

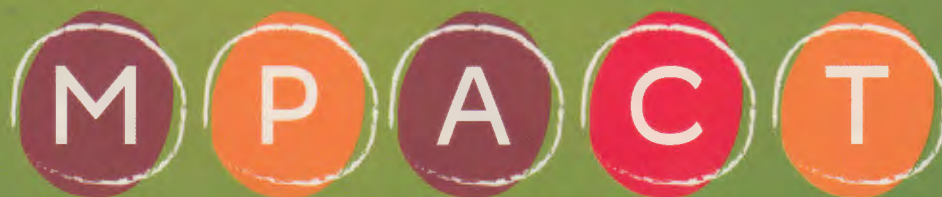
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VISIBLE LEARNING

FOR **MATHEMATICS**

What Works
Best to Optimize
Student Learning

GRADES K-12



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Foreword by Diane J. Briars

CM CORWIN
MATHEMATICS

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