.

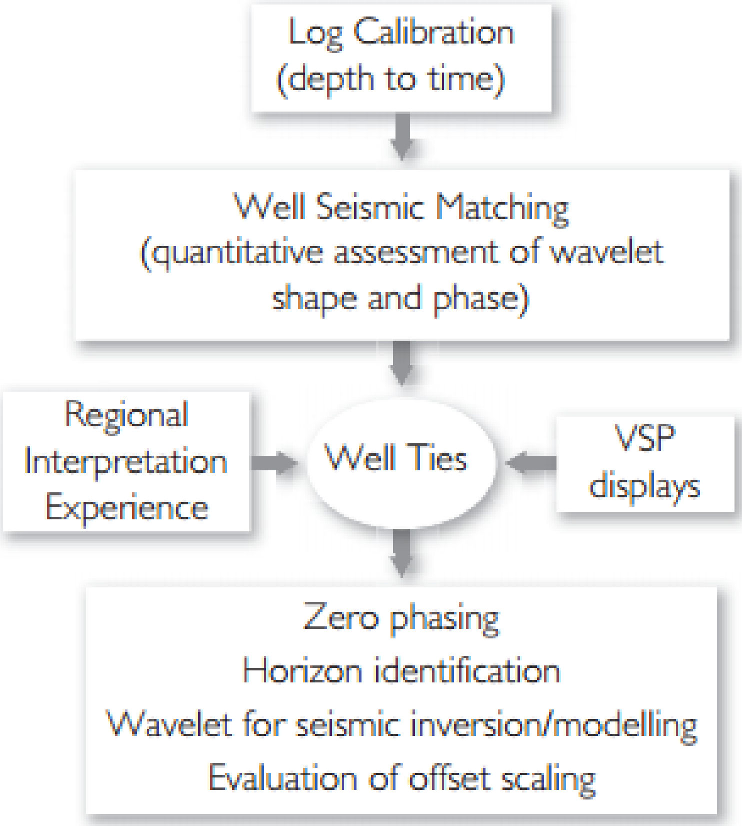

Figure 2.5. The schematic diagram of well seismic tie process (modified from Simm and Bacon, 2014).

One of the key parts of a well-to-seismic tie is to convert depth to time. Apart from VSP (vertical seismic profiling), the typical data involved in this stage is checkshot data. These data are measured from the direct arrival of a seismic wave reaching the receiver which is located inside the bore hole. The checkshot correction, which is often called 'log calibration,' resolves the uncertainty in the integrated sonic log, caused by the lack of sonic logs in the shallow section. In general the workflow is summarized as follows (Figure 2.6) (Simm and Bacon, 2014).

1. Integrate the velocity log from the uppermost or lowermost checkshot that ties the log.
2. Calculate the drift curve which is the difference between the check shots and the integrated sonic log.
3. Quality-control the checkshot data using drift points and checkshot velocities.

4. In case checkshot data is de-selected, it is necessary to come back to step 2.
5. Fit a curve to the drift points expecting the differences of the final time–depth relation and the integrated sonic log are less than approximately 2 ms.
6. Apply the drift correction to the time–depth curve from the integrated sonic log. It is also possible to apply the correction to the velocity log with the aim of generating a calibrated sonic log.
7. Evaluate the corrections effect on the sonic log. In case there is a large difference between the un-calibrated and calibrated sonic logs, it is crucial to double-check every step.

To start a well-seismic-tie process, it is essential to have at least the density and sonic log. These two logs are multiplied to get an acoustic impedance log which is then transformed to reflectivity. Up to this stage, the wavelet is needed. To estimate correct wavelet for horizon interpretation, zero phase phasing design, inversion operator, a pragmatic approach is to estimate the wavelet directly from seismic data (Simm and Bacon, 2004). Once the wavelet has been estimated, the next stage is to convolve it with the reflectivity series becoming synthetic seismogram. This synthetic seismogram is now compared with the composite trace which is extracted from seismic data. Typically a minor bulk shift is needed if the time depth has been corrected with checks hot. In some cases, stretching and squeezing are required to get a best matching between well and seismic data. The interpreter, however, needs a degree of skepticism in regards to modifying the depth-time relationship and should investigate the possible reason for the mistie prior to conducting adjustment. The reason may be due to structural imaging (Simm and Bacon, 2014).

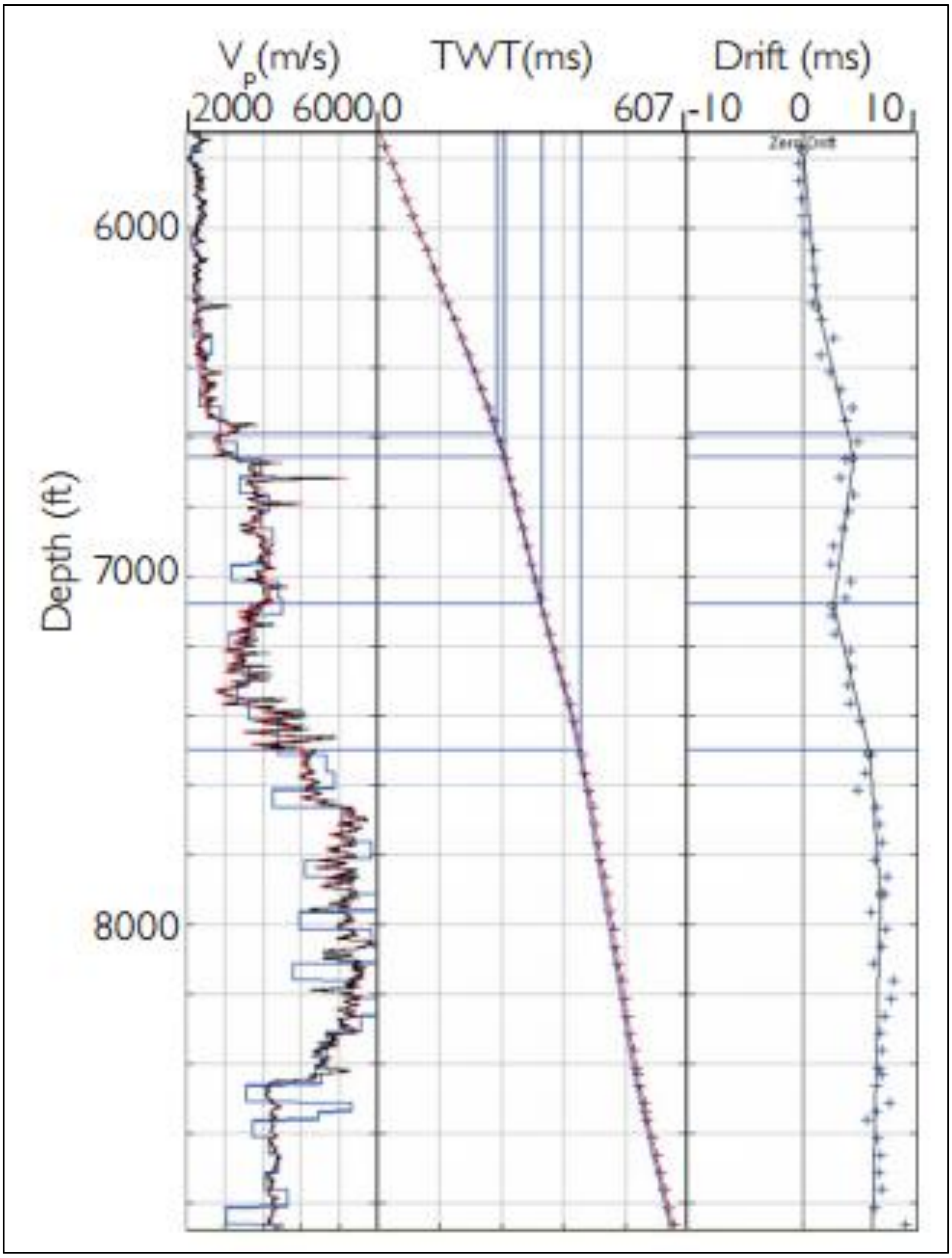

Figure 2.6. Log calibration: column 1, black–Vp log, red –calibrated velocity log, blue–velocities from checkshots; column 2, blue–integrated depth–time curve fromVp, red –calibrated depth-time curve (i.e. with drift applied); column 3, blue crosses–drift points, black–drift curve fitted to the data using linear segments with knee points (modified from Simm and Bacon, 2014).

2.7 Seismic Inversion

The most basic and commonly used one-dimensional model for the seismic trace which is referred to as convolutional model stated that seismic trace is the result of a convolution of the reflectivity of the earth with the seismic source function (wavelet), plus the addition of some noise component (Russel, 1990). This is commonly known as ‘forward modeling’. Mathematically the definition can be written as follows.

$$s(t) = w(t) * r(t) + n(t) \qquad (2.2)$$

s(t) : Seismic trace
w(t) : Seismic wavelet
r(t) : The reflectivity of the earth
n(t) : Noise component

The seismic trace provides information regarding the interface of the layers. It does not help with understanding the property of the layer itself. Therefore there is a need to invert the seismic data back to the original domain that can tell us the rock properties in the subsurface. This process is broadly known as ‘seismic inversion’. The idea is to eliminate (or reduce) the effect of the wavelet while estimating acoustic impedances in layers. This can be more useful than seismic data by presenting impedances in geologically recognizable layers. Inverting the data back to absolute impedance fundamentally relies on well data for its parameterisation. Hence, such work is commonly done for the purpose of reservoir characterization when developing a hydrocarbon accumulation. The illustration of such ‘inverse modeling’ can be seen in Figure 2.7 below.

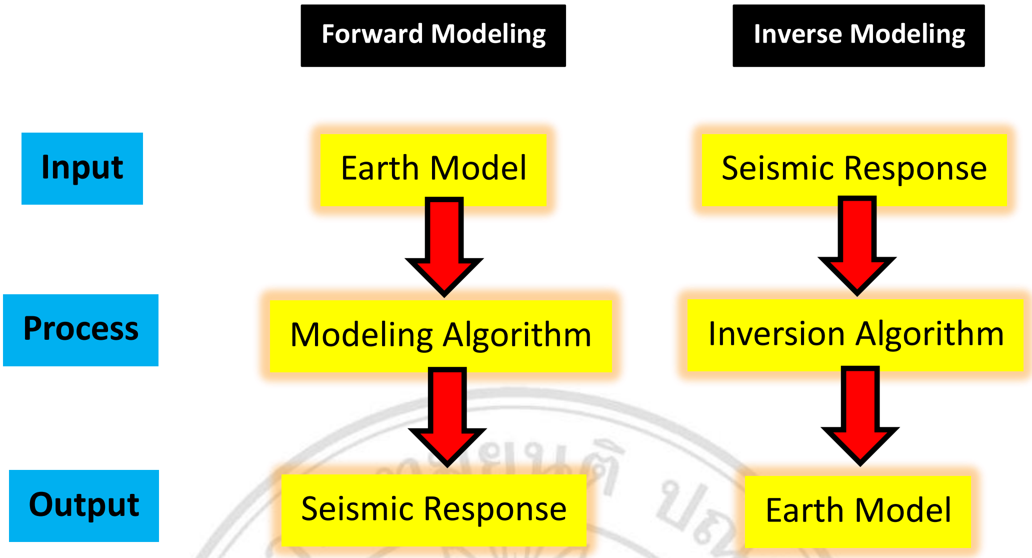

Figure 2.7. The schematic comparison between "Forward" and "Inverse" modeling

2.7.1 Deterministic Inversion

A number of algorithms associated with deterministic inversion have been introduced. Up to now, there is no obvious agreement stated that one particular approach to inversion is better than another. A careful evaluation of every step of the workflow of each type of inversion is perhaps far more important than the type of algorithm used (Simm and Bacon, 2014).

The inversion method that will be emphasized here is model-based inversion, which will be used in this study. This type of inversion is broadly known as the most popular algorithm used in the oil industry. It is available in commercial software such as Hampson-Russell.

This particular inversion uses an iterative forward modeling and comparison procedure (Veeken and Da Silva, 2004). The initial model can come from the interpolation of well data with a low-pass filter applied and guided by horizons to define the geological trend. Alternatively, a seismic stacking or migration velocity cube can be another source for generating this starting model. This interval velocity can be converted to acoustic impedance by assuming an empirical velocity-density relation.

In Hampson-Russell software, two options are available to constrain the impedance solution so that it will be geologically reasonable: hard-constraint and soft-constraint. With hard constraint, it is possible to set the limit as a maximum fractional change from the starting initial model values. In the case when the final inversion result is close to the hard boundary over large 2WT ranges, it will inform the user that the initial model and the seismic data are not consistent. In that case it is crucial to double-check the well-to-seismic tie to determine that it is correct. With soft-constraint, on the other hand, an adjustable weighting parameter is available to determine the relative weight given to deviations of both impedances from the starting point and the synthetic error. To choose the best parameter in this case, it is necessary to do some trials (Simm and Bacon, 2014).

A schematic workflow of model-based inversion is shown in Figure 2.8.

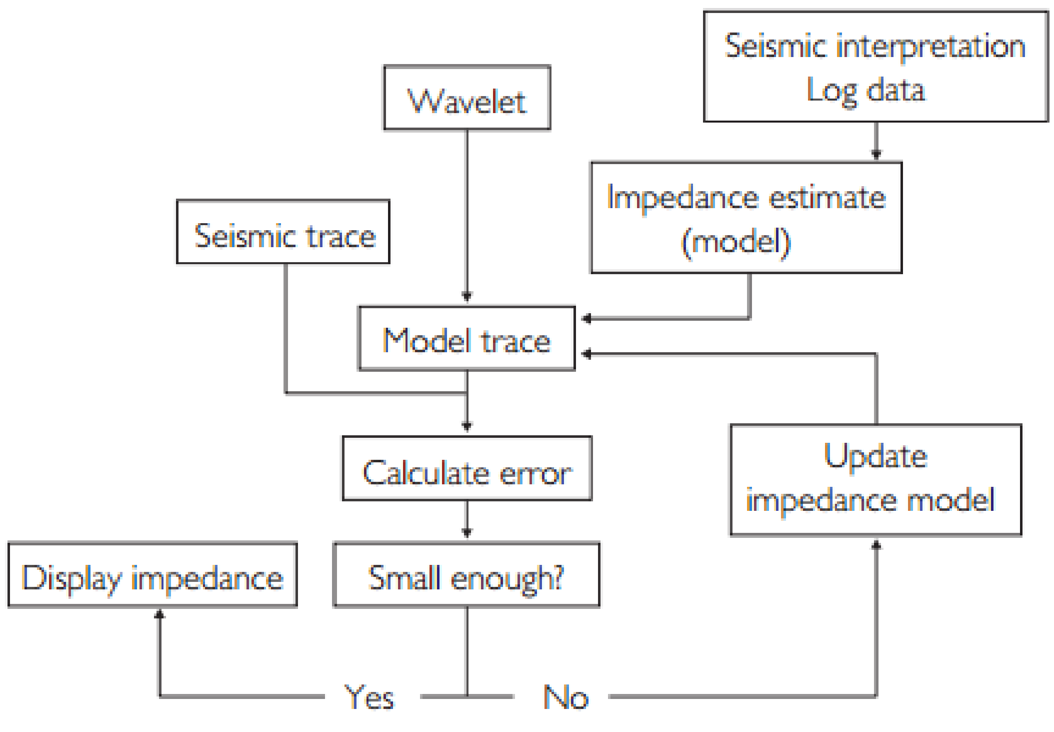

Figure 2.8. The typical workflow of model-based inversion (modified from Simm and Bacon, 2014)

2.7.2 Stochastic Inversion

Unlike deterministic inversion, which tries to get a minimized solution towards the inverse problem, a stochastic inversion algorithm honors the geostatistics (variability) of the well data, statistical properties of impedance, and any spatial constraints. Stochastic inversion is mainly run with the aim of understanding the uncertainty in seismic inversion work and by doing the stochastic inversion, the user is able to see the effect of this physical uncertainty on the lithology, porosity or reservoir volumes in the final result of impedance.

Apart from the low-frequency in the initial model, one of the essential inputs to the stochastic algorithm is the 'error grid'. This is the standard-deviation map obtained by simple kriging of the well locations. This results in a map with values increasing while moving away from the well, up to the radius of influence (the variogram range). Typically, the error grid is intentionally set to be strong (the seismic constraint weak) close the well location, marked by small values of the error grid and vice versa (OpendTect Manual). It is critical that the variogram determines the variance in the vertical direction and the connectivity of the impedance realization in horizontal directions. While the mean and standard deviation within each layer determine the low-frequency one, the vertical variogram constrains the high-frequency information in the realization (Simm and Bacon, 2014).

According to Simm and Bacon (2014), stochastic impedance realization commonly delivers 3 products, as follows.

1. The probability of a certain property at a certain location: Low-impedance oil sands with impedance below a particular threshold, for instance.
2. The geostatistical distributions of volume and area.
3. The connectivity indication of a certain area: Low–impedance gas area, for example.

Acoustic impedance estimated from seismic inversion can be an effective technique for estimating the petrophysical properties of a reservoir. To use this technique, it is crucial to not assume that the seismic signal is stationary. In most cases, however, the wavelet is assumed to be constant. Though the reflectivity series might still be useful, the incorrect assumption will definitely end up with an incorrect estimate. Naghadeh et al. (2017) introduced a technique called 'Stochastic Gabor Reflectivity' to overcome this problem. This method applies non-stationary deconvolution to eliminate time-varying wavelet effects from the signal and to convert the estimated reflection series to absolute AI by getting bias from well logs.

The Gabor transform derives the signal's time frequency analysis and estimated wavelet properties from different windows. The benefit of using different time windows is that to give an ability to generate a time variant-kernel matrix which will be used to eliminate the matrix effects from seismic data. The result of such process is basically the reflectivity not following the stationary assumption. Additionally getting the bias from the well can help to estimate a more reliable AI.

The following equation is introduced with the purpose of estimating AI which is the product of density and velocity from seismic data.

$$J(R) = \lambda_1 \|S - WR\|_2^2 + \lambda_2 \|M - LR\|_2^2 \tag{2.3}$$

Where the weighting factor $\lambda_1 + \lambda_2 = 1$, M is the initial AI model, L is the integration matrix to change RC to AI. Since $R \approx 0.5\Delta \ln(AI)$ also

$$\begin{cases} \Delta = D \\ \ln(AI) = L_z \end{cases} \tag{2.4}$$

Where D is the differential matrix and L_z is the logarithm of AI. Combining the two previous equations will be

$$J(L_z) = \lambda_1 \|S - 0.5WDL_z\|_2^2 + \lambda_2 \|S - 0.5LDL_z\|_2^2 \tag{2.5}$$

The equation above consists of two parts. Minimizing the first part makes a solution to honor seismic data. Minimizing the second part makes a solution to honor AI model. If $\lambda_2 = 0$, the minimizing objective function becomes the constrained model based inversion and hard constraints are then used to bound the minimum and maximum AI prediction. If $\lambda_2 \neq 0$, the minimizing objective function forces the solution to be a stochastic model inversion, soft constraints will be considered.

$$\frac{\partial J}{\partial L_z} = \lambda_1 D^T W^T (S - 0.5WDL_z) + \lambda_2 D^T L^T (S - 0.5LDL_z) \qquad (2.6)$$

Where D^T, W^T and L^T are transposes of D, W and L respectively. To minimize J equation (eq. 2.4) is going to be.

$$\lambda_1 D^T W^T S - 0.5\lambda_1 D^T W^T WDL_z + \lambda_2 D^T L^T M - 0.5\lambda_2 D^T L^T LDL_z = 0 \qquad (2.7)$$

Finally to get L_z

$$L_z = \frac{\lambda_1 D^T W^T S + \lambda_2 D^T L^T M}{0.5\lambda_1 D^T W^T WD + 0.5\lambda_2 D^T L^T LD} \qquad (2.8)$$

To solve equation (eq. 2.6), it is essential to have properties of W and also an initial AI model which is the interpolated low frequency AI between wells guided by horizons.

Moving to the Stochastic Gabor reflectivity and AI inversion, It is possible to introduce an objective function as follows.

$$J(R) = \lambda_1 \|S - WR\|_2^2 + \lambda_2 \|S_r - WR\|_2^2 \qquad (2.9)$$

Where $S_r = WR_r$ and R_r is reference RC at the well location. In order to get the RC from deconvolution very close to that of R_r at the well location then:

$R_r - R = e_r \cong 0, e_r$ is the amount of error due to noise affecting wavelet extraction, hence we would like to attain $e_r = 0$. In order to get stochastic reflectivity inversion, it is essential to minimize the objective function related to RC.

$$\frac{\partial J}{\partial R} = 0 \Rightarrow R = \frac{\lambda_1 W^T S + \lambda_2 W^T S_r}{W^T W + mu \times diag((abs(R_r) + \varepsilon)^{-1})} \qquad (2.10)$$

where *mu* and ε are small positive regularization parameters. The added term will lead the inversion result to be sparse. To get stochastic Gabor AI:

$$J(R) = \lambda_1 \|S - WR\|_2^2 + \lambda_2 \|S_r - WR\|_2^2 \quad (2.11)$$

Using the equation (eq.2.2) to replace R with AI gives:

$$J(L_z) = \lambda_1 \|S - 0.5WDL_z\|_2^2 + \lambda_2 \|S_r - 0.5WDL_z\|_2^2 \quad (2.12)$$

Minimizing the objective function gives

$$L_z = \frac{\lambda_1 D^T W^T S + \lambda_2 D^T W^T S_r}{0.5 D^T W^T WD} \quad (2.13)$$

Since the denominator could be a singular matrix, regularization is useful to stabilize the inversion, so equation (eq.2.10) will be

$$L_z = \frac{\lambda_1 D^T W^T S + \lambda_2 D^T W^T S_r}{0.5 D^T W^T WD + mu \times diag((abs(R_r) + \varepsilon)^{-1})} \quad (2.14)$$