APPENDIX A

Stochastic Process

A.1 Independent of two standard Wiener processes

Let B_t and W_t be two standard Wiener processes with the correlation coefficient ρ. By setting $Z_t = (W_t - \rho B_t)/\sqrt{1-\rho^2}$, we get Z_t is independent of B_t.
Consider

$$\begin{aligned} \text{cov}(B_t, W_t) &= \text{cov}(B_t, \rho B_t + \sqrt{1-\rho^2} Z_t) \\ &= \text{cov}(B_t, \rho B_t) + \text{cov}(B_t, \sqrt{1-\rho^2} Z_t) \\ &= \rho \text{cov}(B_t, B_t) + \sqrt{1-\rho^2} \text{cov}(B_t, Z_t) \\ &= \rho \text{var}(B_t) + \sqrt{1-\rho^2} \text{cov}(B_t, Z_t) \\ &= \rho t + \sqrt{1-\rho^2} \text{cov}(B_t, Z_t), \end{aligned}$$

Then, we obtain that

$$\begin{aligned} \text{corr}(B_t, W_t) &= \frac{\text{cov}(B_t, W_t)}{\sqrt{\text{var}(B_t)}\sqrt{\text{var}(W_t)}} \\ &= \frac{\rho t + \sqrt{1-\rho^2} \text{cov}(B_t, Z_t)}{t}. \end{aligned}$$

Hence, we get

$$\text{cov}(B_t, Z_t) = \frac{t \cdot \text{corr}(B_t, W_t) - \rho t}{\sqrt{1-\rho^2}}.$$

Since $\text{corr}(B_t, W_t) = \rho$, we have $\text{cov}(B_t, Z_t) = 0$.
Therefore, the proof is now completed.

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