CHAPTER 5

Numerical Simulation

In this chapter, the numerical examples are presented to verify the theoretical results in Theorems 4.1.2- 4.2.5.

Many numerical methods have been applied to solve nonlinear fractional differential equations such as the Adams method [83], the predictor corrector method [84], the Adomian decomposition method [85], the extrapolation method [86], and the variational iteration method [87]. However, among these methods, the Adams method is useful for studying the long term behavior of the solutions of nonlinear fractional differential equations [88]. Thus, in this study, the Adams method was used for solving the model with the Matlab software.

For the Model 1, we present four numerical examples to demonstrate the stability of all equilibrium points E_{10}, E_{11}, E_{12} and E_{13} by using the model parameters of cases 1-4 as shown in Table 5.1. The numerical simulations are shown in Figures 5.1-5.8.

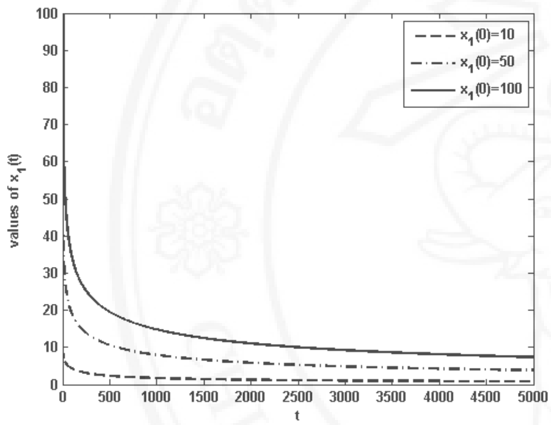
Table 5.1: The parameters of numerical examples for the Model 1

Cases	Parameters							
	r_1	r_2	e_1	e_2	K_1	K_2	b_{12}	b_{21}
1	0.20	0.10	0.30	0.25	200	200	0.10	0.05
2	0.50	0.10	0.30	0.25	200	200	0.10	0.05
3	0.20	0.40	0.30	0.25	200	200	0.10	0.05
4	0.50	0.40	0.30	0.25	200	200	0.10	0.05

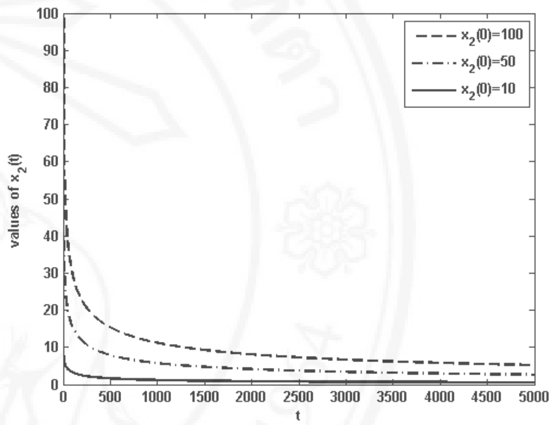
In Figures 5.1, 5.3, 5.5, and 5.7, we set $\alpha = 0.5$ and choose three different initial conditions as follows: $(x_1(0), x_2(0)) = (10, 100), (50, 50), (100, 10)$. In Figures 5.2, 5.4, 5.6, and 5.8, we fix the initial condition $(x_1(0), x_2(0)) = (50, 50)$ and choose various values of $\alpha = 0.3, 0.5$, and 0.7.

Figures (5.1a), (5.1b), (5.2a), and (5.2b) show the relationship between the values of the species x_1 with respect to times t and x_2 with respect to times t, respectively. Figures (5.1c) and (5.2c) present dynamic behaviors of two interacting species.

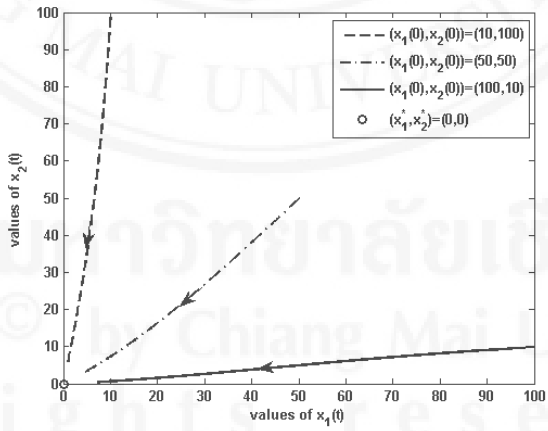
We obtain the relationship between the values of the species x_1, x_2 with respect to times t in Figures 5.3 - 5.8 (a). The relationship between the values of the species x_1 and species x_2 are shown in Figures 5.3 - 5.8 (b).



(5.1a)

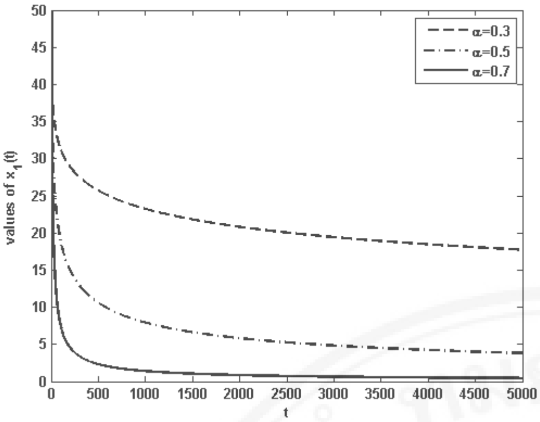


(5.1b)

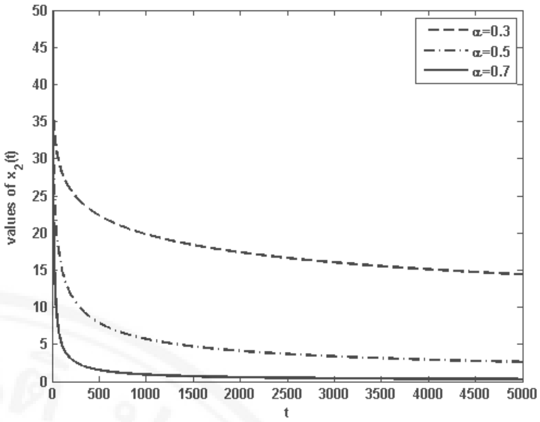


(5.1c)

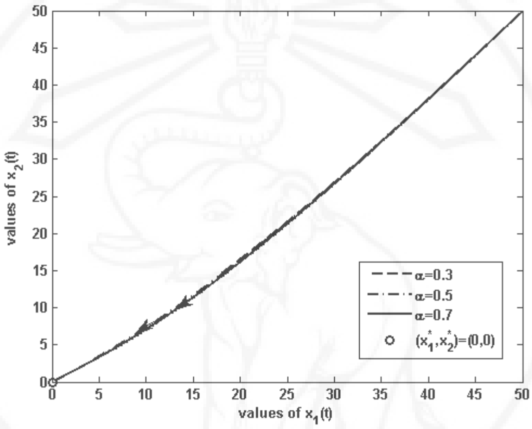
Figure 5.1: (5.1a), (5.1b) The relationship between the numbers of individuals of species x_1 with respect to time t, and the relationship between the values of x_2 with respect to time t, respectively. (5.1c) the dynamic behavior of two interacting species with $\alpha = 0.5$ for $(x_1(0), x_2(0)) = (10, 100), (50, 50)$, and $(100, 10)$.



(5.2a)



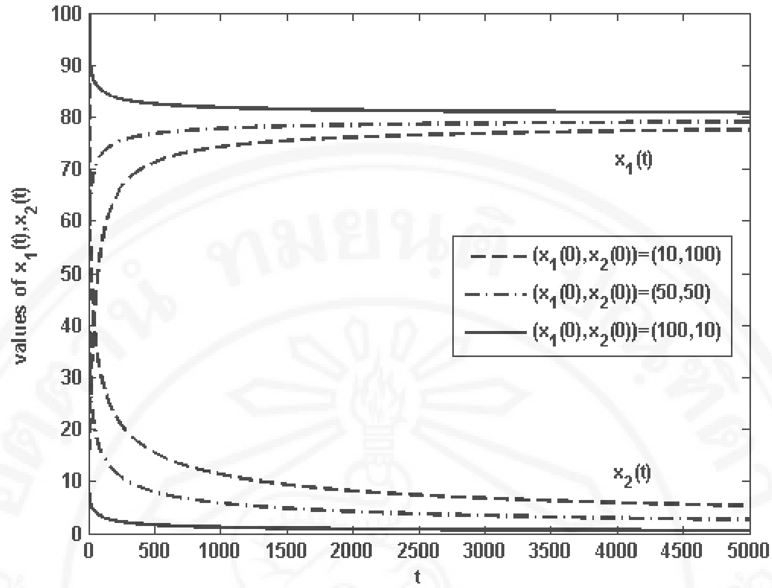
(5.2b)



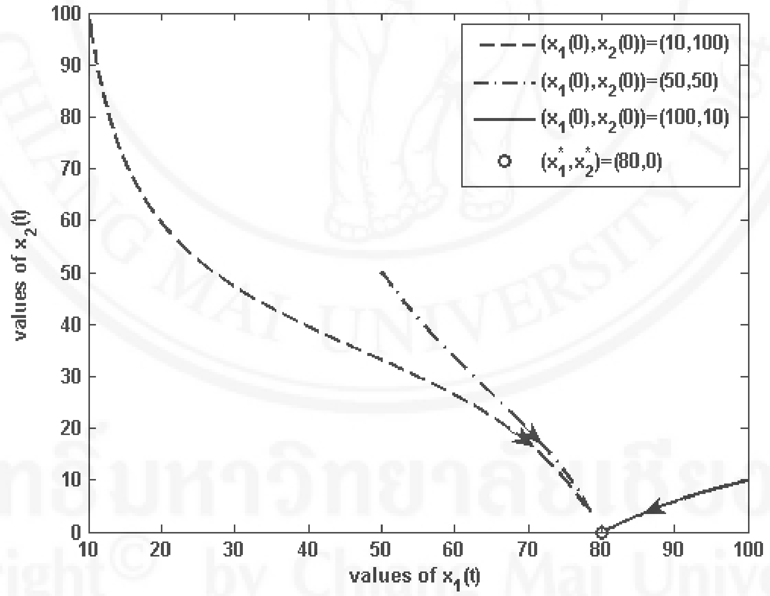
(5.2c)

Figure 5.2: (5.2a), (5.2b) The relationship between the numbers of individuals of species x_1 with respect to time t, and the relationship between the values of x_2 with respect to time t, respectively. (5.2c) the dynamic behavior of two interacting species with initial condition $(x_1(0), x_2(0)) = (50, 50)$ for $\alpha = 0.3, 0.5$ and 0.7.

The model parameters in Case 1 as shown in Table 5.1 satisfy the sufficient conditions of Theorem 4.1.2. Hence, $E_{10}(0, 0)$ is locally asymptotically stable. From Figures 5.1 and 5.2 we can see that all solutions of the system (3.0.7) converge to the equilibrium point $E_{10}(0, 0)$.

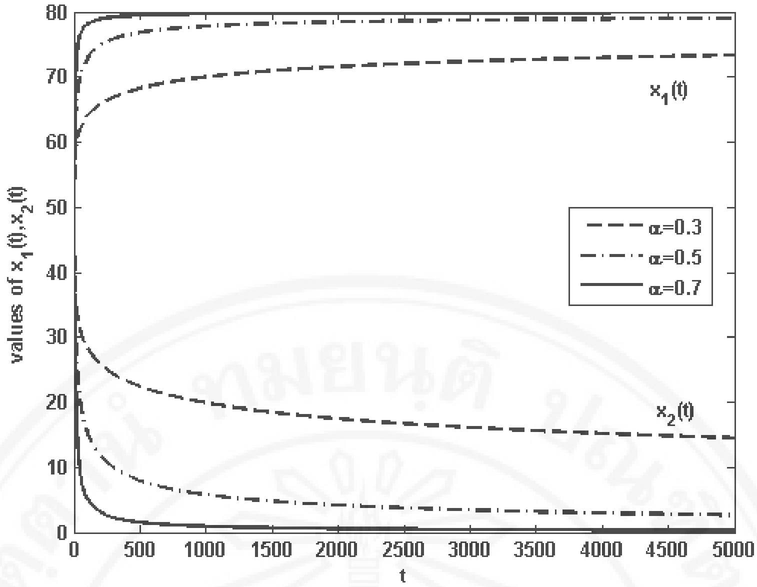


(5.3a)

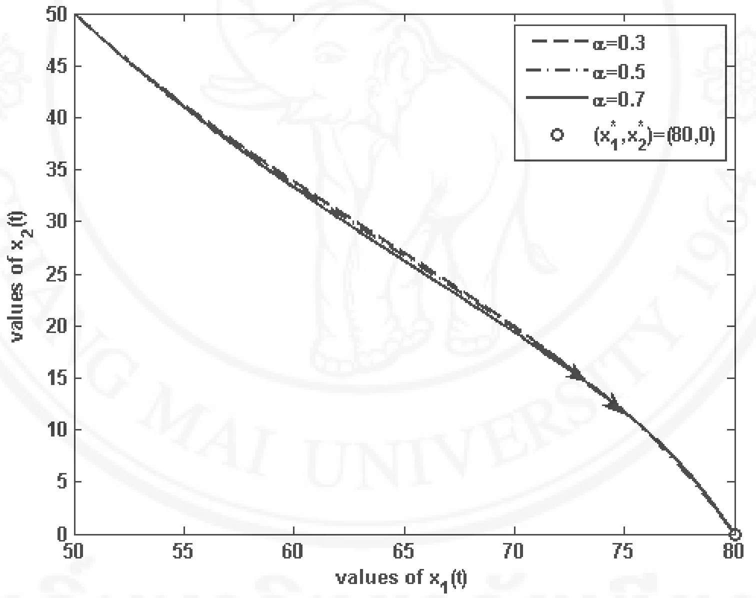


(5.3b)

Figure 5.3: (5.3a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.3b) the dynamic behavior of two interacting species with $\alpha = 0.5$ for $(x_1(0), x_2(0)) = (10, 100), (50, 50), \text{and } (100, 10)$.



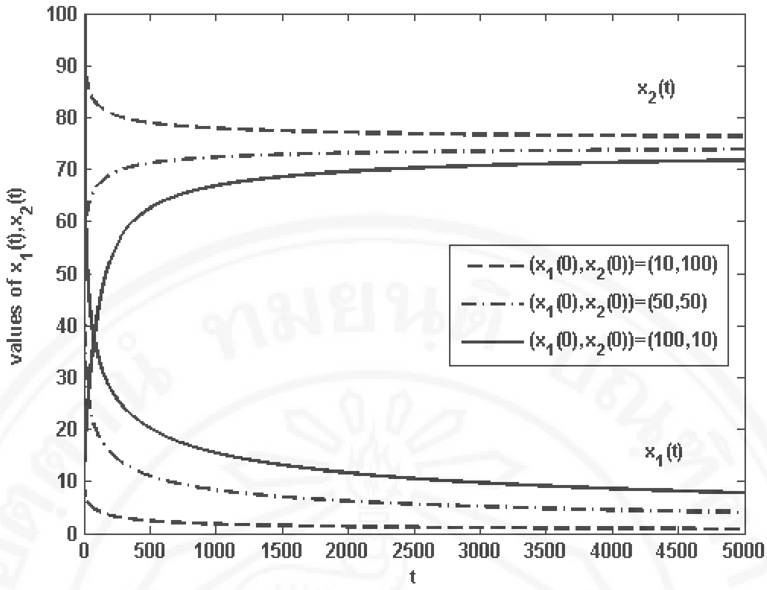
(5.4a)



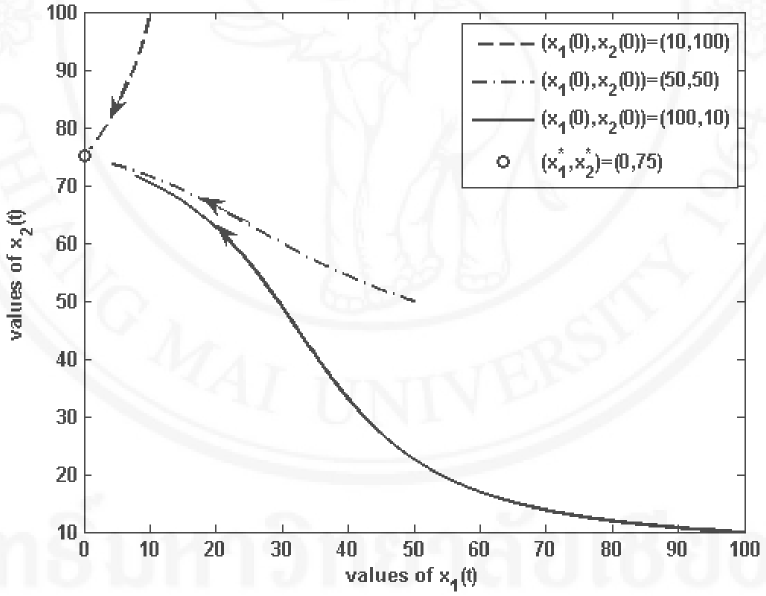
(5.4b)

Figure 5.4: (5.4a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.4b) the dynamic behavior of two interacting species with initial condition $(x_1(0), x_2(0)) = (50, 50)$ for $\alpha = 0.3, 0.5$ and 0.7.

The values of parameters in Case 2 as shown in Table 5.1 satisfy all the conditions of Theorem 4.1.3. From Figures 5.3 and 5.4, we can obtain that all solutions of the system (3.0.7) converge to the equilibrium point $E_{11}(80, 0)$. Thus, $E_{11}(80, 0)$ is locally asymptotically stable.

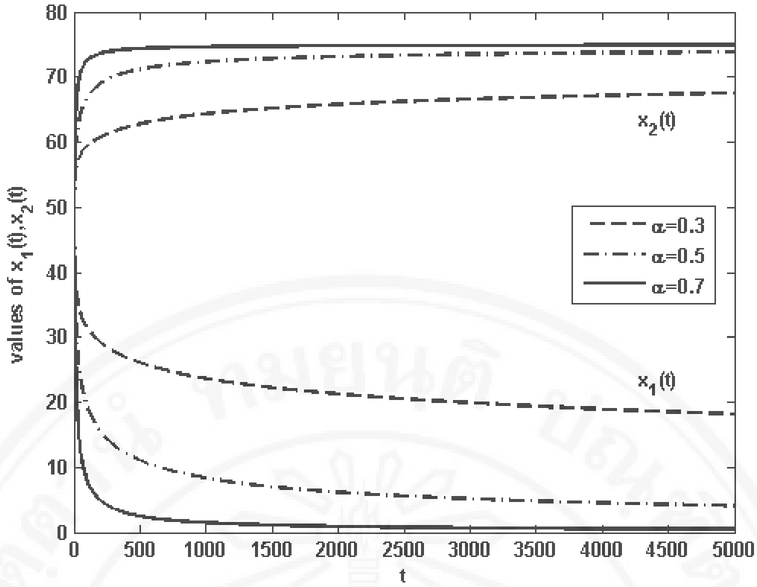


(5.5a)

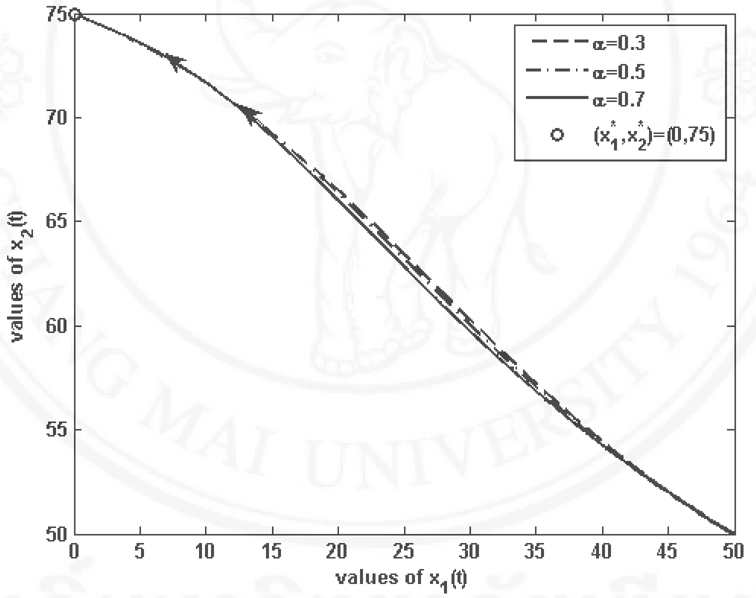


(5.5b)

Figure 5.5: (5.5a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.5b) the dynamic behavior of two interacting species with $\alpha = 0.5$ for $(x_1(0), x_2(0)) = (10, 100), (50, 50)$, and $(100, 10)$.



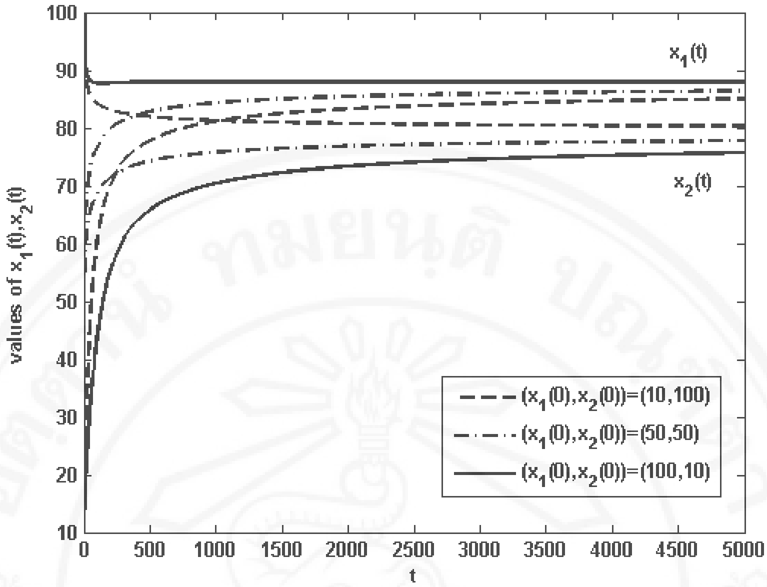
(5.6a)



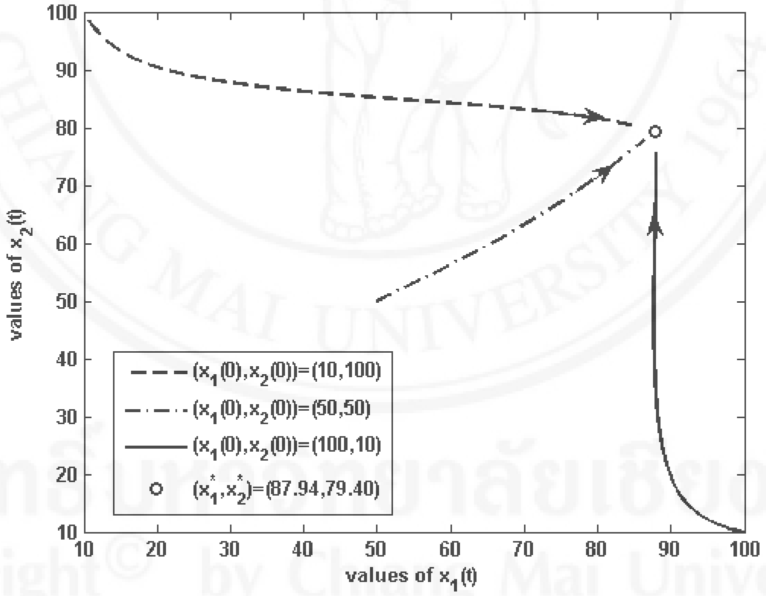
(5.6b)

Figure 5.6: (5.6a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.6b) the dynamic behavior of two interacting species with initial condition $(x_1(0), x_2(0)) = (50, 50)$ for $\alpha = 0.3, 0.5$ and 0.7.

If we assume the model parameters in Case 3 as shown in Table 5.1, which satisfies the sufficient conditions according to Theorem 4.1.4. Then all solutions of the system (3.0.7) converge to the equilibrium point $E_{12}(0, 75)$ as shown in Figures 5.5 and 5.6. Consequently, $E_{12}(0, 75)$ is locally asymptotically stable.

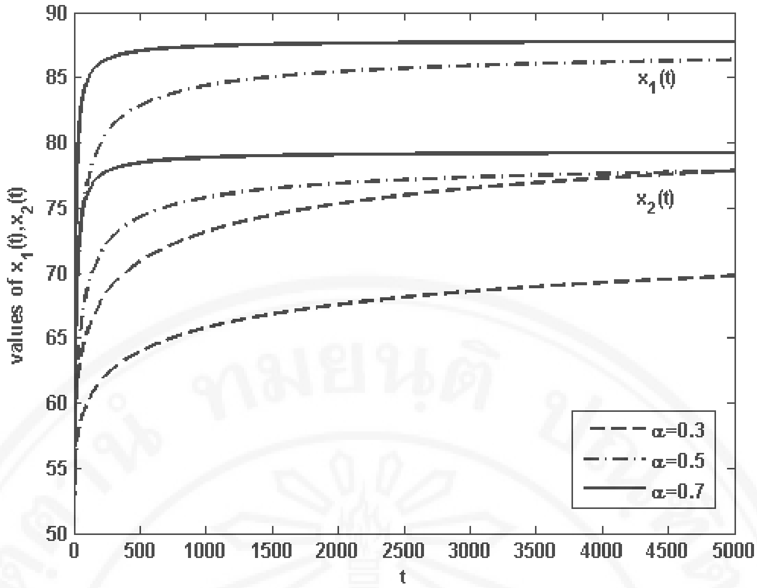


(5.7a)

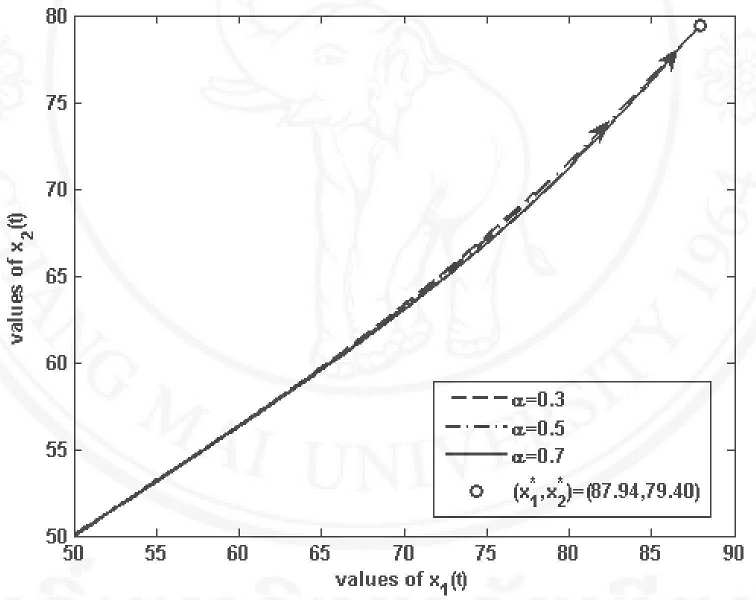


(5.7b)

Figure 5.7: (5.7a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.7b) the dynamic behavior of two interacting species with $\alpha = 0.5$ for $(x_1(0), x_2(0)) = (10, 100), (50, 50),$ and $(100, 10)$.



(5.8a)



(5.8b)

Figure 5.8: (5.8a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.8b) the dynamic behavior of two interacting species with initial condition $(x_1(0), x_2(0)) = (50, 50)$ for $\alpha = 0.3, 0.5$ and 0.7.

The values of parameters in Case 4 in Table 5.1 satisfy all the conditions of Theorem 4.1.5. We can see that all solutions of the system (3.0.7) converge to the equilibrium point $E_{13}(87.9397, 79.3970)$ as shown in Figures 5.7 and 5.8. Therefore, we can conclude that $E_{13}(87.9397, 79.3970)$ is globally uniformly asymptotically stable.

In the same way, the numerical examples on the fractional order two-species facultative mutualism system (3.0.8) to show the stability of all equilibrium points for the Model 2 are carried out. We distinguish the values of all parameters into four cases as presented in Table 5.2. These are employed to obtain numerical simulations on illustration of the approximate solutions which show in Figures 5.9- 5.16.

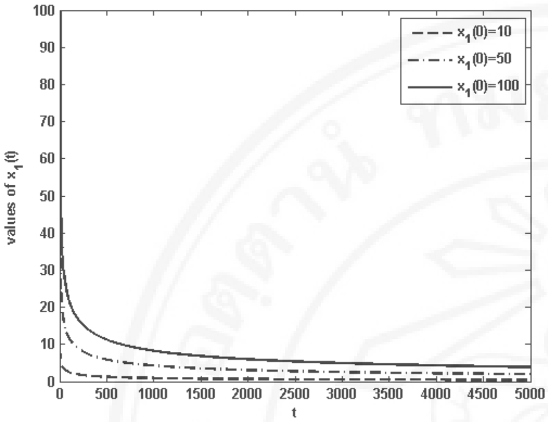
Table 5.2: The parameters of numerical examples for the Model 2

Cases	Parameters							
	r_1	r_2	e_1	e_2	K_1	K_2	b_{12}	b_{21}
5	0.40	0.30	0.60	0.40	250	250	0.20	0.25
6	0.60	0.30	0.40	0.40	250	250	0.20	0.25
7	0.40	0.40	0.60	0.30	250	250	0.20	0.25
8	0.60	0.40	0.40	0.30	250	250	0.20	0.25

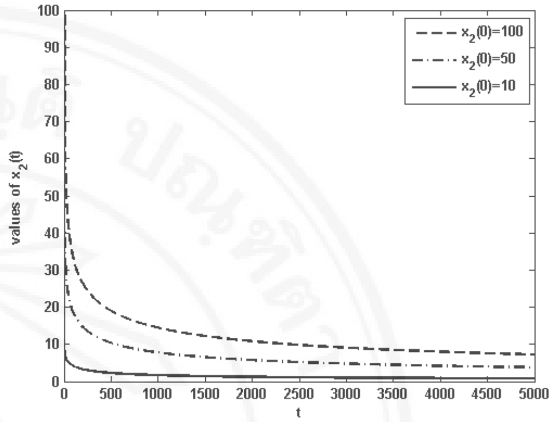
The numerical examples of the system (3.0.8) are shown in Figures 5.9 - 5.16. In Figures 5.9, 5.11, 5.13, and 5.15, we fixed that $\alpha = 0.5$ and three different initial conditions are chosen as follows: $(x_1(0), x_2(0)) = (10, 100), (50, 50), (100, 10)$. In Figures 5.10, 5.12, 5.14, and 5.16, the initial condition $(x_1(0), x_2(0)) = (50, 50)$ is fixed, and various values of $\alpha = 0.3, 0.5$, and 0.7 are chosen.

Figures (5.9a), (5.9b), (5.10a), and (5.10b) show the relationship between the values of the species x_1 with respect to times t and x_2 with respect to times t, respectively. Figures (5.9c) and (5.10c) present descriptions of dynamic behavior of two interacting species.

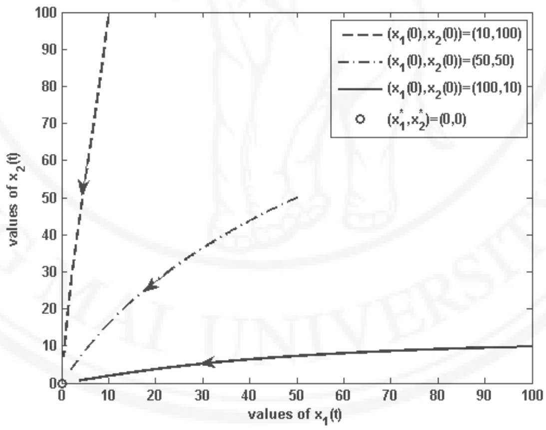
We obtain the relationship between the values of the species x_1, x_2 with respect to times t in Figures 5.11 - 5.16 (a). The relationship between the values of the species x_1 and species x_2 are shown in Figures 5.11 - 5.16 (b).



(5.9a)

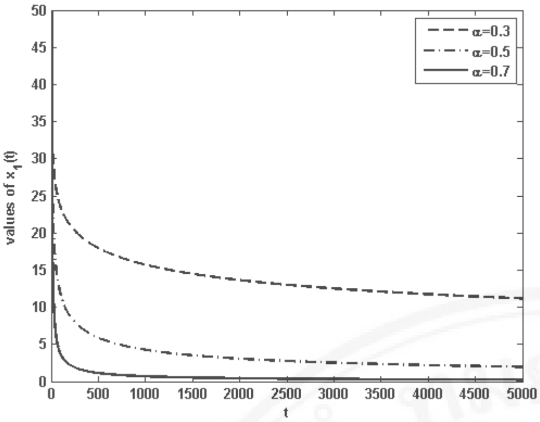


(5.9b)

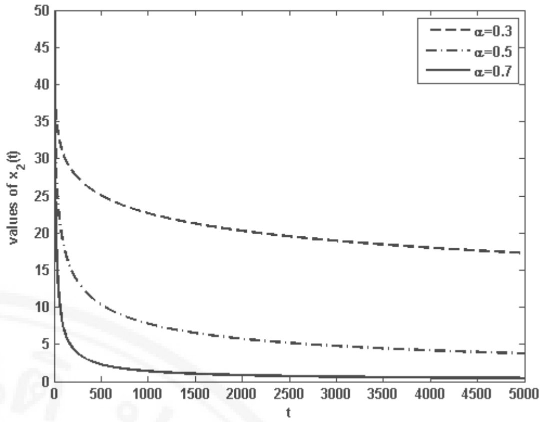


(5.9c)

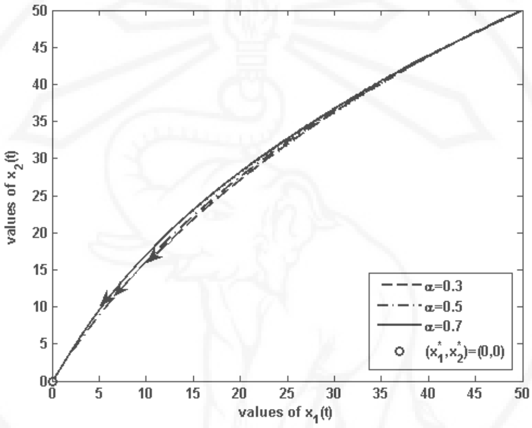
Figure 5.9: (5.9a), (5.9b) The relationship between the numbers of individuals of species x_1 with respect to time t, and the relationship between the values of x_2 with respect to time t, respectively. (5.9c) the dynamic behavior of two interacting species with $\alpha = 0.5$ for $(x_1(0), x_2(0)) = (10, 100), (50, 50)$, and $(100, 10)$.



(5.10a)



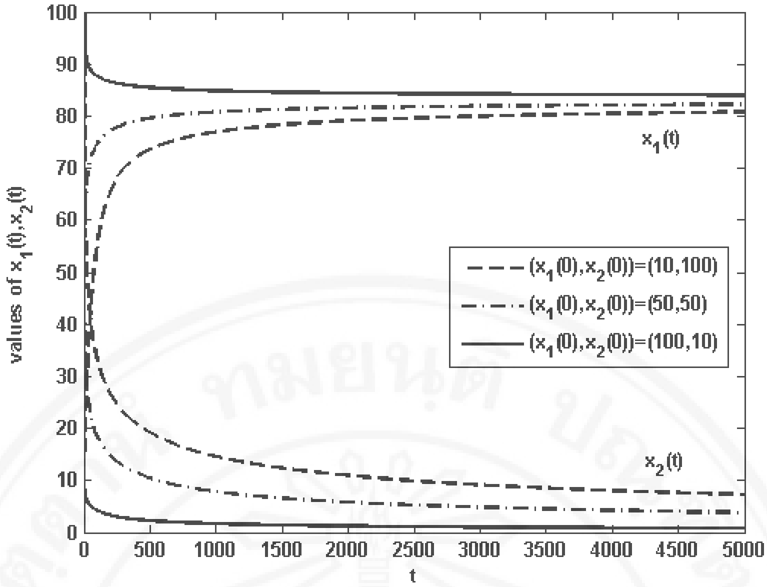
(5.10b)



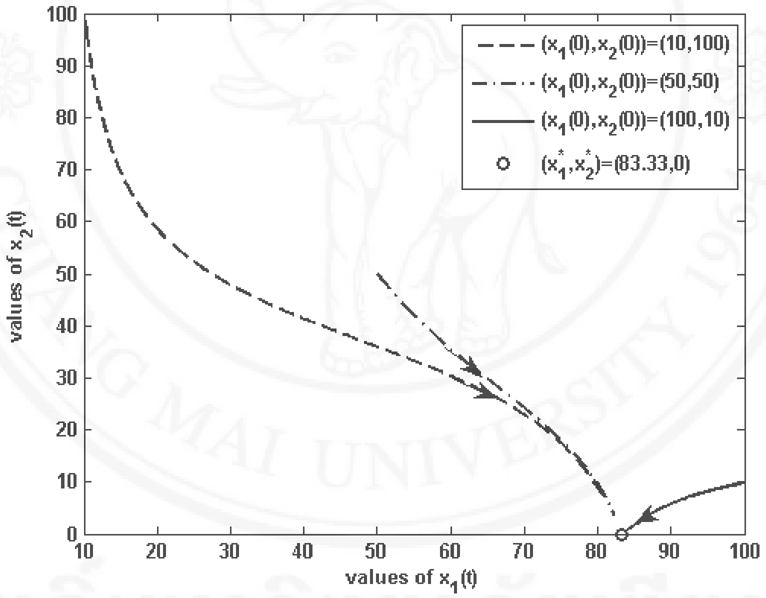
(5.10c)

Figure 5.10: (5.10a), (5.10b) The relationship between the numbers of individuals of species x_1 with respect to time t, and the relationship between the values of x_2 with respect to time t, respectively. (5.10c) the dynamic behavior of two interacting species with initial condition $(x_1(0), x_2(0)) = (50, 50)$ for $\alpha = 0.3, 0.5$ and 0.7.

The model parameters in Case 5 as shown in Table 5.2 satisfy the sufficient conditions of Theorem 4.2.2. From Figures 5.9 and 5.10, we obtain that all solutions of the system (3.0.8) converge to the equilibrium point $E_{20}(0,0)$. Therefore, $E_{20}(0,0)$ is locally asymptotically stable.

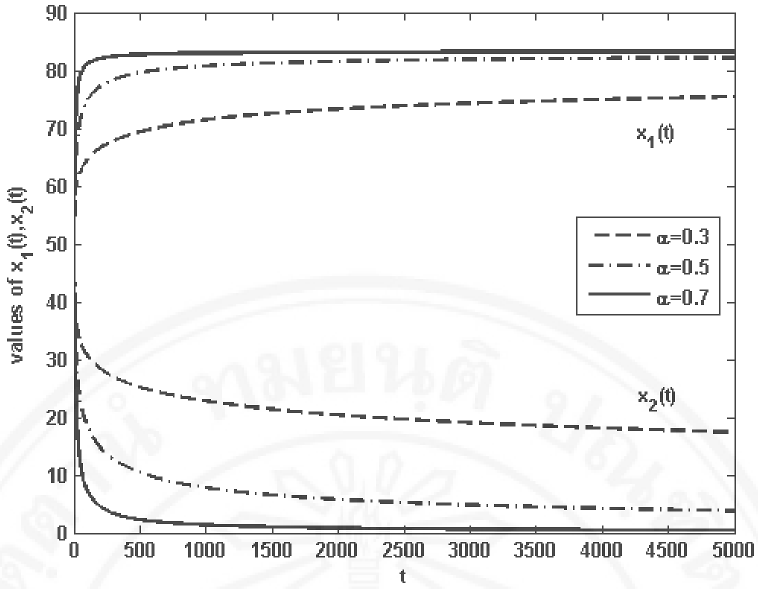


(5.11a)

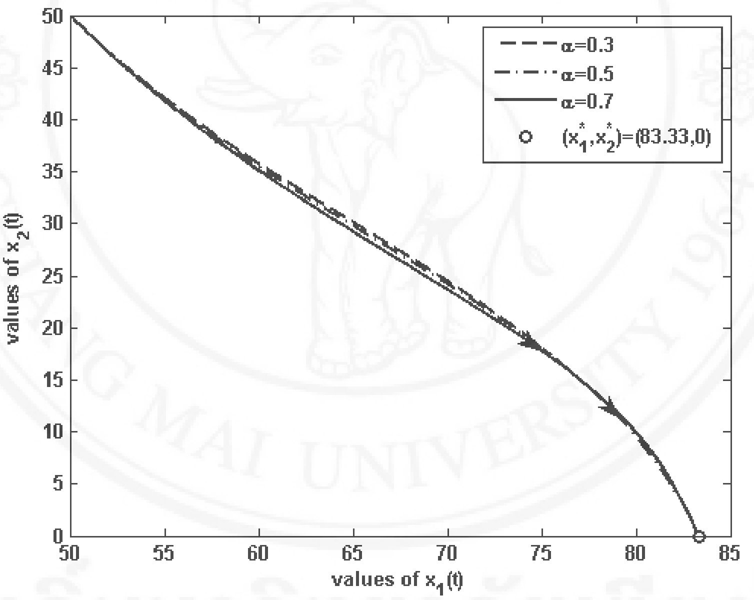


(5.11b)

Figure 5.11: (5.11a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.11b) the dynamic behavior of two interacting species with $\alpha = 0.5$ for $(x_1(0), x_2(0)) = (10, 100), (50, 50),$ and $(100, 10)$.



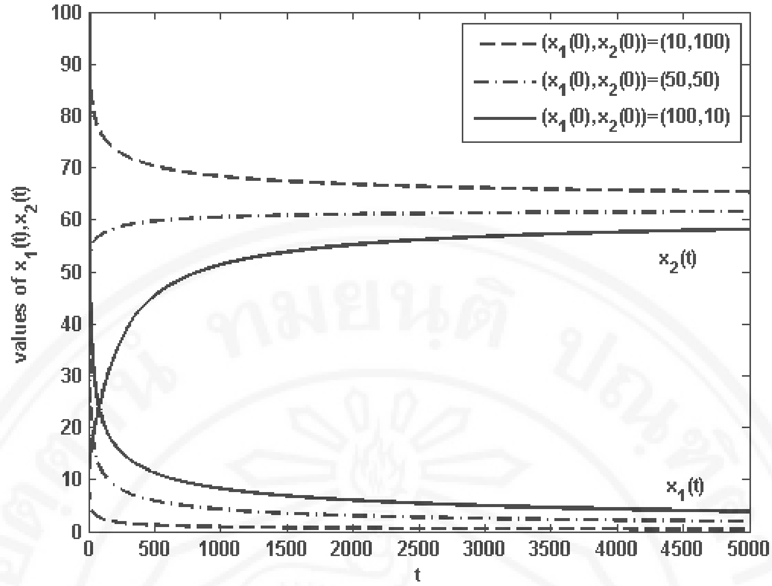
(5.12a)



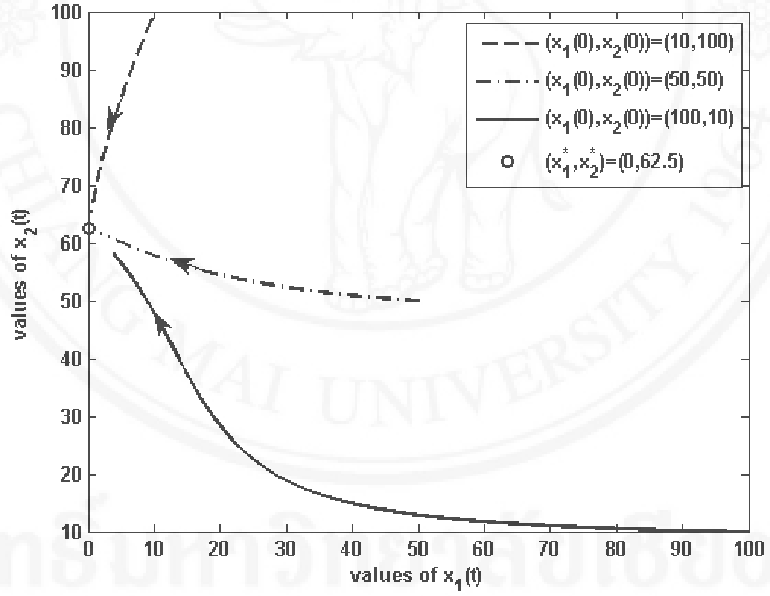
(5.12b)

Figure 5.12: (5.12a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.12b) the dynamic behavior of two interacting species with initial condition $(x_1(0), x_2(0)) = (50, 50)$ for $\alpha = 0.3, 0.5$ and 0.7.

The values of parameters in Case 6 as shown in Table 5.2 satisfy all the conditions of Theorem 4.2.3. From Figures 5.11 and 5.12, we can obtain that all solutions of the system (3.0.8) converge to the equilibrium point $E_{21}(83.3333, 0)$. Hence, $E_{21}(83.3333, 0)$ is locally asymptotically stable.

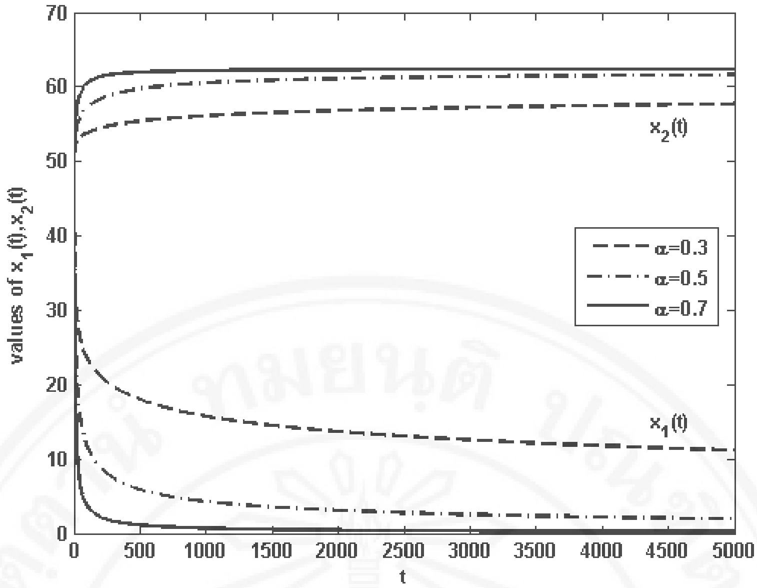


(5.13a)

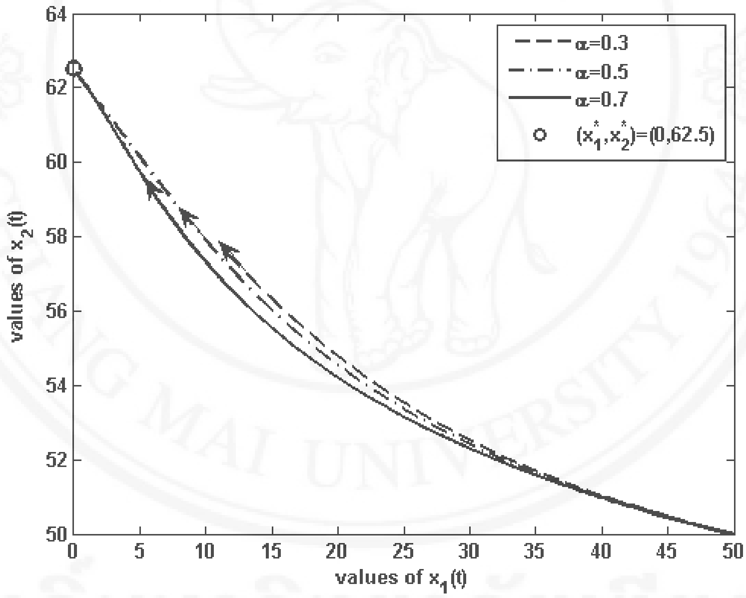


(5.13b)

Figure 5.13: (5.13a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.13b) the dynamic behavior of two interacting species with $\alpha = 0.5$ for $(x_1(0), x_2(0)) = (10, 100), (50, 50), \text{and } (100, 10)$.



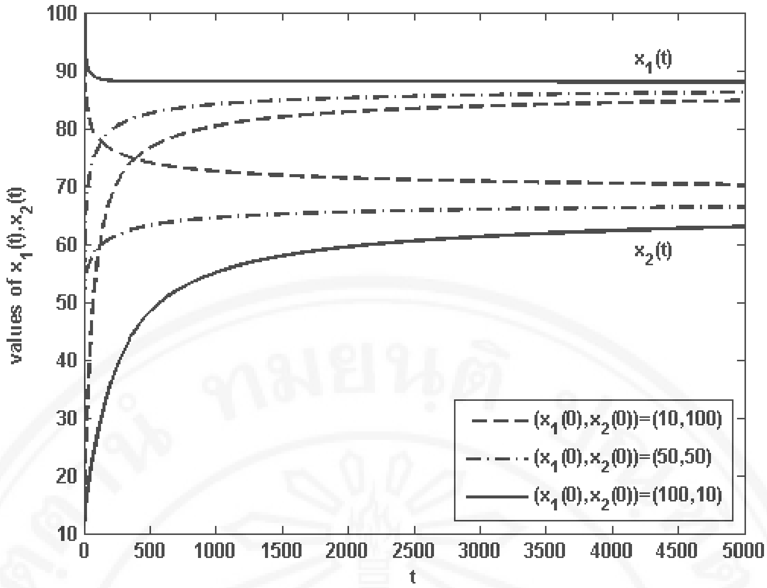
(5.14a)



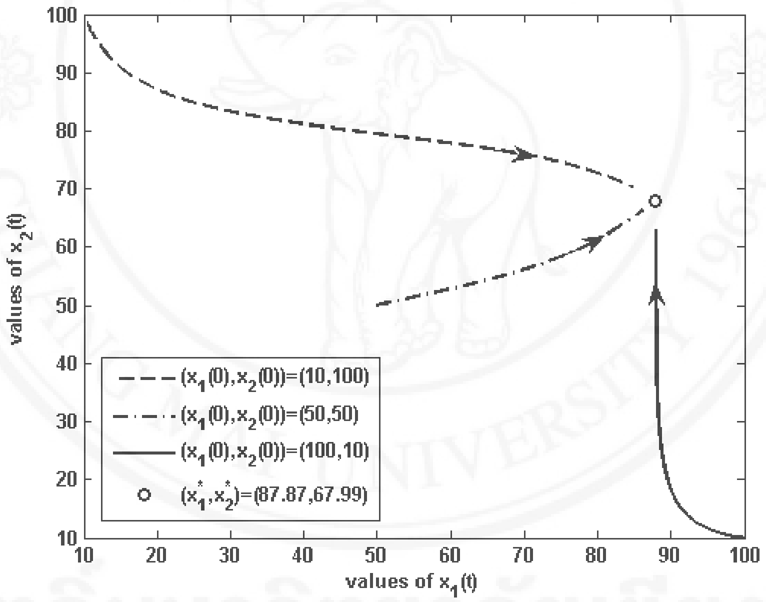
(5.14b)

Figure 5.14: (5.14a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.14b) the dynamic behavior of two interacting species with initial condition $(x_1(0), x_2(0)) = (50, 50)$ for $\alpha = 0.3, 0.5$ and 0.7.

If we assume the model parameters in Case 7 as shown in Table 5.2, which satisfies all conditions according to Theorem 4.2.4. Then all solutions of the system (3.0.8) converge to the equilibrium point $E_{22}(0, 62.50)$ as shown in Figures 5.13 and 5.14. Therefore, $E_{22}(0, 62.50)$ is locally asymptotically stable.

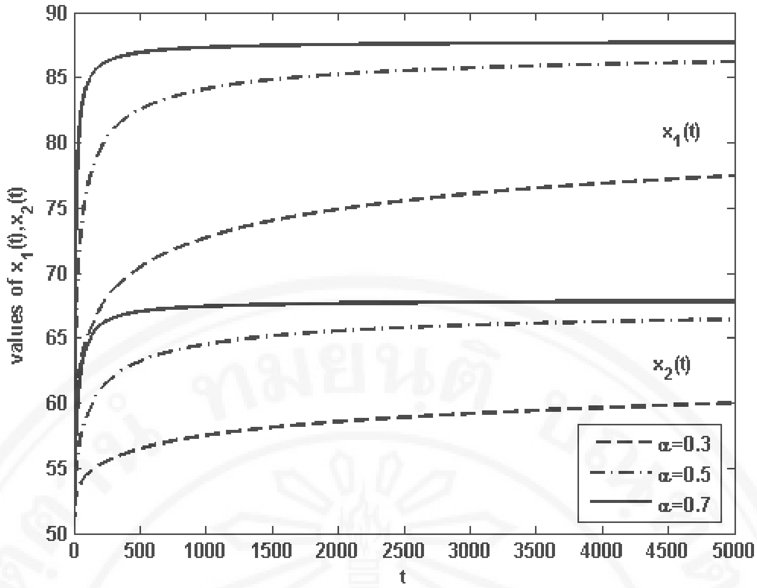


(5.15a)

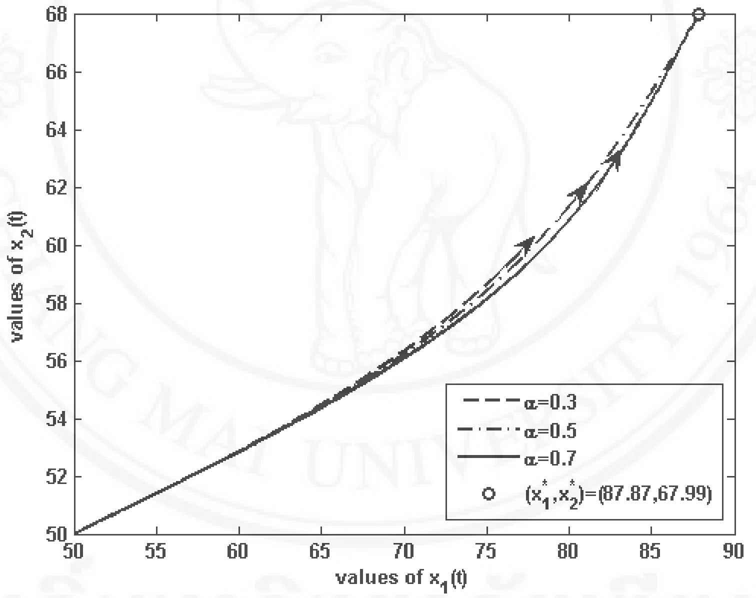


(5.15b)

Figure 5.15: (5.15a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.15b) the dynamic behavior of two interacting species with $\alpha = 0.5$ for $(x_1(0), x_2(0)) = (10, 100), (50, 50),$ and $(100, 10)$.



(5.16a)



(5.16b)

Figure 5.16: (5.16a) the relationship between the numbers of individuals of species x_1, x_2 with respect to time t, and (5.16b) the dynamic behavior of two interacting species with initial condition $(x_1(0), x_2(0)) = (50, 50)$ for $\alpha = 0.3, 0.5$ and 0.7.

From Figures 5.15 and 5.16, we can see that all solutions of the system (3.0.8) converge to the equilibrium point $E_{23}(87.8661, 67.9916)$. The model parameters in Case 8 in Table 5.2 satisfy the sufficient conditions of Theorem 4.2.5. Consequently, the positive equilibriumm point $E_{23}(87.8661, 67.9916)$ is globally uniformly asymptotically stable.