CHAPTER 1

Introduction

Magic squares have been known and interested by mathematicians for a long time. A magic square of order n is a square with n rows and n columns filled with integers such that the sum of these integers in every row, in every column and in each of the two principal main diagonals is the same.

A normal magic square of order 3 has exactly one but, it can be rotated and reflected to produce 8 trivially distinct squares.

In 1675, Bernard Frenicle de Bessey was the first who found that there are exactly 880 normal magic squares of order 4 and it can be generated to $7,040$ different magic squares [6].

In 1973, Richard Schroeppel was the first to compute the number of magic squares of order 5. He found that there are exactly $68,826,306$ squares which can be generated to $275,305,224$ of 5×5 magic squares. However, for the $6{\times}6$ case, it has not been known the exactly number yet but there are estimated to be approximately $1.7745 \pm 0.0016 \times 10^{19}$ squares [10].

In 2015, José M. Pacheco and Isabel Fernández had a trip to Barcelona. They observed a non-normal magic square of order 4 in the Gaudí's Sagrada Famìlia Temple in Barcelona. It is a large artwork over 1m×1m, located on the wall with its magic constant is 33 and it features on rows, columnns, diagonals and 2/2 broken diagonals (not all broken diagonals), and on any 2×2 subsquares [8].

1	14	14	4
11	7	6	9
8	10	10	5
13	2	3	15

Their study emphasised the role of more or less hidden symmetries in preserving the magic constant.

A magic square is said to be a pandiagonal magic square (sometimes diabolic or Nasik) when the magic square has additional properties that its all broken diagonals also add up to the magic constant [6]. For example, the magic square of order 4 below was found in Khajurado, India in 1904.

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

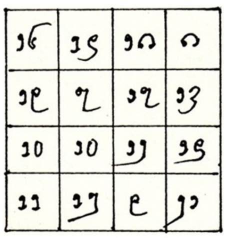
It is a pandiagonal magic square because not only the numbers in rows, columns and principal diagonals add to magic constant 34, but also the numbers in all broken diagonals add to 34, such as 7 6 10 11, 2 12 15 5, 16 13 1 4, 14 2 3 15, 1 11 16 6, 12 8 5 9.

In 1937, Barkley Rosser and R. J. Walker studied on normal 4×4 magic squares. They found that there are 384 pandiagonal magic squares by using Abstract Algebra [9].

Magic squares can also be found in Chiangmai. Magic squares existed in Lanna Yantra, a kind of talismans, from Lanna kingdom which was a kingdom in the 13th to 18th centuries located in northern Thailand and centered in present-day Chiang Mai. Lanna had their own culture, language and letters [7].

Lanna Yantra is talisman of Lanna people. It was recorded by using Lanna letters or Lanna numbers in fabric or thin silver plate or copperplate. There are a lot of Lanna Yantra with different supernatural. Lanna people keep Yantra at home or bring it with themselves.

In 2011, Atichart Kettapun and his research team [5] interested in one of Lanna Yantra which Lanna People believed that its supernatural power is to give safe journeys to owners [1]. They translated Lanna numbers in the Yantra into Arabic numbers as shown below



16	14	18	8
19	7	17	13
10	10	12	14
11	15	9	21

Figure 1.1: Lanna Yantra writen in Lanna numbers and Arabic numbers

16	14	18	8
19	7	17	13
10	10	12	14
11	15	9	21

However, the team agreed that the number in the 3rd row and the 2nd column should be 20. It was from several reasons :

1. It was copied from generation to generation by hand writing so, it can have a mistake.

2. The number 1 and 2 of Lanna language look alike so, it maybe wrong copied from original.

The table below shows the Arabic numbers and Lanna numbers which the number 2 in Lanna language is different from 1 only the length of it.

Arabic numbers	**Lanna Numbers**	**Arabic numbers**	**Lanna Numbers**
1	᪁	6	᪆
2	᪂	7	᪇
3	᪃	8	᪈
4	᪄	9	᪉
5	᪅	0	᪀

Figure 1.2: Lanna numbers and Arabic numbers

3. Because of many of Lanna Yantra are magic squares and if the number in that position is 20, it makes the square be not only a magic square of order 4 but, also a pandiagonal magic square with its magic constant 56. So, the number 10 is possible to be 20.

Thus, the team changed the number 10 in that position to 20 and got the square

16	14	18	8
19	7	17	13
10	20	12	14
11	15	9	21

The square is a magic square of order 4 with magic constant 56 so, the team named it Buddha Khunnung 56 Yantra which means the number of syllables in one of Buddhism prayer [5].

In this thesis, we focus on finding the number of all pandiagonal magic squares of order 4 from Buddha Khunnung 56 Yantra by using a concept of Abstract Algebra, a group action.

However, we reduce the number 7 to 21 in Buddha Khunnung 56 Yantra which has 14 two times and its magic constant is 56 to the number 1 to 15 by substract 6 to every number then it has 8 two times and its magic constant is 32 instead. We reduce the number for making it similar to normal magic square and easier to study. After reducing we get the square

10	8	12	2
13	1	11	7
4	14	6	8
5	9	2	15

We call the square after reducing the number from Buddha Khunnung 56 Yantra to number 1 to 15, **a Lanna Magic Square**. Clearly, a Lanna Magic Square is pandiagonal magic square of order 4 with magic constant 32.

Moreover, we call pandiagonal magic squares of order 4 created by numbers 1 to 15 with repeated number 8 two times, **pandiagonal Lanna Magic Squares** (it means each row, each column, 2 main diagonals and 6 broken diagonals sum to 32).

For example,

2	7	13	10
12	11	1	8
3	6	14	9
15	8	4	5

The purpose of this research is to

1. study the Lanna Magic Square.
2. study a concept of abstract algebra, group action on subgroup of S_{16} to find the total number of pandiagonal Lanna Magic Squares.
3. generate all Lanna Magic Squares using mathematical program (Scilab).

In chapter II, we give some definitions, notations and some known results that will be used in later chapters.

In chapter III, we use a concept of abstract algebra, group action on subgroup of S_{16} to find the number of all pandiagonal Lanna Magic Squares from non-normal magic of order 4 contains with numbers 1 to 15 with repeated 8 twice called the Lanna Magic Square.

In chapter IV, we give Algorithms we used to generated Lanna Magic Squares, pandiagonal Lanna Magic Squares and semi-pandiagonal Lanna Magic Squares.

The conclusion of this research is in chapter V.