CHAPTER 4

Conclusion

Stochastic volatility model is a mathematical finance model that describes behaviours of stock price process where the volatility of price process is assumed to be another stochastic process. Unlike the stock price process, volatility process is a hidden process which normally cannot be observed directly from the market. Hull and White model [16], Ornstein-Uhlenbeck model [24] and Heston model [13] are examples of stochastic volatility model.

In this thesis, we focus on 3/2 volatility model, which can be described by the stochastic differential form as

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dB_t,$$
$$dv_t = \kappa v_t(\theta - v_t)dt + \xi v_t^{\frac{3}{2}} dW_t.$$

where S_t is the stock-price process, v_t is the volatility process, B_t and W_t are two dependent Brownian motions with correlation ρ. The main aims of this thesis are to estimate volatility process and to estimate parameters of the model where only the price process is assumed to be known. We develop the SIS particle filter method to estimate both volatility process and parameters of the model. The idea is to transform system to the log-price process in Lemma 3.1.1. Then, the stochastic system is discretized as described in Lemma 3.1.2. Finally, the posterior distribution is constructed in Proposition 3.1.3. The particle filter algorithm for the estimation is explained afterwards.

The simulation study, see Figure 3.2 - 3.7, shows that; (i) the quality of volatility estimation depends on the choice of parameter distributions, (ii) the quality of parameters estimation is not good. Hence, the proposed method is reliable for volatility estimation only. To improve the quality of parameters estimation, we use the maximum likelihood estimation (MLE) technique where the estimated volatility process from the particle filter method is used as observed data. In other words, the volatility process is "filtered" from the stock-price process by SIS particle filter. The likelihood function is described in Theorem 3.2.4 and the parameters estimation results by MLE are shown in Table 3.1 - 3.4. We can see that the quality of parameters estimation improves significantly.